Current Interactions in HS Theory and Locality

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Higher Spin Theories and Holography 6

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Higher Derivatives in HS Interactions

HS interactions contain higher derivatives

Nonanaliticity in Λ via dimensionless combination $\Lambda^{-\frac{1}{2}}\frac{\partial}{\partial x}$

By a seemingly local field redefinition (Prokushkin, MV 1998) it is possible to get rid of currents from HS field equations: the field transformation is nonlocal having the form

$$\phi \to \phi' = \phi + \sum_n a_{nm}(\rho D)^n \phi \, (\rho D)^m \phi + \dots \, ,$$

ρ is the AdS radius, D is the space-time covariant derivative.

Naive problem setting: find restrictions on a_{nm} distinguishing between truly non-local and generalized local field redefinitions containing an infinite number of terms but a_{nm} decrease fast enough with n and m.

The problems in AdS_d and Minkowski space are essentially different

The most appropriate setup is in the twistor (spinor) space

Plan

”Never solve the problem before knowing the answer”

Attributed to Wigner

0. Answer: 1605.02662

1. Analysis: today

Central On-shell Theorem

Infinite set of integer spins

$\omega(y,\bar{y}\mid x)\,,\quad C(y,\bar{y}\mid x)\quad f(y,\bar{y})=\sum_{n,m=0}^{\infty}\frac{1}{n!m!}f_{\alpha_1\ldots\alpha_n,\dot{\alpha}_1\ldots\dot{\alpha}_m}y^{\alpha_1}\ldots y^{\alpha_n}\bar{y}^{\dot{\alpha}_1}\ldots\bar{y}^{\dot{\alpha}_m}$

The full unfolded system for free bosonic fields is 1989

$$\star\qquad R_1(y,\bar{y}\mid x)=\frac{i}{4}\Big(\eta\overline{H}^{\dot{\alpha}\dot{\beta}}\frac{\partial^2}{\partial\bar{y}^{\dot{\alpha}}\partial\bar{y}^{\dot{\beta}}}\,C(0,\bar{y}\mid x)+\bar{\eta}H^{\alpha\beta}\frac{\partial^2}{\partial y^{\alpha}\partial y^{\beta}}\,C(y,0\mid x)\Big)$$

$$\star\star\qquad \tilde{D}_0 C(y,\bar{y}\mid x)=0$$

Vacuum: $sp(4)\sim o(3,2)$

$$\mathbf{R}_{\alpha\beta}:=\mathbf{d}\omega_{\alpha\beta}+\omega_{\alpha\gamma}\omega_{\beta}{}^{\gamma}-\mathbf{H}_{\alpha\beta}=\mathbf{0}\,,\qquad \mathbf{R}_{\alpha\dot{\beta}}:=\mathbf{d}+\omega_{\alpha\gamma}\mathbf{h}^{\gamma}{}_{\dot{\beta}}+\overline{\omega}_{\dot{\beta}\dot{\delta}}\mathbf{h}_{\alpha}{}^{\dot{\delta}}=\mathbf{0}$$

$$\mathbf{H}^{\alpha\beta}:=\mathbf{h}^{\alpha}{}_{\dot{\alpha}}\wedge\mathbf{h}^{\beta\dot{\alpha}}\,,\qquad \overline{\mathbf{H}}^{\dot{\alpha}\dot{\beta}}:=\mathbf{h}_{\alpha}{}^{\dot{\alpha}}\wedge\mathbf{h}^{\alpha\dot{\beta}}$$

$$R_1(y,\bar{y}\mid x)=D_0^{ad}\omega(y,\bar{y}\mid x)\qquad D_0^{ad}\omega=D^L-h^{\alpha\dot{\beta}}\Big(y_\alpha\frac{\partial}{\partial\bar{y}^{\dot{\beta}}}+\frac{\partial}{\partial y^{\alpha}}\bar{y}_{\dot{\beta}}\Big)$$

$$\tilde{D}_0=D^L+h^{\alpha\dot{\beta}}\Big(y_\alpha\bar{y}_{\dot{\beta}}+\frac{\partial^2}{\partial\mathbf{y}^{\alpha}\partial\bar{\mathbf{y}}^{\dot{\beta}}}\Big)\qquad D^L A=\mathsf{d}_x-\Big(\omega^{\alpha\beta}y_\alpha\frac{\partial}{\partial y^{\beta}}+\bar{\omega}^{\dot{\alpha}\dot{\beta}}\bar{y}_{\dot{\alpha}}\frac{\partial}{\partial\bar{y}^{\dot{\beta}}}\Big)$$

$\star\star$ **implies that higher order terms in y and $\bar{y}$ describe higher-derivative descendants of the primary HS fields**

Fields of the Nonlinear System

Closed formulation of nonlinear equations demands the doubling of spinors and Klein operator

$$\omega(Y|x) \longrightarrow W(Z;Y|x)\,, \qquad C(Y,k|x) \longrightarrow B(Z;Y;k|x)$$

Some of the nonlinear HS equations determine the dependence on the additional variables Z_A in terms of "initial data"

$\omega(Y|x) := W(0;Y|x)$

$C(Y;k|x) := B(0;Y;k|x)$

$S(Z,Y|x) = dZ^A S_A(Z,Y|x)$ is a connection along Z^A

Klein operator k generates chirality automorphisms

$$kf(A) = f(\tilde{A})k\,, \quad A = (a_\alpha\,, \bar{a}_{\dot\alpha}) : \quad \tilde{A} = A = (-a_\alpha\,, \bar{a}_{\dot\alpha})$$

$$P(Y) = P^{\alpha\dot\alpha} y_\alpha \bar{y}_{\dot\alpha} \quad \longrightarrow \quad \tilde{P}(Y) = -P(Y)\,, \qquad \tilde{M}(Y) = M(Y)$$

Nonlinear HS Equations

HS star product

$$(f\star g)(Z,Y)=\int dSdT\,\exp iS_AT^A\,f(Z+S,Y+S)g(Z-T,Y+T)$$

$[Y_A,Y_B]_\star=-[Z_A,Z_B]_\star=2iC_{AB}\,,$ $\qquad Z-Y\,:\,Z+Y$ **normal ordering**

Inner Klein operators:

$$\kappa=\exp iz_\alpha y^\alpha\,,\qquad \bar\kappa=\exp iz_{\dot\alpha}y^{\dot\alpha}\,,\qquad \kappa\star f=\tilde f\star\kappa\,,\qquad \kappa\star\kappa=1$$

$$\begin{cases} \mathrm{d}W+W\star W=0\\ \mathrm{d}B+W\star B-B\star W=0\\ \mathrm{d}S+W\star S+S\star W=0\\ \mathbf{S\star B-B\star S=0}\\ \mathbf{S\star S=i(dZ^AdZ_A+\eta dz^\alpha dz_\alpha B\star k\star\kappa+\bar\eta d\bar z^{\dot\alpha}d\bar z_{\dot\alpha}B\star k\star\bar\kappa)}\end{cases}$$

1992

Real physics is localized in the x-independent twistor sector

Self-dual HS equations where conjectured to be associated with $\eta=0$

Perturbative Analysis

Vacuum solution

$$B_0 = 0\,, \qquad S_0 = dZ^A Z_A\,, \qquad W_0 = \frac{1}{2}\omega_0^{AB}(x) Y_A Y_B$$

$$\mathrm{d}W_0 + W_0 \star W_0 = 0$$

$\omega_0^{AB}(x)$: **describes** AdS_4.

First-order fluctuations

$$B_1 = C(Y)\,, \qquad S = S_0 + S_1\,, \qquad W = W_0(Y) + W_1(Y) + W_0(Y)C(Y)$$

$$[S_0\,, f]_\star = -2i\mathrm{d}_Z f\,, \qquad \mathrm{d}_Z = dZ^A \frac{\partial}{\partial Z^A}$$

Reconstruction of Z^A Variables

Perturbatively, equations containing S have the form

$$\mathsf{d}_Z U_n(Z;Y|dZ) = V[U_{<n}](Z;Y|dZ) \qquad \mathsf{d}_Z V[U_{<n}](Z;Y|dZ) = 0$$

can be solved as

$$U_n(Z;Y|dZ) = \mathsf{d}_Z^* V[U_{<n}](Z;Y|dZ) + h(Y) + \mathsf{d}_Z \epsilon(Z;Y|dZ)$$

For instance

$$\mathsf{d}_Z^* V(Z;Y|dZ) = Z^A \frac{\partial}{dZ^A} \int_0^1 \frac{dt}{t} V(tZ;Y|tdZ)$$

Alternative forms of d_Z^* that differ by d_Z-closed forms can also be used.

This is most important in the context of locality!

Nontrivial space-time equations on $\omega(Y|x)$ and $C(Y|x)$ are in the sector of d_Z-cohomology

Central On-Shell Theorem is reproduced in the lowest order

Conserved Currents and Current Deformation

Gauge invariant conserved currents $J(Y_1,Y_2|x)$ are represented by the bilinears of $C(Y|x)$

$$J(Y_1,Y_2|x) := C(Y_1|x)\tilde{C}(Y_2|x)\,, \qquad \tilde{C}(y,\bar{y}|x) = C(-y,\bar{y}|x)$$

As a consequence of the rank-one equation for $C(Y|x)$,
$J(Y_1,Y_2|x)$ obeys the current equation   Gelfond, MV (2003)

$$\tilde{D}_2 J(Y_1,Y_2|x) = 0\,, \qquad \tilde{D}_2 := D^L - i\lambda h^{\alpha\dot{\beta}}\Big(y_{1\alpha}\bar{y}_{1\dot{\beta}} - y_{2\alpha}\bar{y}_{2\dot{\beta}} - \frac{\partial^2}{\partial y_1^{\alpha}\partial \bar{y}_1{}^{\dot{\beta}}} + \frac{\partial^2}{\partial y_2^{\alpha}\partial \bar{y}_2{}^{\dot{\beta}}}\Big)$$

Current deformation has a form of a linear system

$$D\omega - L(w,C) + \Gamma(w,J) = 0\,,$$

$$\tilde{D}C + \mathcal{H}(w,J) = 0\,, \qquad \tilde{D}_2 J(Y_1,Y_2|x) = 0$$

$$L(w,C) := \frac{i}{4}\Big(\eta\overline{H}^{\dot{\alpha}\dot{\beta}}\frac{\partial^2}{\partial\bar{y}^{\dot{\alpha}}\partial\bar{y}^{\dot{\beta}}}\,\bar{C}(0,\bar{y}|x) + \bar{\eta}H^{\alpha\beta}\frac{\partial^2}{\partial y^{\alpha}\partial y^{\beta}}\,C(y,0|x)\Big)$$

Linear functionals Γ and $\mathcal{H}$ obey the compatibility conditions

Locality and Nonlocality of in HS Gauge Theory

Having infinitely many HS fields with higher derivatives in interactions, the HS Gauge Theory is not local

$\lambda^{-1}D \sim 1$ since $[\lambda^{-1}D\,,\lambda^{-1}D] \sim 1$

A different mass parameter like α' is needed for a low-energy expansion

In HS equations, nonlocality is due to infinite tails of contractions

$$\int \frac{d^4S d^4T}{(2\pi)^4} \exp i[s_\beta t^\beta + \bar{s}_{\dot\beta}\bar{t}^{\dot\beta}] J(y+s,\bar{y}+\bar{s};y+t,\bar{y}+\bar{t})$$
$$= \exp -i[\partial_{1\alpha}\partial_{2\beta}\epsilon^{\alpha\beta} + \partial_{1\dot\alpha}\partial_{2\dot\beta}\epsilon^{\dot\alpha\dot\beta}] J(y,\bar{y};y,\bar{y})$$

# of derivatives in y and $\bar{y}$ infinitely increases for given helicities $\Rightarrow$ infinite tails of space-time derivatives and hence nonlocality

True locality: absence of integration over s and t or $\bar{s}$ and $\bar{t}$

$$\int \frac{d\bar{s}d\bar{t}}{(2\pi)^2} \exp i[\bar{s}_{\dot\beta}\bar{t}^{\dot\beta}] f(y,\bar{y}+\bar{s}) g(y,\bar{y}+\bar{t}) = f(y,\bar{y}) \exp[-i\overleftarrow{\partial}_{\dot\alpha}\overrightarrow{\partial}_{\dot\beta}\epsilon^{\dot\alpha\dot\beta}] g(y,\bar{y})$$

For given helicities carried by g and f, only a single term in the sum contributes hence containing a finite number of derivatives

Locality Criterion from the Perturbative Analysis

To see what is special for the local frame of HS equations reconsider the perturbative analysis in the sector of zero-forms giving

$$S_1 = S_{1\eta} + S_{1\bar{\eta}}\,,$$

$$S_{1\eta}(Z;Y;K|x) = -\eta z^{\beta}\theta_{\beta}\int_0^1 d\tau\tau\,\mathrm{exp}(i\tau z_{\alpha}y^{\alpha})C(-\tau z,\bar{y};K|x) * k$$

Resolution of $[S\,,B]_{\star}=0$ **gives for** B_2 **in the** η**-sector**

$$B_{2\eta} = -\frac{i}{2}\eta\int d^4_{+}\tau\delta(1-\sum_{i=1}^{4}\tau_i)\int d^4U d^4V\,\mathrm{exp}\,i(U_A V^A + (1-\tau_3)z_{\alpha}y^{\alpha})$$
$$\Big[y^{\beta}u_{\beta}\delta(\tau_1)C(\tau_3 y-\tau_1 z+(\tau_3+\tau_4)u,\bar{y}+\bar{u};K)C(\tau_3 y+\tau_2 z+v,\bar{y}+\bar{v};K)$$
$$-y^{\beta}v_{\beta}\delta(\tau_2)C(\tau_3 y-\tau_1 z+u,\bar{y}+\bar{u};K)C(\tau_3 y+\tau_2 z+(\tau_3+\tau_4)v),\bar{y}+\bar{v}\Big] * k * \kappa$$

Equivalently

$$B_{2\eta}=\frac{\eta}{2}\int d_+^4\tau\delta(1-\sum_{i=1}^{4}\tau_i)y^\beta\Big(\delta(\tau_1)\partial_{2\beta}+\delta(\tau_2)\partial_{1\beta}\Big)\exp(X)C(Y_1;K)C(Y_2;K)*k*\kappa|_{Y_1}$$

$$X=i(1-\tau_3)z_\alpha y^\alpha+\tau_3y^\alpha(\partial_{1\alpha}+\partial_{2\alpha})+z^\alpha(\tau_2\partial_{2\alpha}-\tau_1\partial_{1\alpha})+i(\tau_3+\tau_4)\partial_{1\alpha}\partial_2^\alpha+i\bar\partial_{1\dot\alpha}\bar\partial_2^{\dot\alpha}\,,$$

$$\partial_{i\alpha}:=\frac{\partial}{\partial y_i^\alpha}\,,\qquad \bar\partial_{i\dot\alpha}:=\frac{\partial}{\partial \bar y_i^{\dot\alpha}}$$

Since $\tau_4\geq 0$, **the expansion coefficients** $\tau_3+\tau_4$ **in powers of** $\partial_{1\alpha}\partial_2^\alpha$ **are larger than those** (τ_3) **of** $y^\alpha\partial_{i\alpha}$.

By the Schoutens identity

$$z_\alpha y^\alpha\partial_{1\beta}\partial_2^\beta+z^\alpha\partial_{1\alpha}y^\beta\partial_{2\beta}-z^\alpha\partial_{2\alpha}y^\beta\partial_{1\beta}=0$$

$B_{2\eta}$ **can be transformed to a form**

$$B_{2\eta}\quad=-\frac{1}{2}\eta\int d_+^4\tau\Big(iy_\alpha z^\alpha\delta(\tau_4)\delta(1-\sum_{i=1}^{4}\tau_i)+(\delta(\tau_3)-\delta(\tau_4))\delta'(1-\sum_{i=1}^{4}\tau_i)\Big)$$
$$\exp(X)C(Y_1;K)C(Y_2;K)\Big|_{Y_{1,2}=0}*k*\kappa\,.$$

Homotopy Redefinition

The terms with $\delta(\tau_3)$ and $\delta(\tau_4)$ have different meaning:

The part $\Delta C_{2\eta}$ with $\delta(\tau_3)$ is z-independent

$$\Delta C_{2\eta} = -\frac{1}{2}\eta \int d^4_+\tau\delta(\tau_3)\delta'(1-\sum_{i=1}^{4}\tau_i)$$

$$\exp(y^\alpha(\tau_1\partial_{1\alpha}-\tau_2\partial_{2\alpha})+i\tau_4\partial_{1\alpha}\partial_2^\alpha+i\bar{\partial}_{1\dot{\alpha}}\bar{\partial}_2^{\dot{\alpha}})C(Y_1;K)C(Y_2;K)\Big|_{Y_{1,2}=0} * k$$

Since $\Delta C_{2\eta}$ is in the d_Z-cohomology we can define a new homotopy operator

$$\mathsf{d}^*_{loc}B := \mathsf{d}^*_Z B - \Delta C_{2\eta}$$

such that

$$B^{loc}_{2\eta} = -\frac{i}{2}\mathsf{d}^*_{loc}([S_1\,,C]_*)$$

Characteristic Property of the Local Frame

$$B^{loc}_{2\eta} = \frac{1}{2}\eta\int d^3_+\tau\Big(\delta'(1-\sum_{i=1}^{3}\tau_i) - iy_\alpha z^\alpha\delta(1-\sum_{i=1}^{3}\tau_i)\Big)$$
$$\int d^4U d^4V \exp i(U_A V^A + (1-\tau_3)z_\alpha y^\alpha)$$
$$C(\tau_3 y - \tau_1 z + \tau_3 u, \bar{y}+\bar{u}; K)C(\tau_3 y + \tau_2 z + v, \bar{y}+\bar{v}; K) * k * \kappa\,.$$

The coefficient responsible for the index contraction between the first and second factors of C equals to those in front of the y variables, analogously to the star product of Z-independent functions.

$\Delta C_{2\eta}$ represents the nonlinear shift found in 2016 to reduce the nonlocal bilinear corrections to the local form in the sector of x, y-variables.

Original analysis was based on the assumption that, since the left and right parts of d_Z form a bicomplex the dependence on the right(left) spinors remains unchanged in the left(right) sector being represented by the star product.

Functional Classes

The idea is to specify appropriate classes of functions of twistor variables. Appropriate space $V_{0,0}$ was shown (2015) to be spanned by functions of the form

$$f(Z;Y) = \int_0^1 d\tau \phi(\tau Z; (1-\tau)Y; \tau) \exp i\tau Z_A Y^A$$

with $\phi(W;U;\tau)$ regular in W and U and integrable in τ is closed under the HS star product. Being accompanied by the factor of τ and $1-\tau$ the dependence on Z and Y trivializes at $\tau \to 0$ and $\tau \to 1$,

Such behavior is appropriate for the perturbative analysis of HS theory.

The remaining open problem: what is the structure on the pre-exponential factor $\phi(\tau Z; (1-\tau)Y; \tau)$

Nonlinear ϕ

Analysis of locality in the zero-form sector shows that the proper dependence on τ_i in

$$\int d_+^3\tau\rho(\tau)\int d^2u d^2v \exp i(u_\alpha v^\alpha + t z_\alpha y^\alpha) C(\tau_3 y - \tau_1 z + \tau_5 u) C(\tau_4 y + \tau_2 z + v) * k * \kappa$$

$$\tau_3 \leq 1 - t\,, \qquad \tau_4 \leq 1 - t\,, \qquad \tau_1 \leq t\,, \qquad \tau_2 \leq t\,, \qquad \tau_5 \leq 1 - t$$

The new restriction on τ_5 suggests an extension of the 2015 results to the nonlinear case.

Such functions induce minimally nonlocal higher-order corrections!

Special form of the proper nonlinear correction is not visible from the analysis of quadratic corrections at $Z = 0$!

Local Nonequivalence of the Two Frames

The shift induced by $\Delta C_{2\eta}$ the original expression is zero when helicities of the constituent fields have opposite signs while the resulting local expression is not.

Consequence: $\Delta C_{2\eta} \sim$ Green function in the opposite helicities sector

From the locality perspective, the usual homotopy operator d_Z^* is not locally equivalent to d_{loc}^*: in the nonlinear analysis of HS equations one has to use d_{loc}^* whose definition has to be properly extended to higher orders, which is a very interesting open problem for the future.

Conclusion

Classes of functions in the twistor space are identified distinguishing between local and nonlocal field redefinitions in HS theory

To get local or minimally nonlocal results at higher orders one has to use a particular homotopy operator d^*_{loc}. The latter is known in the lowest order but remains to be properly defined in the higher orders

Proper current corrections to HS field equations are uniquely determined by the locality criterion taking into account higher-order corrections and reproduce correctly AdS/CFT results: Gelfond, Didenko, Misuna

The statement that standard homotopy operator d^*_Z is not applicable in the local frame beyond the linear order versus

"...in three and four-dimensions, the g_{s_1,s_2,s_3} couplings were extracted from the unfolded equations, and were shown to be divergent"

Quotation from the recent paper 1704.07859 of Sleight and Taronna