Higher Spin Theory
and Holography-6
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Fermionic continuous spin field in (A)dS

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Plan

1) **Fermionic massless and massive continuous spin field in $R^{d,1}$**

2) **Fermionic continuous spin field in AdS_{d+1}**

3) **Computation of partition function**

of continuous spin field. Modified De Donder gauge

Lagrangian for continuous massless spin

bosonic in $R^{3,1}$

Schuster and Toro **2014**

fermionic in $R^{3,1}$

X.Bekaert, M.Najafizadeh, M.R.Setare, **2016**

Continuous spin field via deformation

of tower decoupled Fang-Fronsdal fields

method by Zinoviev 2001

Field content

Totally symmetric triple-traceless field in $R^{d,1}$

$$\psi^{a_1 \dots a_n}, \qquad n = 0, 1, \dots \infty$$

$$\gamma^a \psi^{abba_4 \dots a_n} = 0$$

Lagrangian for decoupled Fang-Fronsdal fields

$$\mathcal{L} = \sum_{n=0}^{\infty} \mathcal{L}_{\mathbf{n}}$$

$\mathcal{L}_{\mathbf{n}}$ - Fang-Fronsdal Lagrangian for spin n-field

$$[\bar{\alpha}^a, \alpha^b] = \eta^{ab}, \qquad [\bar{v}, v] = 1$$

$$\bar{\alpha}^a|0\rangle = 0 \qquad \bar{v}|0\rangle = 0$$

$$|\psi\rangle = \sum_{n=0}^{\infty} \upsilon^n \alpha^{a_1} \dots \alpha^{a_n} \psi^{a_1 \dots a_n} |0\rangle$$

$$(N_\alpha - N_\upsilon)|\psi\rangle = 0$$

$$N_\alpha \equiv \alpha^a \bar{\alpha}^a \qquad N_\upsilon = \upsilon \bar{\upsilon}$$

gauge transformation parameters

$$\xi^{a_1 \dots a_n}, \qquad n = 0, 1, \dots \infty$$

$$|\xi\rangle = \sum_{n=0}^{\infty} \upsilon^{n+1} \alpha^{a_1} \dots \alpha^{a_n} \xi^{a_1 \dots a_n} |0\rangle \, .$$

gamma-traceless

$$\gamma^a \xi^{a a_2 \dots a_n} = 0$$

Lagrangian for massless fields in $R^{d,1}$

$$\mathrm{i}\mathcal{L} = \langle\psi|E|\psi\rangle$$

$$E = E_{FF}$$

$$E_{FF} \equiv \not\partial - \alpha\partial\gamma\bar{\alpha} - \gamma\alpha\bar{\alpha}\partial + \gamma\alpha\not\partial\gamma\bar{\alpha}$$
$$+\gamma\alpha\alpha\partial\bar{\alpha}^2 + \alpha^2\gamma\bar{\alpha}\bar{\alpha}\partial - \alpha^2\not\partial\bar{\alpha}^2$$

$$\alpha\partial = \alpha^a\partial^a \qquad\qquad \alpha^2 = \alpha^a\alpha^a$$
$$\gamma\alpha = \gamma^a\alpha^a$$

massless in flat: gauge transformations

$$\delta|\psi\rangle = \alpha\partial|\xi\rangle$$

massive in flat

$$\mathrm{i}\mathcal{L} = \langle\psi|E|\psi\rangle$$

$$E = E_{FF} + \mathbf{E_{(0)}}$$

$$\mathbf{E_{(0)}} = (\mathbf{1} - \gamma\alpha\gamma\bar{\alpha} - \alpha^{\mathbf{2}}\bar{\alpha}^{\mathbf{2}})\mathbf{e}_{\mathbf{1}}^{\Gamma}$$
$$+(\gamma\alpha - \alpha^{\mathbf{2}}\gamma\bar{\alpha})\bar{\mathbf{e}}_{\mathbf{1}} + (\gamma\bar{\alpha} - \gamma\alpha\bar{\alpha}^{\mathbf{2}})\mathbf{e}_{\mathbf{1}}$$

$$\delta|\psi\rangle = (\alpha\partial - \mathbf{e}_{\mathbf{1}} + \gamma\alpha\mathbf{e}_{\mathbf{1}}^{\Gamma} - \alpha^{\mathbf{2}}\bar{\mathbf{e}}_{\mathbf{1}})|\xi\rangle$$

$$\begin{aligned}
\delta\psi^{a_1\ldots a_n} &= \partial^{(a_1}\xi^{a_2\ldots a_n)} \\
&+ \mathbf{e_1}\xi^{\mathbf{a_1\ldots a_n}} \\
&+ \mathbf{e_1^\Gamma}\gamma^{(\mathbf{a_1}}\xi^{\mathbf{a_2\ldots a_n})} \\
&+ \bar{\mathbf{e}}_\mathbf{1}\eta^{(\mathbf{a_1 a_2}}\xi^{\mathbf{a_3\ldots a_n})}
\end{aligned}$$

deformation procedure **Zinoviev 2001**

massive in flat

$$e_1^{\Gamma} = m$$

$$e_1 = e_\upsilon \bar{\upsilon} \qquad \bar{e}_1 = \upsilon e_\upsilon$$

$$e_\upsilon = \sqrt{F_\upsilon}$$

$$F_\upsilon = (s - N_\upsilon) m^2$$

Continuous massless in flat

$$e_1^\Gamma = \kappa_0$$

$$e_1 = e_\upsilon \bar{\upsilon} \qquad \bar{e}_1 = \upsilon e_\upsilon$$

$$e_\upsilon = \sqrt{F_\upsilon}$$

$$F_\upsilon = \kappa_0^2$$

Continuous massive in flat

$$e_1^{\Gamma} = \kappa_0$$

$$e_1 = e_\upsilon \bar{\upsilon} \qquad \bar{e}_1 = \upsilon e_\upsilon$$

$$e_\upsilon = \sqrt{F_\upsilon}$$

$$F_\upsilon = \kappa_0^2 - \mu_0 \left(N_\upsilon + \frac{d-1}{2} \right)^2$$

κ_0 and $\mu_0 -$ dimensionfull parameters

Classical unitarity

1)

$$\mathrm{i}\mathcal{L} = \langle\psi|\,\partial\!\!\!/\,|\psi\rangle + \dots$$

2)

$$\mathcal{L} = \mathcal{L}^{\dagger}$$

2) $\Longrightarrow$

$$2a)\quad \mathbf{e}_{\mathbf{1}}^{\mathbf{\Gamma}\dagger} = \mathbf{e}_{\mathbf{1}}^{\mathbf{\Gamma}} \qquad\qquad \mathbf{e}_{\mathbf{1}}^{\dagger} = \bar{\mathbf{e}}_{\mathbf{1}}$$

$$2b)\quad \mathbf{F}_{\upsilon} \geq 0$$

All previously known classically unitary systems turns out to be associated with unitary representations of space-time symmetry algebras

$$\mu_0 = m^2$$

$$\kappa_0 \neq 0$$

$\mu_0 = 0$ **massless continuous** unitary

$\mu_0 > 0$ **massive continuous** - nonunitar

$\mu_0 < 0$ **tachyonic continuous** - unitary

conjecture

massive classically unitary continuous spin field

is associated with

tachyonic UIR of Poincaré algebra

reducible case

$$F_{\upsilon}(s) = 0 \quad \Longrightarrow$$

$$\kappa_{\mathbf{0}}{}^2 = (s + \frac{d-1}{2})^2 \mu_{\mathbf{0}}$$

$$F_{\upsilon} = (s - N_{\upsilon})(s + d - 2 + N_{\upsilon})\mu_{\mathbf{0}}$$

$$|\psi\rangle = |\psi^{0,s}\rangle + |\psi^{s+1,\infty}\rangle$$

$$|\psi^{M,N}\rangle \equiv \sum_{n=M}^{N} \upsilon^{n}\alpha^{a_1}\ldots\alpha^{a_n}\psi^{a_1\ldots a_n}|0\rangle$$

$$\mathcal{L}(\psi) = \mathcal{L}(\psi^{0,s}) + \mathcal{L}(\psi^{s+1,\infty})$$

$$\mu_0 > 0$$

$\psi^{0,s}$ – classically **unitary**

$\psi^{s+1,\infty}$ – classically **non-unitary**

Fermionic continuous spin field in (A)dS$_{d+1}$

$$\mathrm{i}\mathcal{L} = \langle\psi|E|\psi\rangle$$

$$E = E_{FF} + \mathbf{E_{(0)}}$$

$$\mathbf{E_{(0)}} = (\mathbf{1} - \gamma\alpha\gamma\bar{\alpha} - \alpha^{\mathbf{2}}\bar{\alpha}^{\mathbf{2}})\mathbf{e}_{\mathbf{1}}^{\Gamma}$$

$$+(\gamma\alpha - \alpha^{\mathbf{2}}\gamma\bar{\alpha})\bar{\mathbf{e}}_{\mathbf{1}} + (\gamma\bar{\alpha} - \gamma\alpha\bar{\alpha}^{\mathbf{2}})\mathbf{e}_{\mathbf{1}}$$

$$\delta|\psi\rangle = (\alpha D - \mathbf{e}_{\mathbf{1}} + \gamma\alpha\mathbf{e}_{\mathbf{1}}^{\Gamma} - \alpha^{\mathbf{2}}\bar{\mathbf{e}}_{\mathbf{1}})|\xi\rangle$$

$$e_1^{\Gamma} = \kappa_0$$

$$e_1 = e_\upsilon \bar{\upsilon} \qquad \bar{e}_1 = \upsilon e_\upsilon$$

$$e_\upsilon = \sqrt{F_\upsilon}$$

$$F_\upsilon = \kappa_0^2 - \mu_0 \left(N_\upsilon + \frac{d-1}{2} \right)^2 - \rho \left(\mathbf{N}_\upsilon + \frac{d-1}{2} \right)^4$$

$$\rho = -1/R^2 \qquad for \quad AdS$$

$$\rho = +1/R^2 \qquad for \quad dS$$

Analyse $F_v(n) \geq 0$

For **dS** there are **NO** classically unitary solution

many solutions for **AdS**

reducible cases

One root

$$F_{\upsilon}(s) = 0$$

$$\Longrightarrow massive + infinite\ comp.\ fields$$

two roots

$$F_{\upsilon}(s_1) = 0, \qquad F_{\upsilon}(s_2) = 0$$

$$\Longrightarrow massive + partial\ massless$$

$$+ infinite\ component\ fields$$

Partition function

Partition function of fermionic continuous spin is equal to 1

$$Z = 1$$

$$Z^{-1} = \prod_{n=0}^{\infty} Z_n^{-1}$$

$$Z_n^{-1} = \frac{\mathcal{D}_{n-1}(\mathbf{M_{n-1}^2})\mathcal{D}_{n-1}(\mathbf{M_{n-1}^2})}{\mathcal{D}_n(\mathbf{M_n^2})\mathcal{D}_{n-2}(\mathbf{M_{n-2}^2})}$$

$$\mathcal{D}_n(M^2) = \sqrt{\det{}_n(-\not\Box + M^2)}$$

$$\not\Box \equiv \not{D}\not{D}$$

$$M_n^2 \equiv \mu_0 + \rho\left(n + \frac{d-1}{2}\right)^2$$

$\mathcal{D}$ on space of gamma-traceless fields

Modified de Donder gauge condition

$$\bar{\mathbf{C}}_{\mathrm{mod}}|\psi\rangle = \mathbf{0}$$

$$\bar{\mathbf{C}}_{\mathrm{mod}} \equiv \bar{C} - (\gamma\bar{\alpha} + \gamma\alpha\bar{\alpha}^2)\mathbf{e}_1^{\Gamma} + \bar{\alpha}^2\mathbf{e}_1 - \Pi\bar{\mathbf{e}}_1$$

$$\bar{C} \equiv \bar{\alpha}D - \frac{1}{2}\alpha D\bar{\alpha}^2$$

$$\Pi \equiv 1 - \alpha^2\frac{1}{2(2N_\alpha + d + 1)}\bar{\alpha}^2$$

Simple equations of motion

$$(\not{\Box} - \mathbf{M^2} + \rho\alpha^{\mathbf{2}}\bar{\alpha}^{\mathbf{2}})|\psi\rangle = \mathbf{0}$$

$$M^2 \equiv \mu_0 + \rho(N_v + \frac{d-1}{2})^2$$

Left-over gauge symmetry of EOM

$$(\not{\Box} - \mathbf{M_\xi^2})|\xi\rangle = \mathbf{0}$$

$$M_\xi^2 \equiv \mu_0 + \rho(N_v + \frac{d-3}{2})^2$$

Decompose physical field into γ-traceless fields

$$|\psi\rangle = |\psi_{\mathbf{I}}\rangle + \gamma\alpha|\psi_{\mathbf{\Gamma}}\rangle + \alpha^{\mathbf{2}}|\psi_{\mathbf{II}}\rangle$$

$$\gamma\bar{\alpha}|\psi_\tau\rangle = 0\,, \qquad \tau = I, \Gamma, II$$

$$(\not\Box - \mathbf{M_n}^2)\psi_\tau^{a_1 \cdots a_n} = 0$$

$$(\not\Box - \mathbf{M_n}^2)\xi^{a_1 \cdots a_n} = 0$$