# Type-B Formal Higher Spin Gravity

Maxim Grigoriev

*Lebedev Physical Institute, Moscow, Russia*

*Based on: M.G. Skvortsov, 1804.03196*

*earlier related works: Bekaert, M.G. Skvortsov 2017*

*Alkalaev, M.G. Skvortsov 2014*

*M.G. 2012, 2006*

June 6, 2018,
HSTH-7, Moscow

# Motivation

- HS theories are interesting on their own as nontrivial extensions of gravity, which share background independence, relation to geometry, etc.

- AdS holographic duals of simple CFTs (free scalar)

- Known examples of HS theories *Vasiliev* are essentially limited to totally symmetric fields in the bulk and to conformal scalar on the boundary.

- We lack first-principle derivation of Vasiliev system.

- Hologaphic duality between HS in the bulk and free CFT on the boundary is not manifest

- Known holographic reconstruction *Bekaert et. all, Taronna* works only perturbatively and does not tell much about HS geometry

## CFT with HS symmetry

Consider as a simplest and standard example free conformal scalar

$$\Box\phi = 0$$

Symmetries:

$$[\Box, A] = B\Box\,, \qquad A, B- \text{ differential operators}$$

Associated conserved HS currents:

$$J_{a_1\ldots a_s} = \bar{\phi}\partial_{a_1}\ldots\partial_{a_s}\phi + \ldots$$

Sources:

$$\langle h, J\rangle = \sum_s J_{a_1\ldots a_s} h^{a_1\ldots a_s}$$

$$\partial_{a_1} J^{a_1 a_2\ldots a_s} = 0 \quad \to \quad \delta h^{a_1\ldots a_s} = \partial^{(a_1}\xi^{a_2\ldots a_s)} - \text{traces}$$

These sources are infinitesimal in the sense that the action

$$S[\phi,h]=\langle\phi,\Box\phi\rangle+\langle J,h\rangle$$

is only invariant under gauge transformations

$$\delta h=\partial\cdot\xi+\eta\omega\,,\qquad \delta\phi=0\,.$$

at $\Box\phi=0$. This symmetry is not enough to fix the correlation functions.

The enhanced gauge symmetry can be found using the different base for the currents and the sources (background fields) *Segal 2002*

$$S[\phi,H]=\langle\phi,H\phi\rangle\,,$$

$$H=\sum_s H^{a_1\dots a_s}\partial_{a_1}\dots\partial_{a_s}\,,\qquad \delta H=HU+U^\dagger H\,,\quad U=U(x,\partial)\,.$$

Writing $H=\Box+h'$ note that $h'$ is related to $h$ through a nontrivial and nonlinear redefinition.

Integrating out the scalar results in the effective action:

$$e^{W[H]} = \int D\phi\, D\bar{\phi}\, e^{-\int\langle\phi, H\phi\rangle}\,, \qquad W[H] = -\mathrm{tr}\log H\,,$$

Its invariance with respect to

$$\delta H = HU + U^{\dagger}H\,, \quad U = U(x,\partial)\,.$$

encodes HS invariance of the correlation functions of $J_{a\ldots}$, which in known to fix them *Maldacena, Zhiboedov*. HS algebra show up as the algebra of Killings.

It follows, all the information about free CFT is encoded in the gauge theory of background fields (finite sources).

- These fields are off-shell
- Subject to nonlinear gauge symmetries

Usual understanding of HS holography:

Nonlinear HS in the bulk $\Leftrightarrow$ CFT with HS symmetry

The idea is to replace it with:

Nonlinear HS in the bulk $\Leftrightarrow$ Nonlinear background fields

It's easier to relate objects of the same nature.

Somewhat implicitly, was employed in the case of Type-A HS theory (Vasiliev theory) in *Bekaert, M.G., Skvortsov, 2017*
Earlier relevant developments: *Alkalaev, M.G. Skvortsov 2014, Bekaret, MG 2013, MG 2012, MG 2006*

## Background fields from constrained system

Given a quantum constrained system:

$$\widehat{F}_i|\Phi\rangle = 0\,, \qquad (\text{More generally: } F_a|\Phi\rangle = 0\,, \quad |\Phi\rangle \sim F_\alpha|\Xi\rangle)$$

The consistency condition (switching to star product notations):

$$[F_i, F_j]_\star = U_{ij}^k \star F_k$$

Natural equivalence transformations for the constraints:

$$\delta F_i = \lambda_i^j \star F_j + [\epsilon, F_i]_*$$

$F_i(x,p)$ can be seen as a generating function of background fields:

$$F_i = F_i(x) + F_i^a(x)p_a + F_i^{ab}(x)p_a p_b + \dots$$

while the above consistency and the equivalence, as resp. equations of motion and gauge symmetries.

# Background fields for a scalar

Phase space:

$$[x^a, p_b]_* = \delta^a_b$$

First class constraint $F(x,p)$. Although the consistency condition is trivial the gauge symmetries are

$$\delta F = [F, \xi]_* + F \star \omega$$

These are precisely gauge symmetries for background fields for a conformal scalar *Segal 2002*.

Upon linearizing around $F = p^2$ one gets linear gauge symmetries equivalent to those of Fradkin-Tseytlin fields. Type-A HS algebra arise as that of global reducibility parameters.

# Background fields for a spinor

Phase space:

$$[x^a, p_b]_* = \delta^a_b \qquad [\theta^a, \theta^b]_* = \eta^{ab} \,.$$

First class constraints: fermionic $F(x,p,\theta)$ and bosonic $H(x,p,\theta)$.
The consistency condition:

$$[F,F]_* = 2H \,, \qquad [F,H]_* = 0$$

Vacuum solution:

$$F^0 = \theta^a p_a \qquad H^0 = p^2$$

In the representation space (spinor fields) Dirac equation:

$$\widehat{\theta}^a \partial_a \psi = 0 \,, \qquad \Box \psi = 0$$

Linearized gauge symmetries:

$$\delta f = -(p\cdot\partial_\theta - \theta\cdot\partial_x)\epsilon + 2\alpha(p\cdot\theta) + 2\beta(p\cdot p) + \dots$$

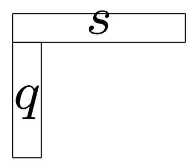
$\alpha, \beta$ can be used to set all traces to zero. Then one can reach the partial gauge $\theta\cdot\partial_p f = 0$. In terms of

$$\phi = p\cdot\partial_\theta f$$

one has:

$$\partial_p\cdot\partial_p\phi = \partial_p\cdot\partial_\theta\phi = 0\,, \qquad p\cdot\partial_\theta\phi = 0\,, \qquad \delta\phi = \Pi((p\cdot\partial_x)\epsilon)$$

The field content:

$$\phi_{a_1\dots a_s,b_1\dots q}(x) \sim$$ 


"Hook-type conformal background fields".
Studying the gauge symmetries preserving the vacuum solution $F^0 = \theta^a p_a$ one arrives at Type-B HS algebra. *Nikitin 1991, Vasiliev 2004*

# Ambient description of the conformal spinor

In terms of ambient space $\mathbb{R}^{d+2}$ with coordinates $X^A$, flat metric $\eta^{AB}$:

$$(X\cdot\partial_X+\frac{d-2}{2})\Psi=0\,, \qquad \Psi\sim\Psi+X^2\alpha+(X\cdot\theta)\beta\,,$$

$$\partial_X\cdot\partial_X\Psi=0\,, \qquad \theta\cdot\partial_X\Psi=0\,,$$

The above operators

$$X^2\,, \qquad \partial_X^2\,, \qquad (X\cdot\partial_X+\tfrac{d+2}{2})\,, \qquad \theta\cdot X\,, \qquad \theta\cdot\partial_X\,.$$

form $osp(1|2)$.

# Background Fields in Ambient Space

Phase space:

$$[X^A, P_B]_* = \delta^A_B\,, \qquad\qquad [\theta_A, \theta_B]_* = 2\eta_{AB}$$

Generating functions:

$$F_i(X, P, \Theta)\,, \qquad i = \{++, 0, --, +, -\}$$

Equations:

$$[F_i, F_j]_\star = U^k_{ij} \star F_k\,, \qquad U^k_{ij} - osp(1|2) \text{ structure constants}$$

Gauge symmetries:

$$\delta F_i = \lambda^j_i \star F_j + [\epsilon, F_i]_*$$

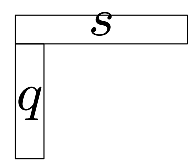
Natural vaccuum solution:

$$F^0_{++} = \tfrac{1}{2}P^2\,, \quad F^0_0 = X\cdot P\,, \quad F^0_{--} = \tfrac{1}{2}X^2\,, \quad F^0_+ = \theta\cdot P\,, \quad F^0_- = \theta\cdot X\,.$$

# Type-B Theory at Free Level

Field content:

$$(f_S)_{\mu_1\ldots\mu_s,\nu_1\ldots\nu_q}(x) \sim$$




Ambient space description: *Alkalaev, MG 2009*

$$(X\cdot\partial_P)f_S = 0\,, \quad (X\cdot\partial_\theta)f_S = 0$$

$$(P\cdot\partial_\theta)f_S = 0\,, \quad (X\cdot\partial_X - P\cdot\partial_P + 2)f_S = 0$$

$$(\partial_X\cdot\partial_X)f_S = (\partial_X\cdot\partial_P)f_S = (\partial_P\cdot\partial_P)f_S = (\partial_X\cdot\partial_\theta)f_S = (\partial_P\cdot\partial_\theta)f_S = 0$$

$$\delta f_S = (P\cdot\partial_X)\xi_S$$

Boundary values of these fields coincide with the linearized background conformal fields (modulo holographic anomaly in even $d$). Obvious using the technique of *Chekmenev, MG 2015*.

# Holographic reconstruction

The theory is determined by its on-shell gauge transformations
*Vinogradov, Barnich-Brandt-Henneaux, Vasiliev, Barnich-Grigoriev*

In the case at hand background fields on the boundary are 1:1 with on-shell bulk fields . Moreover, we know nonlinear gauge transformations for background fields!

We can in some sense reconstruct the bulk theory.

In the ambient formalism bulk/boundary relation amounts to considering the same system either around $X^2 = -1$ or $X^2 = 0$ *Bekaert, MG 2012*.

The proposal for Type-B HS theory is the same $osp(1|2)$ system

$$[F_i, F_j]_\star = U_{ij}^k \star F_k \,, \qquad \delta F_i = \lambda_i^j \star F_j + [\epsilon, F_i]_*$$

considered in the vicinity of the hyperboloid $X^2 = -1$.

Clearly requires regularization if considered around the vacuum $F^0_{--} = X^2$. Can be analyzed by switching to the parent formulation.

# Off-shell bulk system

Field content:

$$A = dx^\mu A_\mu(x|Y,P,\theta)\,, \qquad F_i = F_i(x|Y,P,\theta)\,,$$

"Internal" ambient space:

$$[Y^A, P_B]_* = \delta^A_B\,, \qquad [\theta^A,\theta^B]_* = 2\eta^{AB}$$

The system:

$$dA = \tfrac{1}{2}[A,A]_\star\,, \qquad \delta A = d\xi - [A,\xi]_\star\,,$$

$$dF_i = [A,F_i]_\star\,, \qquad \delta F_i = [\xi, F_i]_\star\,,$$

$$[F_i,F_j]_\star = C^k_{ij}F_k\,,$$

AdS Vacuum:

$$A^0 = dx^\mu \omega_{\mu A}{}^B T^A_B \,, \qquad V^A = const^A \text{– compensator}$$

$$F^0_- = (Y^A + V^A)\theta_A \,, \qquad F^0_+ = P^A \theta_A \,,$$

$$F^0_{--} = \frac{1}{2}(Y+V)\cdot(Y+V)\,, \quad F^0_{++} = \frac{1}{2} P \cdot P \,, \quad F^0_0 = (Y+V)\cdot P \,,$$

$$T^{AB} = -(Y^A + V^A)\cdot P^B + \frac{1}{4}\theta^A \theta^B - (A \rightleftarrows B)\,,$$

Linearized system (after homological reduction):

$$D_0 a = 0 \,, \qquad \delta a = D_0 \xi \,,$$

$$D_0 f_+ = [a, F^0_+]_* \,, \qquad \delta f_+ = [F^0_+, \xi]_* \,,$$

$$[F^0_-, f_+]_* = [F^0_0, f_+]_* - f_+ = 0 \,, \qquad [F^0_-, a]_* = [F^0_0, a]_* = 0 \,,$$

If $a, f_+$ were totally traceless this is precisely the system from *Alkalaev, MG, 2009* descriing the hook-type fields on AdS.

If we could use gauge symmetry:

$$\delta f_i = \lambda_i^j \star F_j^0\,, \qquad \delta a = \lambda^i F_i^0$$

to set $f_i, a$ totally traceless, we would conclude that the proposed theory properly describes free limit.

For the linearized system it's true provided we pick a proper functional class $\mathfrak{C}$: polynomials in $P, \theta$, formal series in $Y$ such that

$$(\partial_Y \cdot \partial_Y)^l \phi = 0$$

Then there is a twisted traceless projector $\Pi'$:

$$\phi = \phi_0 + (Y + V_0)^2 \phi_{10} + (Y + V) \cdot P \phi_{11} + \dots\,, \qquad \phi_{\dots} - \text{ totally traceless}$$

$$\Pi' \phi = \phi_0$$

In this class $Tr.(a) = Tr.(f_i) = 0$ is a legitimate gauge condition.

# HS-flat connection

Full on-shell system:

$$dA - \frac{1}{2}[A, A]_* = u^i \star F_i\,, \qquad \delta A = d\xi - [A, \xi]_* + \lambda^j \star F_j\,,$$
$$dF_i - [A, F_i]_\star = u_i^j \star F_j\,, \qquad \delta F_i = [\xi, F_i]_* + \lambda_i^j \star F_j\,,$$
$$[F_i, F_j]_* - C_{ij}^k F_k = u_{ij}^k \star F_k\,.$$

The system admits vacuum solution (belonging to $\mathfrak{C}$):

$$F_i^0, \qquad A^0(x, P, Y, \theta)$$

$$[F_i^0, A^0]_* = 0 \qquad dA^0 - \frac{1}{2}[A^0, A^0]_* = u^i \star F_i^0\,,$$

i.e. $A_0$ is a flat connection of Type-B HS algebra.

Type-B higher spin algebra $\mathsf{hs}_B$: *Nikitin 1991, Vasiliev 2004*

$$\mathsf{hs}_B = \{\mathsf{a} \in \mathfrak{C} : [\mathsf{a}, F_i^0]_* = 0\,, \quad \mathsf{a} \sim \mathsf{a} + \lambda^i \star F_i^0\}\,,$$

Representatives can be taken traceless. $\star$-descends to $\mathsf{hs}_B$.

Linearized system (after homological reduction):

$$\begin{aligned} D_0 a &= 0\,, & D_0 f_+ &= [a, F_+^0]_*\,, \\ \delta a &= D_0 \xi\,, & \delta f_+ &= [\xi, F_+^0]_*\,, \\ [F_-^0, f_+]_* = [F_-^0, a]_* &= 0\,, & [F_0^0, f_+]_* - f_+ &= [F_0^0, a]_* = 0\,, \end{aligned}$$

where $D_0 \bullet \equiv d \bullet - \Pi'[A^0, \bullet]_\star$.

This gives a concise formulation of the multiplet of hook-type fields propagating on the background of generic flat connection of the Type-A higher spin algebra.

This can be reduced to unfolded form (e.g. as in *Barnich, MG (2006)*). It should have the structure:

$$d\bar{a} + \Pi'([A_0, \bar{a}]) = \mu(A_0, A_0, C) \qquad \text{note: } A_0 \in \mathrm{hs}_B$$

$P \cdot \partial_Y \bar{a} = 0$ and $C$ parametrises the quotient $f_+ \sim f_+ + P \cdot \partial_Y \epsilon$.

Recent work by *Sharapov, Skvortsov* shows that such $\mu$ is a Hochshild cocycle of the HS algebra and it fully determines the complete deformation (the deformation is unobstructed due to the absence of higher cohomology).

This gives an extra argument in support that the proposed system "knows everything" about the Type-B HS gravity.

# Conclusions

- Concise consistent system of equations describing formal Type-B HS gravity

- Build in terms of boundary conformal spinor. HS holography automatically built in thanks to the ambient formalism Non-linear CHS fields are reproduced on the boundary. Classical version of holographic reconstruction?

- $osp(1|2)$ and HS generalization of the *Fefferman-Graham* construction. Proper language for HS geometry?

- Unifies metric-like and frame-like formalism. In particular, $F_+$ is an ambient version of the metric-like HS field

- Likely to provide a framework for studying nonlocality issue at more invariant level