Failure of mean field approximation in weakly coupled dilaton gravity*

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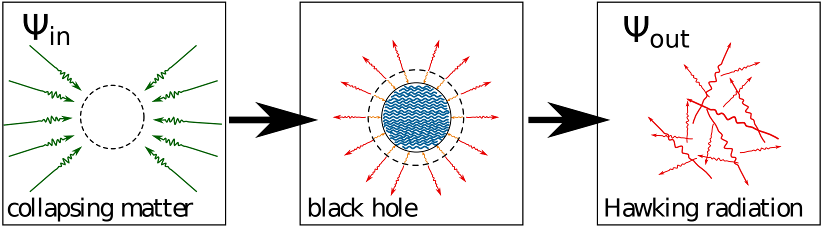
Motivation

- Information paradox: apparent loss of quantum coherence.

Hawking, 1975



$$\hat{\rho}_{in} = |\Psi_{in}\rangle\langle\Psi_{in}| \quad \mapsto \quad \hat{\rho}_{out} = \mathrm{Tr}_{BH}\left(|\Psi_{ext}\rangle|\Psi_{BH}\rangle\langle\Psi_{BH}|\langle\Psi_{ext}|\right), \quad \mathrm{Tr}(\hat{\rho}^2_{out}) < 1$$

  - Responses: AdS/CFT correspondence, "complementarity", etc.
  - AMPS-firewall. Unitarity versus Equivalence principle.

Almheiri et al, 2012

We need useful solvable models!

- Toy models: 2D dilaton gravity. Period of activity: $t \in (1991, 1996)$.
  Problem: apparent non-unitarity persists.

- Idea: revive 2d gravity with new semi-classical methods!
  - S-matrix as path integral: complex classical solutions.

Outline

1. Weakly coupled dilaton gravity
2. Failure of mean field theory
3. Semiclassical S-matrix elements

Weakly coupled dilaton gravity

$$S = \int d^2x\sqrt{-g}\left[e^{-2\phi}\left(R + 4(\nabla\phi)^2 + 4\lambda^2\right) - \frac{1}{2}(\nabla f)^2\right]$$

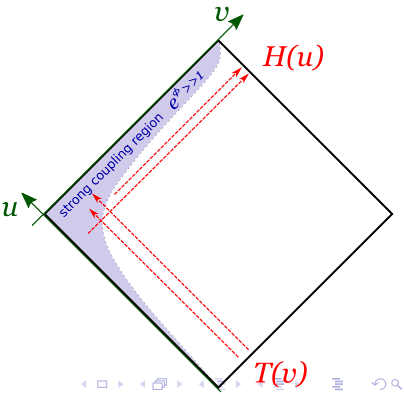
ArXiv:9111056 [hep-th] C. Callan, S. Giddings, J. Harvey, A. Strominger, 1991

In the bulk:

$ds^2 = -e^{2\phi}dvdu$, $f(v,u) = f_{out}(u) + f_{in}(v)$

$e^{-2\phi} = -\lambda^2 vu - \mathcal{T}(v) - \mathcal{H}(u)$

$\partial_v^2\mathcal{T} = (\partial_v f_{in})^2/2$, $\partial_u^2\mathcal{H} = (\partial_u f_{out})^2/2$



Weakly coupled dilaton gravity

$$S = \int d^2x\sqrt{-g}\left[e^{-2\phi}\left(R + 4(\nabla\phi)^2 + 4\lambda^2\right) - \frac{1}{2}(\nabla f)^2\right] + 2\int_{\partial\mathcal{M}} d\tau\, e^{-2\phi}(K + 2\lambda)$$

ArXiv:9111056 [hep-th] C. Callan, S. Giddings, J. Harvey, A. Strominger, 1991

ArXiv:1702.02576 [hep-th] M.F., D. Levkov, Y. Zenkevich, 2017

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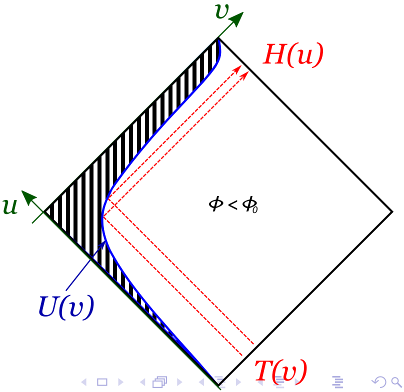
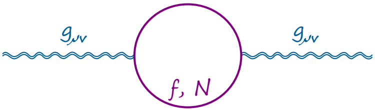
- Weak coupling: $g_{gr} = e^{\phi} \leq e^{\phi_0} \ll 1$
- Minimal black hole mass $M_{cr} = 2\lambda e^{-2\phi_0}$
- Reflecting condition $f_{out}(U(v)) = f_{in}(v)$.



One-loop effective action

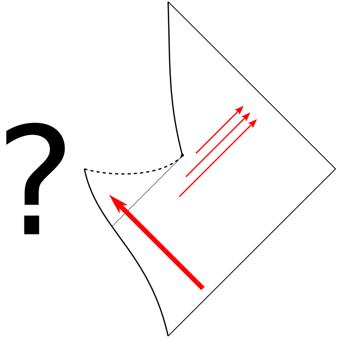


Includes Hawking radiation and backreaction on metric.

One loop from $N = 24Q^2$ scalars f_i $\Rightarrow$ Liouville-Polyakov action

$$S_{PL} = -\frac{Q^2}{2}\int dx\sqrt{-g}\int dy\sqrt{-g}\,R\frac{1}{\Box}R$$

$$S_{PL} \underbrace{=}_{on-shell} \int d^2x\sqrt{-g}\left[-\frac{1}{2}(\nabla\chi)^2 + Q\chi R\right]$$



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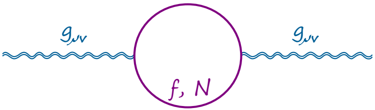

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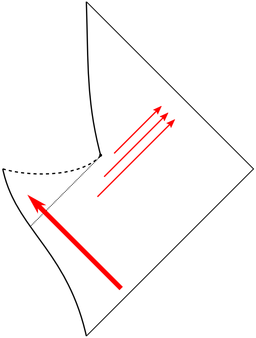
To restore solvability: *ArXiv:9206070 [hep-th]*

J. Russo, L. Susskind, L. Thorlacius, 1992

$$\Delta S_{RST} = -Q^2 \int d^2x \sqrt{-g} \phi R$$

?



Boundary terms: fixed by Wess-Zumino

$$\Delta S_{boundary} = 2 \int d\tau \left[\left(-Q^2\phi + Q\chi \right) K + \lambda Q^2 \right]$$

M.F., D. Levkov, Y. Zenkevich, in preparation

Correspondence $\mathcal{Q}\,RST \;\simeq\; \mathcal{Q}\,JT$

Conjecture: correct quantization via Weyl rescaling: $g_{\mu\nu} = e^{2\phi}\hat{g}_{\mu\nu}$.
No contribution from ghosts.

A. Strominger, 1992

It yields $S[g_{\mu\nu}, \phi, f_j] \;\mapsto\; \hat{S}[\hat{g}_{\mu\nu}, \phi, f_j] + S_A$, S_A - quantum anomaly term:

$$\boxed{\int [\mathcal{D}f_j]_g \; e^{iS_f[f_j, g_{\mu\nu}]} = \int [\mathcal{D}f_j]_{\hat{g}} \; e^{iS_A + iS_f[f_j, \hat{g}_{\mu\nu}]}\;,} \qquad S_A = \int \hat{R}\Box^{-1}\hat{R} - \int R\Box^{-1}R\;.$$

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With field redefinition $\Omega = e^{-2\phi} + Q^2\phi$ we obtain Jackiw-Teitelboim gravity with boundary

$$\hat{S}[\hat{g}_{\mu\nu}, \Omega, f_j] = \int d^2x\,\sqrt{-\hat{g}}\left[\Omega\hat{R} + 4\lambda^2 - \frac{1}{2}\sum_{j=1}^{N}\hat{g}^{\mu\nu}\partial_\mu f_j \partial_\nu f_j\right] + \text{boundary terms}\,.$$

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Statement: JT model minimally coupled to arbitrary matter
$S_{JT} = \int d^2x\, \sqrt{-\hat{g}} \left(\hat{\phi}\hat{R} - \Lambda \right) + S_{matter}[\hat{g}_{\mu\nu}, \hat{f}]$ has unitary S-matrix.

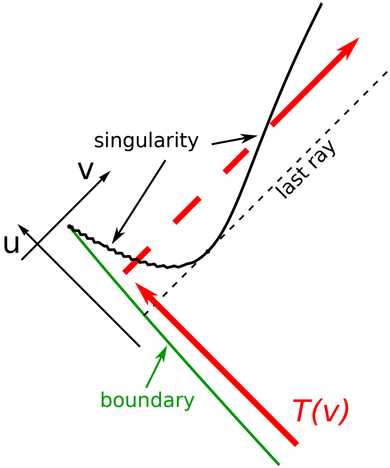
ArXiv:1706.06604 [hep-th] S. Dubovsky, V. Gorbenko, M. Mirbabayi, 2018

Solution for black hole

$U(v)$ - boundary in light-cone coordinates u, v; $\mathcal{T}(v)$ - incoming matter f_{in}.

Boundary condition: $\partial_v U = \text{const}\left(\partial_v \mathcal{T} + \lambda^2 U + \frac{Q^2 \partial_v^2 U}{2\partial_v U}\right)^2$

$E > E_{cr} \Rightarrow$ 2nd branch $\bar{U}(v) \neq U(v)$ of $\phi(v, u) = \phi_0$ is the singularity.



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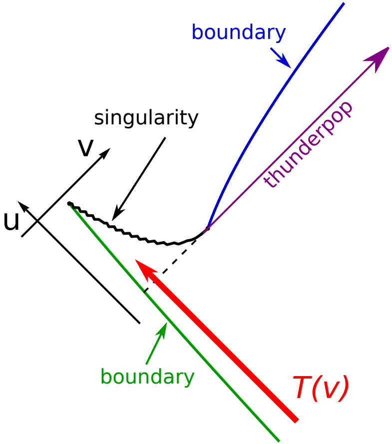
Impose new boundary condition $\Rightarrow$ new boundary.



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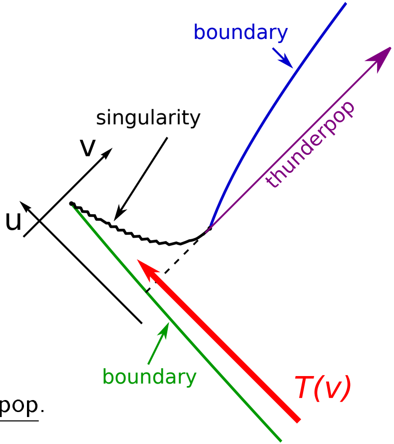
Impose new boundary condition $\Rightarrow$ new boundary.

Problems:

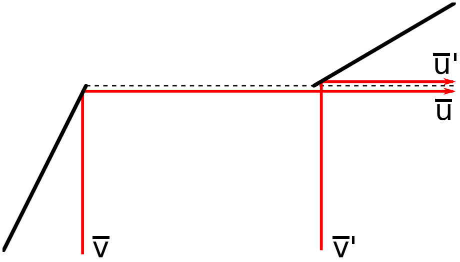
- Non-analyticity.
- Ambiguity: $\partial_v U\Big|_{end\ point}$ - ?
- Thunderpop: $E_{th} \sim -\lambda Q^2$.

$\Rightarrow$ has to introduce smearing around thunderpop.



Failure of mean field theory

Vacuum correlation function $G_{vac}(\bar{v}, \bar{v}') \equiv \langle f(\bar{v}) f(\bar{v}') \rangle \propto \ln |\bar{v} - \bar{v}'|$



Near the thunderpop $\bar{u}, \bar{u}' \simeq \bar{u}_{end}$,

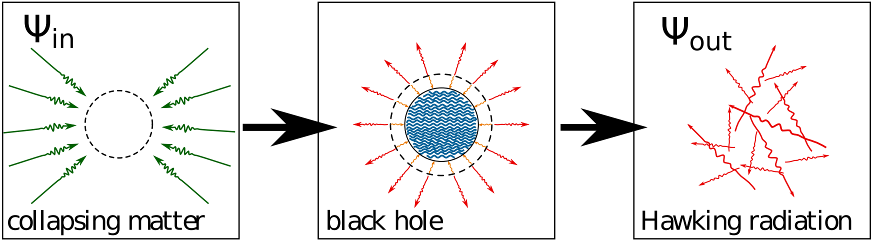
$$G(\bar{u}, \bar{u}') = \langle f_{out}(\bar{u}) f_{in}(\bar{u}') \rangle = \langle f_{in}(\bar{v}(\bar{u})) f_{in}(\bar{v}(\bar{u}')) \rangle \neq G_{vac}(\bar{u}, \bar{u}')$$

$\Rightarrow$ thunderbolt: particles with arbitrary large momenta $k \sim \Delta\bar{u}^{-1}$.

Strominger, 1994

Energy conservation $\Rightarrow \Delta\bar{u} \simeq Q/M_{cr}$ - characteristic size of quantum area where semiclassics always fails.

Failure of mean field theory



$$\langle \Psi_{out} | \hat{S} | \Psi_{in} \rangle = \int \mathcal{D}\Phi \, \Psi^*_{out} \Psi_{in} \exp\{\frac{i}{\hbar} S[\Phi]\} \, , \quad \Phi = \{g_{\mu\nu}, \phi, f\}$$

Semiclassics $\Rightarrow \frac{\delta}{\delta\Phi} S = 0 \Rightarrow$ saddle point Φ_s - can not be singular.

- MFA fails at 0th order: glued solutions are incorrect saddle points!
- Another possible answer: stiff boundary condition is inconsistent.
  - Analogy: Klein paradox in QM. Second quantization of the boundary?

Semiclassical S-matrix elements

- But we want to consider the whole solution Φ_s corresponding to $\Psi_{in} \mapsto \Psi_{out}$ (asymptotically flat to asymptotically flat).
  - At $E < Ecr \Rightarrow$ semiclassical S-matrix exists: $\langle \Psi_{out} | \hat{S} | \Psi_{in} \rangle \approx \exp\{\frac{i}{\hbar} S[\Phi_c]\}$
  - At $E > E_{cr} \Rightarrow$ no flat asymptotics of classical solutions at $t \to +\infty \Rightarrow$ ill-defined S-matrix.
- Idea: obtain physical solutions at $E > E_{cr}$ via analytic continuation.
  - Problem: many possibilities for deformation: suppressed, unphysical solutions. We need leading contribution!

We need a criterion to chose physical branch at $E > E_{cr}$, $\Im m\, E \to 0$.

Criterion: start from the “shell” model

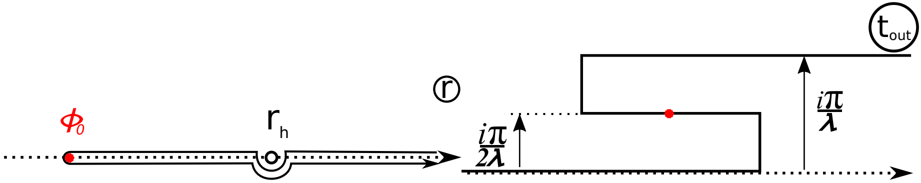
$$S = -m \int d\tau \sqrt{-g_{\mu\nu} \frac{dy^{\mu}}{d\tau} \frac{dy^{\mu}}{d\tau}^{\nu}} + S_{gravity}$$

Junction condition on shell: $\left(\frac{dr}{d\tau}\right)^2 - \left(\frac{M}{m} + \frac{m}{8\lambda} e^{-2\lambda r}\right)^2 + 1 = 0$

Well-known analytic continuation: $M \mapsto M + i\varepsilon\,, \quad \varepsilon \to +0$

ArXiv:9907001 [hep-th] M. Parikh, F. Wilczek, 2000

ArXiv:1503.07181 [hep-th] F. Bezrukov, D. Levkov, S. Sibiriakov, 2015

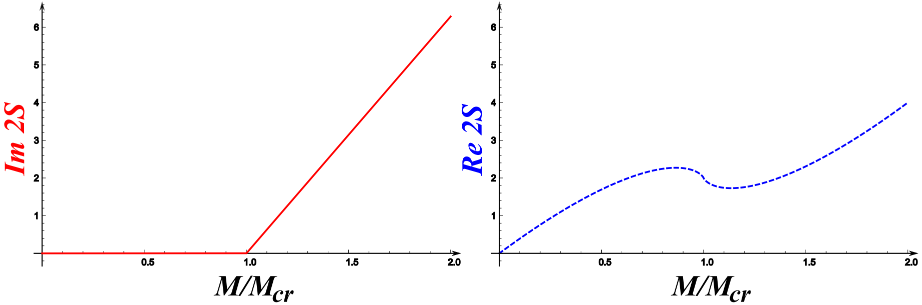


Can be generalized to field theory.

"Shell" model: result

$$2\Im m\, S_{tot} = S_{BH} - S_{cr}\ , \qquad S = \frac{M}{T_H} = \frac{2\pi}{\lambda} M$$

Plots for $m = 0$:



Probability: $\boxed{\mathcal{P}_{fi} \approx \exp\left(-S_{BH} + S_{cr}\right)}$.

Non-entropic suppression: Nature abhors discontinuity.

Conclusions

- Mean field theory:
  - Either not a good approximation...
  - ...or models with stiff boundaries are not self-consistent.
- Complex semiclassical method:
  - Reliable analytic continuation for shells.
  - Relevant solutions for fields.
  - Non-entropic suppression.

Thank you for attention!