Quotients of Multiparticle Higher-Spin Algebra.

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The algebra which describes higher spin equations is the infinite dimensional higher-spin algebra [Fradkin, Vasiliev, 1987].
It can be realized with the help of Weyl algebra with the basis elements:

$$f(Y) = \sum_{n=0}^{\infty} f^{i_1,\dots,i_n} Y_{i_1} \dots Y_{i_n}$$

and the product:

$$f(Y) \star g(Y) = f(Y) \exp(i(\overleftarrow{\partial}_i \epsilon^{ij} \overrightarrow{\partial}_j)) g(Y)$$

$$\epsilon_{ij} = -\epsilon_{ji}$$

We will speak about linearized free equations in AdS_4 background where the fields can be written in spinorial form:

$$C(Y) \equiv C(y, \bar{y}|x) = \sum_{n,m=0}^{\infty} C^{i_1,\dots,i_n,j_1,\dots,j_m}(x) y_{i_1} \dots y_{i_n} \bar{y}_{j_1} \dots \bar{y}_{j_m}$$

$$\omega_1(Y) \equiv \omega_1(y, \bar{y}|x) = \sum_{n,m=0}^{\infty} \omega_1^{i_1,\dots,i_n,j_1,\dots,j_m}(x) y_{i_1} \dots y_{i_n} \bar{y}_{j_1} \dots \bar{y}_{j_m}$$

As shown in [Vasiliev, 1989], the equations based on higher spin algebra are:

$$R_1 \equiv d\omega - \omega_0 \star \omega_1 + \omega_1 \star \omega_0 =$$

$$= h^{\gamma\dot\beta} \wedge h_\gamma^{\dot\alpha} \frac{\partial}{\partial \overline{y}^{\dot\alpha}} \frac{\partial}{\partial \overline{y}^{\dot\beta}} C(0, \overline{y}|x) + h^{\alpha\dot\gamma} \wedge h_{\dot\gamma}^{\beta} \frac{\partial}{\partial y^{\alpha}} \frac{\partial}{\partial y^{\beta}} C(y, 0|x)$$

$$\mathcal{D}_0 C \equiv dC - \omega_0 \star C + C \star \tilde{\omega}_0 = 0, \quad \tilde{\omega}(y, \overline{y}) = \tilde{\omega}(-y, \overline{y})$$

$$\mathcal{D}_0 C(y, \overline{y}|x) = D^L C(y, \overline{y}|x) + i\lambda h^{\alpha\dot\beta} \left(y_\alpha \overline{y}_{\dot\beta} - \frac{\partial}{\partial \overline{y}^{\alpha}} \frac{\partial}{\partial \overline{y}^{\dot\beta}} \right) C(y, \overline{y}|x)$$

$$D^L C(y, \overline{y}|x) = dC(y, \overline{y}|x) - \left(\omega^{\alpha\beta} y_\alpha \frac{\partial}{\partial y^{\beta}} + \overline{\omega}^{\dot\alpha\dot\beta} \overline{y}_{\dot\alpha} \frac{\partial}{\partial \overline{y}^{\dot\beta}} \right) C(y, \overline{y}|x)$$

Here ω_0 and h are the connection and the frame field of AdS_4.

- There is a fundamental difference between String Theory and Higher-Spin Theory, which is the presence of mass.
- To find the connection between those two theories it is important to overcome this difference.
- In [Vasiliev, 2012] there was introduced a way to extend higher spin algebra so that mixed symmetry type fields could appear which could bring current-like terms to the equations.

Suppose we have an associative algebra A with product $a_i \star a_j = f_{ij}^k a_k$ and unit element $e_\star$. From this algebra we can derive a Lie algebra $l(A)$ using bracket:

$$[a, b]_\star = a \star b - b \star a, \quad a, b \in A$$

The extension of algebra $M(A)$ is isomorphic to the universal enveloping algebra $U(l(A))$, which can be written using basis of polynomials:

$$F(\alpha) = \sum_{n=0}^{\infty} F^{i_1,\dots,i_n} \alpha_{i_1} \dots \alpha_{i_n}$$

and product

$$F(\alpha) \circ G(\alpha) = F(\alpha) \exp\left(\overleftarrow{\partial}^i f_{ij}^n \alpha_n \overrightarrow{\partial}^j \right) G(\alpha), \quad \alpha \in A,\ F(\alpha), G(\alpha) \in M(A)$$

There is a family of two sided ideals on $M(A)$

$$\mathcal{I}^N : \alpha^N \circ G(\alpha), \quad \alpha \text{ linear polynomial in M(A)}, G(\alpha) \in M(A)$$

These ideals generate a family of quotients

$$M_N(A) \equiv M(A)/\mathcal{I}^{N+1}$$

The basis of these quotients consists only of polynomials with degree smaller or equal to N

$$F(\alpha) = \sum_{n=0}^{N} F^{i_1,\dots,i_n} \alpha_{i_1} \dots \alpha_{i_n}$$

If we take $N = 2$ after factorization we get an algebra M_2 of polynomials:

$$F(\alpha) = F^0 + F^{i_1}\alpha_{i_1} + F^{i_1,i_2}\alpha_{i_1}\alpha_{i_2}$$

With product law:

$$\alpha_1 \circ \mathbf{1} = \alpha_1$$

$$\alpha_1 \circ \alpha_2 = \alpha_1\alpha_2 + \alpha_1 \star \alpha_2$$

$$\alpha_1 \circ (\alpha_2\alpha_3) = (\alpha_1 \star \alpha_2)\alpha_3 + (\alpha_1 \star \alpha_3)\alpha_2$$

$$(\alpha_1\alpha_2) \circ (\alpha_3\alpha_4) = (\alpha_1 \star \alpha_3)(\alpha_2 \star \alpha_4) + (\alpha_1 \star \alpha_4)(\alpha_2 \star \alpha_3)$$

The remaining algebra has two unit elements: 1 and $e_\star$ for two products $\circ$ and $\star$. The natural idea is to identify those elements with each other. It can be done by factorization of the algebra by its ideal:

$$\mathcal{I}_{e_\star - 2\cdot\mathbf{1}} : (e_\star - 2\cdot\mathbf{1})\circ G(\alpha), \quad G(\alpha)\in M(A)$$

In such a way we can derive the family of quotients

$$M^2(A) = M_2(A)/\mathcal{I}_{e_\star - 2\cdot\mathbf{1}}$$

The same procedure for $N=1$ would give $M^1(A)\sim A$

It can be shown that $M^2(A)$ consists of elements

$$\alpha_i\alpha_j, \quad \alpha_i, \alpha_j \in A$$

with product

$$(\alpha_i\alpha_j) \circ (\alpha_k\alpha_l) = (\alpha_i \star \alpha_k)(\alpha_j \star \alpha_l) + (\alpha_i \star \alpha_l)(\alpha_j \star \alpha_k)$$

The Lie algebra $l(A)$ is a subalgebra of the Lie algebra $l(M^2(A))$.

Currently we are investigating the higher-spin equations based on $M^2(A)$ with A taken to be the higher-spin algebra.
The expected form of equations should be roughly

$$R_1(Y^1, Z) = h \wedge h\partial^2 C(Y^1|x) + \mathcal{O}J(Y^1, Y^2|x)$$

$$J(Y^1, Y^2|x) = \sum_{n,m}^{\infty} C^{i_1,\dots,i_n,j_1\dots,j_m} Y^1_{i_1} \dots Y^2_{i_n} Y^2_{j_1} \dots Y^2_{j_m}$$

For the equations to be consistent the J term has to have properties of a current. This gives a possibility for the theory to get massive particles via spontaneous breaking mechanism.

Conclusions

- Higher-spin algebra A was extended to its universal enveloping algebra $M(A)$.
- There is a family of quotients of this algebra $M^N(A)$, which are "bigger" than the original algebra A. Particularly $M^1 \sim A$
- The new terms in equations of motion should look like currents so spontaneous breaking mechanism can take place.
- It is a possibility that the theory, constructed with the help of the algebra $M^2(A)$ will bring mixed-symmetry type fields to the higher-spin theory in higher dimensions $D > 4$.

Thank you for your attention!

$$\omega_0 = \frac{1}{4i}(\omega_0(x)^{\alpha\beta}y_\alpha y_\beta + \overline{\omega}_0(x)^{\alpha\dot{\beta}}\overline{y}_{\dot{\alpha}}\overline{y}_{\dot{\beta}} + 2\lambda h_0^{\dot{\alpha}\dot{\beta}}(x)y_\alpha\overline{y}_{\dot{\beta}})$$

$$h_{\underline{n}}^{\alpha\dot{\beta}}(x) = z^{-1}\sigma_{\underline{n}}^{\alpha\dot{\beta}}$$

$$\omega_{\underline{n}}^{\alpha\alpha} = -\lambda^2 z^{-1}\sigma_{\underline{n}}^{\alpha\dot{\beta}}x_{\dot{\beta}}^{\alpha}$$

$$z = 1 + \lambda^2 x^2$$

[V.E. Didenko, E.D. Skvortsov, 2014, arXiv:1401.2975]

$$\sigma^{a}_{\alpha\dot{\beta}} = (I, \sigma^{a})$$

$$\overline{\sigma}^{a\dot{\alpha}\beta} = (I, -\sigma^{a})$$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$v_{\alpha\dot{\beta}} = v^{m}\sigma_{m\alpha\dot{\beta}}$$

$$v^{m} = -\frac{1}{2} v_{\alpha\dot{\beta}}\overline{\sigma}^{m\dot{\beta}\alpha}$$