Conformal higher spin fields and cosmology

A.O.Barvinsky

Theory Department, Lebedev Physics Institute, Moscow

based on works with:
A.Kamenshchik
D.Nesterov
A.Starobinsky
C.Steinwachs

Higher Spin Theory and Holography-6
Lebedev Institute, June 1, 2017

Plan

Cosmological initial conditions:

- microcanonical density matrix of the Universe;
- cosmology dominated by quantum matter conformally coupled to gravity
- thermal cosmological instantons and the range of Λ

A.B. & A.Kamenshchik, JCAP, 09, 014 (2006) Phys. Rev. D74, 121502 (2006); A.B., Phys. Rev. Lett. 99, 071301 (2007)

A.B. & A.Kamenshchik, and D.Nesterov, JCAP, 01, 036 (2016)

Conformal higher spin fields (CHS):

- solution of hierarchy problem;
- stability of quantum corrections below the gravitational cutoff

A.B, arXiv:1511.07625 Phys.Rev. D93 (2016) 103530

Microcanonical ensemble in cosmology

$$|\Psi\rangle \to \hat{\rho}, \qquad \hat{H}_\mu\,\hat{\rho} = 0$$

Microcanonical density matrix – projector onto subspace of quantum gravitational constraints

$$\hat{\rho} = e^{\Gamma}\,\prod_\mu \delta(\hat{H}_\mu)$$

$$e^{-\Gamma} = \mathrm{Tr}\,\prod_\mu \delta(\hat{H}_\mu)$$

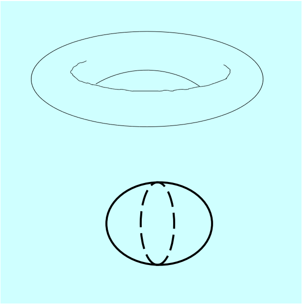
A.B., Phys. Rev. Lett. 99, 071301 (2007)

Cosmological initial conditions – microcanonical density matrix of the Universe and its statistical sum:

$$e^{-\Gamma} = \int_{\text{periodic}} D[\,g_{\mu\nu},\Phi\,]\; e^{-S[\,g_{\mu\nu},\Phi\,]}$$

on $S^3 \times S^1$ (thermal)

including as a limiting (vacuum) case S^4



Motivation: aesthetic (minimum of assumptions – Occam razor)

A simple analogy -- a system with a conserved Hamiltonian in the microcanonical state of a fixed energy *E*

$$\widehat{\rho} \sim \delta(\widehat{H} - E) \quad \Rightarrow \quad \widehat{\rho} \sim \prod_{\mu} \delta(\widehat{H}_\mu)$$

Spatially closed cosmology does not have *freely specifiable* constants of motion. The only conserved quantities are the Hamiltonian and momentum constraints H_μ, all having a particular value --- zero

$$\hat{\rho} = \sum_{\text{all } |\Psi\rangle} |\,\Psi\,\rangle\langle\,\Psi\,|$$

sum over "everything" that satisfies the Wheeler-DeWitt equation

An ultimate equipartition in the full set of states of the theory --- "*Sum over Everything*". Creation of the Universe from *Everything* is conceptually more appealing than creation from *Nothing*, because the democracy of the microcanonical equipartition better fits the principle of the Occam razor than the selection of a concrete state.

Application to CFT driven cosmology -- Universe dominated by quantum matter conformally coupled to gravity :

$$S[\,g_{\mu\nu},\Phi\,] = -\frac{M_P^2}{2}\int d^4x\,g^{1/2}\,(R-2\Lambda) + S_{CFT}[\,g_{\mu\nu},\Phi\,]$$

Λ -- primordial cosmological constant

⇓ **Omission of graviton loops**

$$S_{\rm eff}[g_{\mu\nu}] = -\frac{M_P^2}{2}\int d^4x\,g^{1/2}(R-2\Lambda) + \Gamma_{CFT}[g_{\mu\nu}],$$

$$e^{-\Gamma_{CFT}[\,g_{\mu\nu}]} = \int D\Phi\, e^{-S_{CFT}[\,g_{\mu\nu},\Phi\,]}$$

Saddle point of the path integral over 4-metrics – the solution of *effective* Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \frac{2}{M_P^2}\frac{\delta\Gamma_{CFT}}{\delta g^{\mu\nu}}$$

Local conformal invariance of S_{CFT} $\rightarrow$

recovery of $\Gamma_{CFT}[\,g_{\mu\nu}^{FRW}\,] = \Gamma_{CFT}[\,a, N\,]$ on a generic FRW background by a conformal map onto static Einstein Universe:

i) contribution of the conformal anomaly associated with this map;
ii) contributions of the Casimir energy and free energy on a static periodically identified Einstein Universe

$$g_{\mu\nu}\frac{\delta\Gamma_{CFT}}{\delta g_{\mu\nu}} = \frac{1}{4(4\pi)^2}g^{1/2}\left(\alpha\Box R + \beta E + \gamma C^2_{\mu\nu\alpha\beta}\right)$$

Gauss-Bonnet term

Weyl term

A.A.Starobinsky (1980);
Fischetty,Hartle,Hu;
Riegert; Tseytlin;
Antoniadis, Mazur, Mottola;
……
A.B. & A.Kamenshchik,
JCAP, 09, 014 (2006)
Phys. Rev. D74, 121502 (2006)

The coefficient of the topological Gauss-Bonnet term

$$\beta = \sum_s \beta_s\,\mathbb{N}_s,$$

$\mathbb{N}_s$ -- number of fields of spin *s*,
β_s -- spin-dependent coefficients

Effective Friedmann equation for saddle points of the path integral:

anomaly contribution

$$\frac{1}{a^2} - \frac{\dot{a}^2}{a^2} - B\left(\frac{\dot{a}^4}{2a^4} - \frac{\dot{a}^2}{a^4}\right) = \frac{\Lambda}{3} + \frac{C}{a^4},$$

$$C = \frac{B}{2} + \frac{1}{6\pi^2 M_P^2} \sum_\omega \frac{\omega}{e^{\omega\eta} \pm 1}$$

$$\eta = \int_{S^1} \frac{d\tau N}{a}$$

Casimir energy and radiation energy constant; energy of CFT particles – sum over field oscillators with frequencies ω on S^3

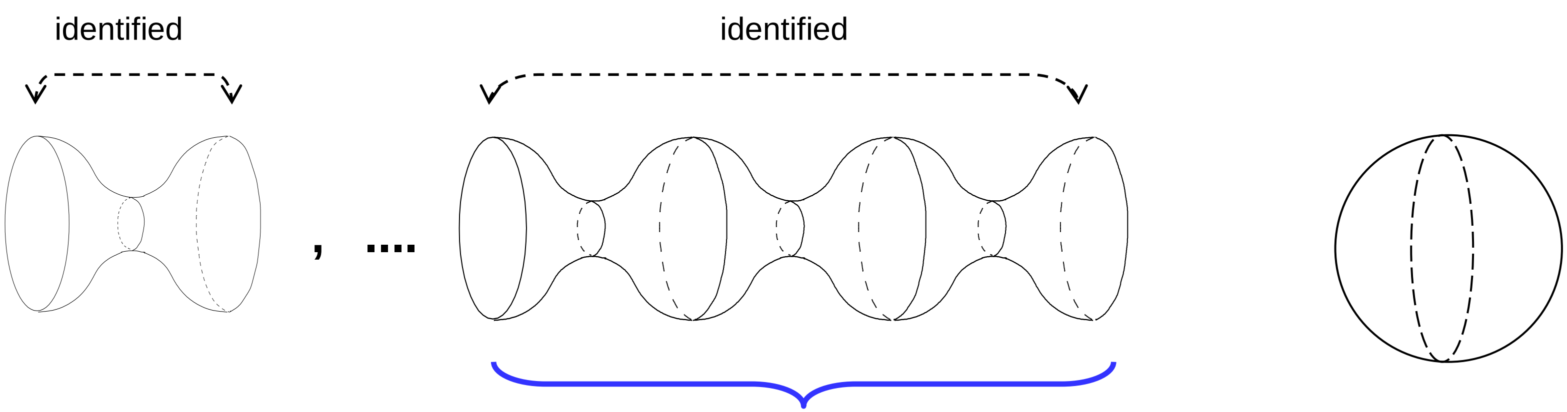
Inverse temperature in units of conformal time period on S^1

$$B = \frac{\beta}{8\pi^2 M_P^2}$$

-- coefficient of the Gauss-Bonnet term in the conformal anomaly

A.B. & A.Kamenshchik,
JCAP, 09, 014 (2006)
Phys. Rev. D74,
121502 (2006)

Saddle point solutions --- set of periodic (thermal) garland-type instantons with oscillating scale factor ($S^1 \times S^3$) and the vacuum Hartle-Hawking instantons (S^4)



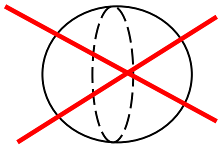
1- fold, *k*=1 **–** ***k*- folded garland, *k*=1,2,3,…** **–** **S^4**

1) Limited range of Λ – subplanckian domain (limiting the string vacua landscape?):

$$\Lambda_{\min} \le \Lambda \le \Lambda_{\max} = \frac{12\pi^2 M_P^2}{\beta}$$

$$g_{\mu\nu}\frac{\delta\Gamma}{\delta g_{\mu\nu}} = \frac{1}{4(4\pi)^2} g^{1/2}\left(\alpha\Box R + \beta E + \gamma C^2_{\mu\nu\alpha\beta}\right)$$

2) No-boundary instantons S^4 are ruled out by ***infinite positive*** Euclidean action – elimination of infrared catastrophe



3) Generalization to inflationary model, $\Lambda \to V(\phi)$ – selection of inflaton potential $V(\phi)$ ***maxima*** (new type of hill-top inflation) – quantum origin of the Starobinsky model and Higgs inflation model at $V(\phi) \sim \Lambda_{max}$. Employs **the mechanism of hill shape** inflaton potential!

4) Potentially observable signatures of thermal corrections to CMB power spectrum

5) Hidden sector of conformal higher spin fields (CHS): solution of the hierarchy problem and stabilization of the theory against the inclusion of graviton loop corrections

Hierarchy problem

Starobinsky *R^2*-model and non-minimal Higgs inflation model at *$V(\phi) \sim \Lambda_{max}$*

$$10^{-11} M_P^4 \simeq V_{\mathrm{inflation}} \sim \Lambda_{max} = \frac{12\pi^2}{\beta} M_P^4$$

***E*-coefficient of total conformal anomaly**

$$\beta \simeq 10^{13}$$

Impossible in Standard model with low spins *s=0,1/2,1* and $N_s \sim 100$

$$\beta = \frac{1}{180}\left(\mathbb{N}_0 + 11\mathbb{N}_{1/2} + 62\mathbb{N}_1\right)$$

Hidden sector of infinitely many massive fields in string theory – no conformal invariance

Hypothesis of string theory as a broken phase of Vasiliev theory of higher spin gauge fields

Vasiliev 1990, 1992, 2003

AdS/CFT correspondence tests with conformal higher spin (CHS) fields – totally symmetric rank *s* tensors (bosons) and fermionic spintensors with higher derivatives

Giombi, Klebanov, Pufu, Safdi, and Tarnopolsky 2013; Tseytlin 2013

$$S_{CHS} = \int d^4x \left(h^{\mu_1 \cdots \mu_s} \Box^s h_{\mu_1 \ldots \mu_s} + ,,, \right)$$

Recent progress in HS field theory (Vasiliev) and CHS theory (Klebanov, Giombi, Tseytlin, etc) arXiv:1309.0785 – a-anomalies and #'s of polarizations:

$$\beta_s = \frac{1}{360} \nu_s^2 (3 + 14\nu_s), \quad \nu_s = s(s+1), \quad s = 1, 2, 3, \ldots$$

$$\beta_s = \frac{1}{720} \nu_s (12 + 45\nu_s + 14\nu_s^2), \quad \nu_s = -2\left(s + \frac{1}{2}\right)^2, \quad s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots.$$

$$\beta_{\text{boson}} = \sum_{s=1}^{S} \beta_s \simeq \frac{S^7}{180} \sim 10^{13} \quad \Rightarrow \quad S \sim 100$$

$$\mathbb{N}_{\text{boson}} = \sum_{s=1}^{S} N_s \sim 10^6$$

We need a hidden sector of CHS with the tower of spins to S∼ 100 and # of polarizations ∼ 10^6

This number of hidden sector fields gives a red thermal correction to CMB spectral index in the third (potentially observable) decimal order:

$$n_s^{\text{observable}} = 0.96,$$
$$\Delta n_s^{\text{thermal}} \sim -0.001$$

Stability of quantum "corrections" and gravitational cutoff

Inflation scale

$$\Lambda_I = \frac{M_P}{\sqrt{\beta}} \ll M_P$$

Gravitational cutoff for $\mathrm{N} \gg 1$ quantum species (from smallness of graviton loops)

$$\Lambda = \frac{M_P}{\sqrt{\mathbb{N}}}$$

Veneziano (2002); G.Dvali et al (2002); G.Dvali and M.Redi (2008); G.Dvali (2010)

Critical feature of CHS fields: $\beta \sim s^6 \gg \mathrm{N} \sim s^2$ – smallness of graviton loop effects relative quantum matter loops!

Individual spin $s \gg 1$:

$$\mathbb{N}_s = \nu_s \sim s^2, \qquad \frac{\Lambda_{I,s}}{\Lambda_s} = \sqrt{\frac{\mathbb{N}_s}{\beta_s}} \simeq \frac{5}{s^2} \sim 10^{-4}$$

Tower of spins of the height S:

$$\mathbb{N} = \sum_s \nu_s \simeq \frac{S^3}{3}, \qquad \frac{\Lambda_{I,S}}{\Lambda_S} = \sqrt{\frac{\mathbb{N}}{\beta}} \simeq \frac{\sqrt{60}}{S^2} \sim 10^{-3}$$

Justification of approximation scheme: effective field theory for nonrenormalizable graviton sector below the cutoff and CHS matter sector beyond perturbation theory

Conclusions and discussion

Microcanonical density matrix of the Universe and its application to the CFT driven cosmology with a large # of quantum species – a limited range of Λ -- elimination of IR dangerous no-boundary states

Solution of hierarchy problem via CHS fields, stability of quantum corrections below the gravitational cutoff

Problems:

Consistent theory with a nonvanishing Weyl anomaly?

Consistent HS theory requires $S=\infty$

Conformal symmetry breaking at $s \geq S$ --- string theory is a symmetry breaking phase of HST?