Higher-Spin Theories & Locality

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Based on: arXiv: 1607.04718, 1701.05772 and 1704.07859 (with Charlotte Sleight)

A standard field theory approach: Noether procedure

Starting point: the Fronsdal Lagrangian [Fronsdal '78]

$$S^{(2)} = \sum_s \int \frac{1}{2}\,\varphi^{\mu_1 \dots \mu_s} \Box \varphi_{\mu_1 \dots \mu_s} + \dots$$

$$\delta^{(0)} \varphi_{\mu(s)} = \nabla_\mu \xi_{\mu(s-1)}$$

Consider a **weak field expansion** of a would be non-linear action and enforce gauge invariance:

$$S = S^{(2)} + S^{(3)} + S^{(4)} + \dots$$
$$\delta\varphi = \delta^{(0)}\varphi + \delta^{(1)}\varphi + \dots$$

$$\Longrightarrow$$

$$\delta^{(0)} S^{(2)} = 0$$
$$\delta^{(1)} S^{(2)} + \delta^{(0)} S^{(3)} = 0$$
$$\delta^{(2)} S^{(2)} + \delta^{(1)} S^{(3)} + \delta^{(0)} S^{(4)} = 0$$
$$\dots$$

Becomes more and more **involved** beyond the cubic order (Locality?)

[Boulanger, Leclercq, Sundell 2008, M.T. 2011; Boulanger, Kessel, Skvortsov & M.T. 2015; Bekaert, Erdmenger, Ponomarev & Sleight 2015; M.T. 2016, 2017; ...]

Particular and homogeneous solutions

With no-locality prescription the Noether procedure is empty:

$$\delta^{(0)}S^{(4)}+\boxed{\delta^{(1)}S^{(3)}}\approx 0$$

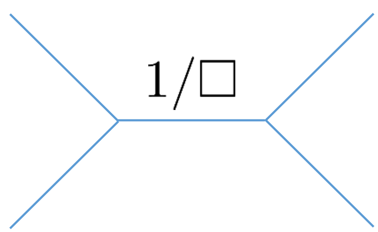
$\delta^{(0)}$ Is a linear operator

Non-homogeneous term

Indeed, at any order, a particular solution is given by to be minus the exchange

$$S^{(4)}=S^{(4)}_{\text{homo}}+S^{(4)}_p$$

$S_p^{(4)} = -\Big($



$+\ \text{t, u-channels}\Big)$

Once the above solution is identified Noether procedure reduces to:

$$\boxed{\delta^{(0)}S_h^{(n)}\approx 0}$$

[Barnich, Henneaux '93; M.T. '11, '17; C.Sleight & M.T. '17]

Yang Mills example

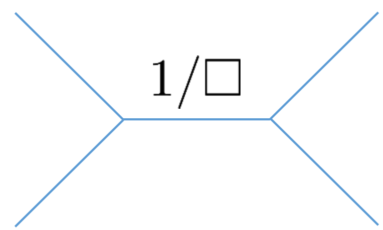
In the 1-0-0-0 example it is very easy to solve the homogeneous solution:

$$S_h^{(4)} = g_{1000}(\partial_{x_1}\cdot\partial_{x_2},\partial_{x_1}\cdot\partial_{x_4})\,\underbrace{(\partial_{x_1}\cdot\partial_{x_4}\,\partial_{u_1}\cdot\partial_{x_2}-\partial_{x_1}\cdot\partial_{x_2}\,\partial_{u_1}\cdot\partial_{x_4})}_{\text{Curl-type structure: } \delta^{(0)}S_h^{(4)}=0}\,A_1(x_1,u_1)\phi_2(x_2)\phi_3(x_3)\phi_4(x_4)$$

M.T. ‘11, C.Sleight & M.T. ‘17

No locality requirement ➡ Noether does not constrain the coefficient (at any order)

$$S^{(4)} = S_h^{(4)} - \sum_i g_{10i}\,g_{i00}\Big[\ \cdots\ + \text{t, u-channels}\Big]$$



Locality is what fixes the g_{1000} in terms of the cubic coupling constants with the requirement of removing all $1/\Box$

s-0-0-0

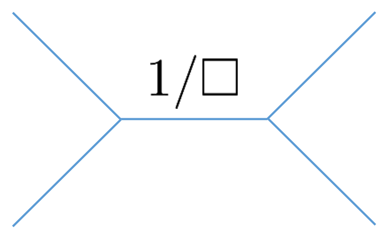
In the s-0-0-0 example the homogeneous solution is also very simple:

M.T. '11, C.Sleight & M.T. '17

$$S_h^{(4)} = g_{s000}(\partial_{x_1}\cdot\partial_{x_2},\partial_{x_1}\cdot\partial_{x_4})\,(\partial_{x_1}\cdot\partial_{x_4}\,\partial_{u_1}\cdot\partial_{x_2}-\partial_{x_1}\cdot\partial_{x_2}\,\partial_{u_1}\cdot\partial_{x_4})^s\,\varphi_1(x_1,u_1)\phi_2(x_2)\phi_3(x_3)\phi_4(x_4)$$

If we do not require locality the coefficient above is, again, not constrained at any order by Noether procedure:

$$S^{(4)} = S_h^{(4)} - \sum_i g_{10i}g_{i00}\Bigg($$



$$+\ \mathsf{t},\ \text{u-channels}\Bigg)$$

But, among the above, does a local solution possibly exist? $\mathsf{s} = \partial_{x_1}\cdot\partial_{x_2}$

$$S^{(4)}\Big|_{\mathcal{O}(1/\Box)} = \frac{(\partial_{u_1}\cdot\partial_{x_2})^{s_1}}{\mathsf{s}}\Bigg[(-1)^{s_1-1}\sum_{n\geq 0}\Big[g^{[n]}_{s_1 000}+(-1)^{s_1-1}g_{s_1,0,n+s_1-1}\,g_{n+s_1-1,0,0}\Big]\left(\frac{\mathsf{t}-\mathsf{u}}{4}\right)^{n+s_1-1}$$

$$+\sum_{r=0}^{s_1-2} g_{s_1,0,r}\,g_{r,0,0}\left(\frac{\mathsf{t}-\mathsf{u}}{4}\right)^r\Bigg] + \mathsf{t},\mathsf{u}$$

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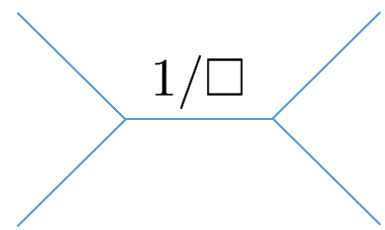

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$$+\sum_{r=0}^{s_1-2}g_{s_1,0,r}\,g_{r,0,0}\left(\frac{\mathsf{t}-\mathsf{u}}{4}\right)^r\Bigg]+\mathsf{t},\mathsf{u}$$

Locality gives a condition on the coupling constants...

s-0-0-0

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M.T. ‘11, C.Sleight & M.T. ‘17

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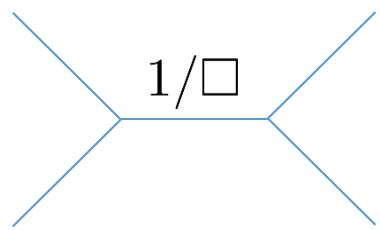
If we do not require locality the coefficient above is, again, not constrained at any order by Noether procedure:

$$S^{(4)} = S_h^{(4)} - \sum_i g_{10i}\,g_{i00}\left(\vphantom{\sum}\right.$$



$$\left.+\ \text{t, u-channels}\right)$$

But, among the above, does a local solution possibly exist? $\mathsf{s} = \partial_{x_1}\cdot\partial_{x_2}$

$$S^{(4)}\Big|_{\mathcal{O}(1/\Box)} = \frac{(\partial_{u_1}\cdot\partial_{x_2})^{s_1}}{\mathsf{s}}\left[(-1)^{s_1-1}\sum_{n\geq 0}\left[g^{[n]}_{s_1000} + (-1)^{s_1-1} g_{s_1,0,n+s_1-1}\,g_{n+s_1-1,0,0}\right]\left(\frac{\mathsf{t}-\mathsf{u}}{4}\right)^{n+s_1-1}\right.$$

$$\left.\boxed{+\sum_{r=0}^{s_1-2} g_{s_1,0,r}\,g_{r,0,0}\left(\frac{\mathsf{t}-\mathsf{u}}{4}\right)^{r}}\,\right] + \mathsf{t},\mathsf{u}$$

...and an obstruction for higher-spins s1>2

To sum up so far:

- All known local field theories are recovered: YM, Gravity, SUGRA, ...
- As soon as s>2 are included or the graviton is colored we get $1/\Box$
- Notice that for HS external legs $1/\Box$ **cannot** be removed adding auxiliary fields
- Adding ghosts may be an option but it is not clear if the procedure would ever stop
- Non-localities of quartic is not completely unrelated with the issue of field redefinitions (higher time derivatives...)

$$\frac{1}{\partial_t^2 - \nabla^2 + a}$$

- Requiring that the redefinition removing higher-time derivatives is legal (e.g. to quantise the theory in the bulk) means that $1/\Box$ redefinitions **should** be used. What is the prescription?

Locality and Weinberg theorem

Locality does not play a direct role in Weinberg theorem

In the soft-limit $q \to 0$ HS Ward identities force the observable to be trivial:

[Tseytlin et al '16; C.Sleight & M.T. '16]

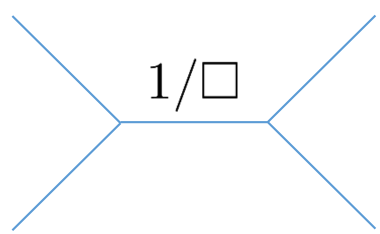
$$\left(\sum_{i=1}^{n} g_i\, p^{(i)}_{\mu_1}\cdots p^{(i)}_{\mu_{s-1}}\right) S_h^{(n)}(p^{(1)},\ldots,p^{(n)}) = 0$$

$$\boxed{S_h^{(4)} \sim \delta(\mathsf{s}) + \ldots}$$

Weinberg argument **does not** force g=0!!!

We can still solve the Noether procedure:

$$S^{(4)} = \delta(\mathsf{s}) - \sum_i g_{s0i}\, g_{i00} \Bigg[$$



$$+\ \mathsf{t},\ \mathsf{u}\text{-channels}\Bigg]$$

This solution does not differ from the free theory at the level of observables, but requiring the **HS symmetry to be gauged** forces arbitrary non-localities in the Lagrangian!

Light-cone (d=4)

The non-locality we uncover are not off-shell artifacts (or effects of auxiliary fields)!
They are non-trivial in the light-cone gauge:

$$x^{\pm} \equiv \frac{x^0 \pm x^3}{\sqrt{2}} \qquad z = \frac{x^1 + i x^2}{\sqrt{2}} \qquad \bar{z} = \frac{x^1 - i x^2}{\sqrt{2}}$$

Upon gauge fixing the non-local solutions found do not disappear:

$$\mathsf{u}\,\partial_{u_1} \cdot p_2 - \mathsf{s}\,\partial_{u_1} \cdot p_4 \;\longrightarrow\; \mathsf{s}\,\mathsf{u}\left[\partial_{u_1}\left(\frac{\partial_2^+}{P_{12}} + \frac{\partial_4^+}{P_{14}}\right) + \bar{\partial}_{u_1}\left(\frac{\partial_2^+}{\bar{P}_{12}} + \frac{\partial_4^+}{\bar{P}_{14}}\right)\right]$$

$$P_{ij} = \partial_i\,\partial_j^+ - \partial_j\,\partial_i^+$$

These non-localities are **different** from off-shell non-localities which vanish on-shell (like those obtained from integrating out auxiliary fields)! **The latter only generate contact terms in amplitudes.** Examples of such are given by:
Unconstrained HS (Francia, Sagnotti et al.), Integrating out trace in EH action, ...

Pseudo-locality in AdS

It is sometimes stated that AdS evades the obstructions arising in flat space:

- IR cut-off allows to soften the non-localities!

$$\frac{1}{\Box + \#/L^2} \qquad \to \qquad \frac{L^2}{(L^2\Box - a) + (\# + a)} \sim L^2 \sum c_n (L^2\Box - a)^n$$

- However this is not much different than flat space upon introducing a length scale:

$$\frac{1}{\Box} \qquad \to \qquad \frac{\lambda}{(\lambda\Box - a) + a} \sim \lambda \sum_n c_n (\lambda\Box - a)^n$$

- Both expansions have a common feature: they have a **finite radius** of convergence regardless the background (!)

- Furthermore, both in AdS and flat, observables uniquely fixed by (boundary) Ward identities (see Weinberg!)

Locality in Vasiliev's theory

Computations in Vasiliev's theory produce infinite expansions in derivatives (locality not built in at cubic):

$$\Box\Phi_{\mu_1\mu_2} + \ldots = J_{\mu_1\mu_2} \equiv \sum_l a_l\,\Lambda^{-l}\left(\nabla_{\mu_1\mu_2\nu(l)}\Phi\,\nabla^{\nu(l)}\Phi + \ldots\right)$$

[Boulanger, Kessel, Skvorstov & M.T]

$$a_l \sim \frac{1}{l^3}\frac{1}{(l!)^2}$$

However, only a **finite number** of coefficients is physical in the above series!

Identifying such finite number of coefficients is equivalent to fix a representative of the above non-local couplings:

$$J_{\mu_1\mu_2} = g\,\mathbf{J}_{\mu_1\mu_2} + \delta J_{\mu_1\mu_2}$$

Locality is what gives a meaning to the above splitting

Locality in Vasiliev's theory

$$\Box\Phi_{\mu_1\mu_2} + \ldots = J_{\mu_1\mu_2}$$

Performing field redefinitions on-shell is **dangerous**! Field redefinitions should be performed off-shell at the action level...

$$\int \frac{1}{2}\partial_\mu\Phi\,\partial^\mu\Phi + m^2\Phi^2 \quad\longrightarrow\quad \int \delta\Phi(-\Box + m^2)\Phi + \int_\partial n^\mu\left[(\delta\Phi)\partial_\mu\Phi\right]$$

A non-local redefinition will contribute a non-local boundary term!

Trick: do not perform any redefinition! Find a splitting which preserves the observables:

$$\int_{AdS} h^{\mu_1\mu_2} J_{\mu_1\mu_2} = \int_{AdS} h^{\mu_1\mu_2}\,[g\,\mathbf{J}_{\mu_1\mu_2} + \underbrace{\delta J_{\mu_1\mu_2}}_{=0}] = g\int_{AdS} h^{\mu_1\mu_2}\mathbf{J}_{\mu_1\mu_2}$$

Locality in Vasiliev's theory

Enforcing this idea for the explicit backreaction extracted from Vasiliev's equations we arrive to:

$$J_{\mu_1\mu_2} = -\frac{1}{12}\left(\sum_l l\right)\left[\left(\nabla_{\mu_1\mu_2}\Phi\right)\Phi + \ldots\right] + \delta J_{\mu_1\mu_2}$$

- The coefficient g is extracted unambiguously: $g = -\frac{1}{12}\sum_{l=0}^{\infty} l$

$$\frac{1}{\Box} = \sum_n (1-\Box)^n \sim \left(\sum_n 1\right) + \ldots$$

- The splitting preserves the Witten diagram computation (no subtlety of boundary terms)

$$\int_{AdS} h^{\mu_1\mu_2}\delta J_{\mu_1\mu_2} = 0$$

- The coefficient of the non-trivial representative for the current is formally infinite...

New modified equations

Last year a different splitting of the current was proposed at the level of zero-form equations (Vasiliev 2016):

$$\widetilde{D}C^{(2)} = \omega \star C - C \star \pi(\omega) + \mathcal{V}(h, C, C)$$

$$\mathcal{V}(h,C,C) = \int_0^1 d\tau \int \frac{d\bar{u}d\bar{v}}{(2\pi)^2}\, e^{i\bar{u}_{\dot\beta}\bar{v}^{\dot\beta}}\, h_{\alpha\dot\alpha} y^\alpha \left[\tau \bar{u} + (1-\tau)\bar{v}\right]^{\dot\alpha} C(\tau y, \bar{y}+\bar{u})C((1-\tau)y, \bar{y}+\bar{v}) + \text{c.c}$$

$$+\, \delta\mathcal{V}(h,C,C)$$

All **non-localities** have been **moved** in $\delta\mathcal{V}$ but we should check if the Witten diagram gives zero otherwise we can get any result... (boundary terms!)

The local part has been **engineered** to get the following scalar equation:

M.T. ‘16 (unpublished)

$$(\Box - 4)\Phi(x) = \frac{2i(-1)^{\frac{s_1-s_2}{2}}}{\Gamma(s_1+s_2)\Gamma(s_1-s_2)}\, \omega^{\partial\bar\partial}_{\alpha(s_1+s_2)\dot\alpha(s_1-s_2)} C^{\alpha(s_1+s_2)\dot\alpha(s_1-s_2)} + 2i\,\frac{(-1)^s}{\Gamma(2s)}\, C^{\alpha(2s)}C_{\alpha(2s)} + c.c$$

$$\Downarrow$$

$$(\Box - 4)\Phi(x) = \frac{2i}{\Gamma(s_1+s_2)}\left[(\nabla^{\alpha\dot\beta})^{s_2}\, \phi^{\alpha(s_1)\dot\alpha(s_1-s_2)}{}_{\dot\beta(s_2)}\right]\left[(\nabla_{\alpha\dot\alpha})^{s_1-s_2}(\nabla_{\alpha\dot\gamma})^{s_2}\phi^{\alpha(s_2)\dot\gamma(s_2)}\right] + c.c$$

Matches the 4d metric–like result:
[C. Sleight & M.T. ’16]

$$g_{s_1,s_2,s_3} = \frac{1}{\Gamma(s_1+s_2+s_3)}$$

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$$+\,\delta\mathcal{V}(h,C,C)$$

All **non-localities** have been **moved** in $\delta\mathcal{V}$ but we should check if the Witten diagram gives zero otherwise we can get any result... (boundary terms!)

The field redefinition that removes $\delta\mathcal{V}$ is non-local and therefore it is not safe to perform it (boundary terms...). One can e.g. evaluate the following Witten diagram:

$$\int_{AdS} \Phi\, \delta\mathcal{V}(C,C)$$

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The field redefinition that removes $\delta\mathcal{V}$ is non-local and therefore it is not safe to perform it (boundary terms...). One can e.g. evaluate the following Witten diagram:

$$\int_{AdS} \Phi\, \delta\mathcal{V}(C,C) = \infty$$

(M.T. 2016 unpublished)

- The above redefinition appears to be **not admissible**
- Performing such redefinition would generate **non-local boundary terms** which have not been yet analysed (Sezgin, Skvortsov & Zhu; Didenko, Vasiliev)

Non-admissible Redefinitions

To have a grasp on admissible vs non-admissible redefinitions it is useful compare the redefinition for the modified form of Vasiliev's equations with the redefinitions which removes the stress tensor

$$h_{\mu_1\mu_2} \longrightarrow h_{\mu_1\mu_2} + \sum_l a_l\,\Lambda^{-l}\left(\nabla_{\mu_1\mu_2\nu(l)}\Phi\,\nabla^{\nu(l)}\Phi + \ldots\right)$$

Removing the stress tensor requires (M.T. 2016):

$$a_l = \frac{4\alpha}{(l+1)^2(l+2)(l+3)^2} \sim \frac{1}{l^5} \qquad \left(\frac{1}{l^{2s+1}}\right)$$

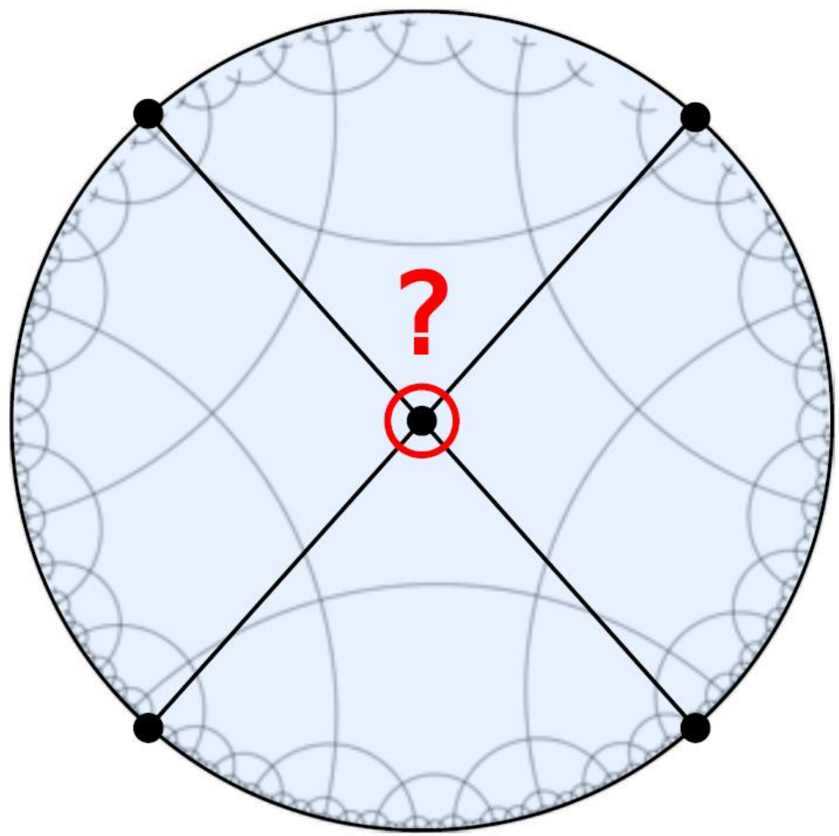
Re-def to obtain modified equations:

$$a_l = \frac{(2+l(l+3)+2)}{(l+1)^2(l+2)(l+3)^2} \sim \frac{1}{l^3}$$

The redefinition which removes the stress tensor is **less singular**!

[M.T. 2016]

Locality and holography?

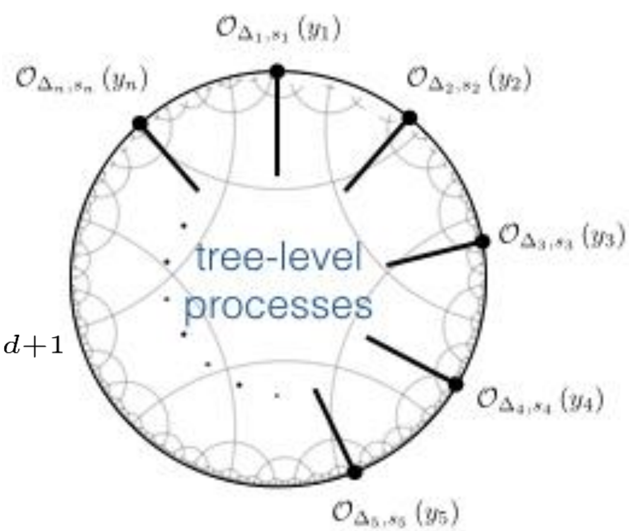


Holographic Approach

Higher-spin theory on AdS$_{d+1}$ ⟷ Free O(N) vector model

[Sezgin-Sundell, Klebanov-Polyakov, '02]

$$\int_{\mathrm{AdS}_{d+1}}$$



$$= \langle \mathcal{O}_{\Delta_1,s_1}(y_1)\dots\mathcal{O}_{\Delta_n,s_n}(y_n)\rangle$$

$$\approx -\frac{1}{G}\prod_{i=1}^{n}\frac{\delta}{\delta\bar{\varphi}_{s_1}(y_1)}\,S_{AdS}[\varphi_i,\varphi_i|_{\partial AdS=\bar{\varphi}_i}]$$

Solve the above equation for the bulk vertices $\mathcal{V}(X)$ and check that the CFT gives a solution to the Noether procedure

Locality at quartic order

Let us present the s-0-0-0 example: (1704.07859, C. Sleight & M.T.)

CFT:

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} \qquad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2} \qquad z_1^2 = 0$$

$$\int_{AdS} \mathcal{A}^{(4)} = \langle \mathcal{J}_{s_1}(y_1; z_1) \mathcal{O}(y_2) \mathcal{O}(y_3) \mathcal{O}(y_4) \rangle_{\text{conn.}}$$

$$= \frac{1}{N} \frac{1}{(y_{12}^2 y_{34}^2)^\tau} \Bigg[u^{\tau/2} \left(\frac{z_1 \cdot y_{13}}{y_{13}^2} - \frac{z_1 \cdot y_{12}}{y_{12}^2} \right)^{s_1} + \left(\frac{u}{v} \right)^{\tau/2} \left(\frac{z_1 \cdot y_{12}}{y_{12}^2} - \frac{z_1 \cdot y_{14}}{y_{14}^2} \right)^{s_1}$$

$$+ u^{\tau/2} \left(\frac{u}{v} \right)^{\tau/2} \left(\frac{z_1 \cdot y_{14}}{y_{14}^2} - \frac{z_1 \cdot y_{13}}{y_{13}^2} \right)^{s_1} \Bigg]$$

$$= \mathcal{H}_{(s_1,0|d-2|0,0)}(y_1, y_2; y_3, y_4) + \frac{1}{N} \frac{1}{(y_{12}^2 y_{34}^2)^\tau} u^{\tau/2} \left(\frac{u}{v} \right)^{\tau/2} \left(\frac{z_1 \cdot y_{14}}{y_{14}^2} - \frac{z_1 \cdot y_{13}}{y_{13}^2} \right)^{s_1} \sim \frac{1}{\Box}$$

Exchange sum:

$$\sum_{r=0}^{\infty} \mathcal{E}_r^{(\mathrm{s})} = \mathcal{H}_{(s_1,0|d-2|0,0)}(y_1, y_2; y_3, y_4) + \text{contact} \sim \frac{1}{\Box} + \text{contact}$$

Locality at quartic order

Contact term:

$$S^{(4)} = \int \left[\mathcal{A}^{(4)} - \sum_r (\mathcal{E}_r^{(\mathsf{s})} + \mathcal{E}_r^{(\mathsf{t})} + \mathcal{E}_r^{(\mathsf{u})}) \right]$$

$$= -\frac{1}{2} \underbrace{\left(\mathcal{H}_{(s_1,0|d-2|0,0)}\left(y_1, y_2; y_3, y_4\right) - \mathcal{H}_{(s_1,0|d-2|0,0)}\left(y_1, y_3; y_2, y_4\right) - \mathcal{H}_{(s_1,0|d-2|0,0)}\left(y_1, y_4; y_3, y_2\right) \right)}_{\text{It contains } \frac{1}{\Box}} + \text{contact}$$

NO-GO: either $S^{(4)}$ or the improvement (auxiliary field) terms in the exchange or both contain sums over spin and derivatives with **finite radius of convergence**

Locality at quartic order

Contact term:

$$S^{(4)} = \int \left[\mathcal{A}^{(4)} - \sum_r (\mathcal{E}_r^{(\mathsf{s})} + \mathcal{E}_r^{(\mathsf{t})} + \mathcal{E}_r^{(\mathsf{u})}) \right]$$

$$= -\frac{1}{2} \underbrace{\left(\mathcal{H}_{(s_1,0|d-2|0,0)}\left(y_1,y_2;y_3,y_4\right) - \mathcal{H}_{(s_1,0|d-2|0,0)}\left(y_1,y_3;y_2,y_4\right) - \mathcal{H}_{(s_1,0|d-2|0,0)}\left(y_1,y_4;y_3,y_2\right)\right)}_{\text{It contains } \frac{1}{\Box}} + \text{contac}$$

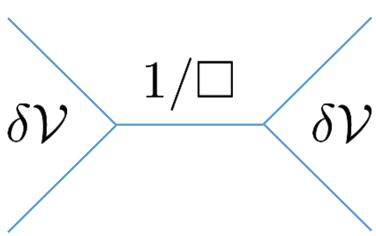
NO-GO: either $S^{(4)}$ or the improvement (auxiliary field) terms in the exchange or both contain sums over spin and derivatives with **finite radius of convergence**

Non-convergent local improvements (C.Sleight & M.T.):

c_s Coeff is arbitrary!

$$\mathcal{V}_{0,0,s} \quad \rightarrow \quad \mathcal{V}_{0,0,s} + c_s \delta\mathcal{V}_{0,0,s}\,, \quad \delta\mathcal{V}_{0,0,s} \approx 0$$

$$\sum_{s=0}^{\infty}$$



$$\sim \sum_{s=0}^{\infty} c_s^2 \Box^s \sim \frac{1}{\Box} \qquad c_s = (-1)^s s! \rightarrow e^{1/\Box}\Gamma(0,\Box)$$

(1704.07859, C. Sleight & M.T.)

Locality at quartic order

Contact term:

$$S^{(4)} = \int \left[\mathcal{A}^{(4)} - \sum_r (\mathcal{E}_r^{(\mathsf{s})} + \mathcal{E}_r^{(\mathsf{t})} + \mathcal{E}_r^{(\mathsf{u})}) \right]$$

$$= -\frac{1}{2} \underbrace{\left(\mathcal{H}_{(s_1,0|d-2|0,0)}\left(y_1,y_2;y_3,y_4\right) - \mathcal{H}_{(s_1,0|d-2|0,0)}\left(y_1,y_3;y_2,y_4\right) - \mathcal{H}_{(s_1,0|d-2|0,0)}\left(y_1,y_4;y_3,y_2\right) \right)}_{\text{It contains } \frac{1}{\Box}} + \text{contact}$$

NO-GO: either $S^{(4)}$ or the improvement (auxiliary field) terms in the exchange or both contain sums over spin and derivatives with **finite radius of convergence**

Non-convergent local improvements (C.Sleight & M.T.):

c_s Coeff is arbitrary!

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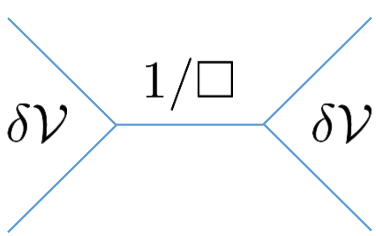
$\sum_{s=0}^{\infty}$ 

$$\sim \sum_{s=0}^{\infty} c_s^2 \, \Box^s \sim \frac{1}{\Box}$$

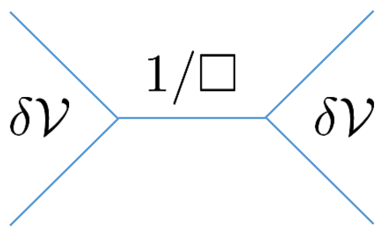
Even local redefinitions are subtle!
(non-local cubic couplings in spin)

(1704.07859, C. Sleight & M.T.)

Speculations beyond Field Theory?

We have seen that local improvement which are not convergent in spin can generate arbitrary singularities

$$\sum_{s=0}^{\infty}$$

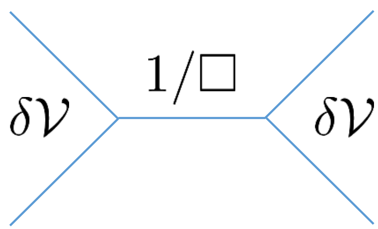


$$= \sum_{s,l} c_{s,l} (\phi \nabla^s \phi) \Box^l (\phi \nabla^s \phi)$$

Local improvements

Speculations beyond Field Theory?

We have seen that local improvement which are not convergent in spin can generate arbitrary singularities



$$\sum_{s=0}^{\infty} \delta\mathcal{V} \quad 1/\Box \quad \delta\mathcal{V} \qquad = \sum_{s,l} \boxed{c_{s,l}} (\phi\nabla^s\phi)\Box^l(\phi\nabla^s\phi)$$

Local improvements

With a local improvement at fixed spin we can balance any contact term

If the sums over spins and derivatives do **not converge**, we can find (fine tuning the improvements) local off-shell vertices such that:

$$S^{(4)} = 0$$

Are HS theories cubic theories like string theory? Maybe we should not expand in spin but work with **string fields** (or analogues)**!**

[C. Sleight & M.T '17]

Summary & Outlook

- We have reviewed the open problem of locality & HS both in flat and AdS spaces
- The problem arise at quartic order where sum over spins and derivatives are shown to have a finite radius of convergence (in the **strict** tensionless limit)
- How can we define HS interactions without invoking string theory (string fields)? **HS quantum effective actions**?
- Is there a prescription to deal with 1/□ interactions within Noether procedure?
- The problem is quite subtle as non-localities allow to go ``on-shell'' even ``off-shell''... While a on-shell action is a boundary term, a too non-local action can be written as a boundary term also off-shell...

$$\int d^dx[(\partial_\mu\Phi)(\partial^\mu\Phi)+\Phi J]=\int d^dx\,\Phi\underbrace{(-\Box\Phi+J)}_{\approx 0}+\int d^{d-1}x\;n^\mu(\Phi\partial_\mu\Phi)$$

$$\Phi\longrightarrow\Phi+\frac{1}{\Box}J$$