AdS/CFT in Fractional Dimension and Higher-Spins at One Loop

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May, 31st, 2017

General Motivation

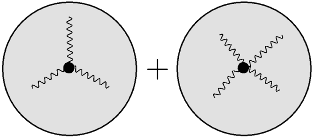
CFT's exist in non-integer dimensions: Famous $4-\epsilon$ expansion introduced by Wilson-Fisher allows one to make sense of critical vector model in fractional dimensions. Large-N expansion is defined for all d. Numerical bootstrap also shows that some observables are smooth.

Critical vector model should be dual to higher-spin theories. Do they make sense in $AdS_{d-\epsilon}$ or AdS_d for non-integer d? Can we perform some test of the duality in fractional dimension? (Klebanov, Polyakov).

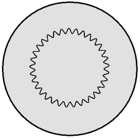
One-loop vacuum corrections have recently been computed on the both sides of the duality. We can make a general proof of these results and extend them to fractional dimensions.

Higher-Spin Theories

There are encouraging results on the action principle: (Kessel, Lucena-Gómez, E.S., Taronna; Bekaert, Ponomarev, Sleight, Erdmenger); in particular, the full cubic action is known in any d for the dual of Free Boson (Sleight, Taronna) and $0-0-0-0$ in $4d$ (Bekaert, Ponomarev, Sleight, Erdmenger)

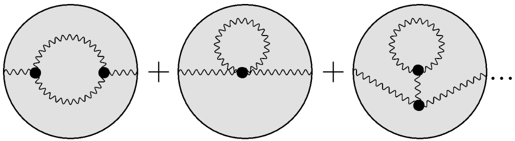


Vacuum one-loop corrections (Giombi, Klebanov, Safdi,Tseytlin and many others) can be computed if spectrum is known

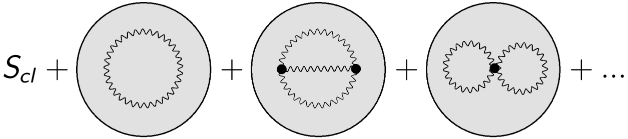


One-Loop

The part of the action known at present does not allow to compute enough of legged diagrams.



Neither do we know the classical action on AdS



But the one-loop determinant can be computed! (Giombi, Klebanov, Safdi, Tseytlin, Beccaria, Joung, Lal, Bekaert, Boulanger, Gunaydin, Tung, E.S., ...)

Free Energy AdS/CFT Expansion

On the both sides of the duality we expect

$$F_{AdS} = \frac{1}{G} F^0_{AdS} + F^1_{AdS} + \ldots$$
$$F_{CFT} = N F^0_{CFT} + F^1_{CFT} + \ldots$$

Free Energy AdS/CFT Expansion

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$$F_{AdS} = \frac{1}{G} F^0_{AdS} + F^1_{AdS} + \dots$$
$$F_{CFT} = N F^0_{CFT}$$

- On general grounds $G^{-1} \sim N$;
- The first term is not available in AdS;
- In free CFT's the second and others vanish identically;
- Generally we would expect to find boring $0 = 0$

$$F^1_{AdS} \sim \sum_s \log\det[\Box + m_s^2] = 0$$

- On contrary, in many cases $F^1_{AdS} \neq 0$ and is an integer multiplet of F^0_{CFT}, which naturally leads to (Giombi, Klebanov)

$$G^{-1} = a(N + \text{integer})$$

What Determinant Can Tell

One-loop determinant can be computed as

$$F = -\zeta(0)\log \Lambda l - \frac{1}{2}\zeta'(0)$$

Whenever the first term is non-zero, the finite part is ill-defined. Depending on the background, various information can be extracted.

Let's restrict to Euclidian AdS vs. free energy F on a sphere:

$$F^{even} = a \log R \qquad\qquad F^{odd} = \text{number}$$

In AdS there is also a volume divergence:

$$\mathrm{vol}\, AdS_{2n+1} \sim \log R \qquad\qquad \mathrm{vol}\, AdS_{2n+2} \sim \text{const}$$

Sum Rules

Instead of something like supermultiplet sum rules:

$$\sum_s (-)^{2s} d(s) s^p = 0$$

we have infinite sums that may run over bosonic fields only, so no usual SUSY cancellation is possible.

The recipe that was shown to work in many examples is to use (Hurwitz) zeta-function. For example, from Giombi, Klebanov:

$$\frac{1}{360} + \sum_s \left(\frac{1}{180} - \frac{s^2}{24} + \frac{5s^4}{24} \right) = 0$$

Spectral Zeta Function

Thanks to Camporesi and Higuchi, we can derive the zeta-function for arbitrary-spin field in Euclidian *AdS* space:

$$\zeta = \frac{\mathrm{vol}(\mathbb{H}^{d+1})}{\mathrm{vol}(S^d)} v_d g(s) \int_0^\infty d\lambda \, \frac{\mu(\lambda)}{\left[\frac{1}{4}(d-2\Delta)^2+\lambda^2\right]^z}$$

where $g(s)$ counts the number of field components and $\mu(\lambda)$ is a spectral density. In flat space $\mu \sim p^{d-1}$, but in AdS_{2k+1}:

$$d \text{ even}: \quad \mu^B(\lambda) = w_d \left(\left(\frac{d-2}{2}+s\right)^2+\lambda^2\right) \prod_{j=0}^{\frac{d-4}{2}} (j^2+\lambda^2)$$

$$d \text{ odd}: \quad \mu^B(\lambda) = w_d \lambda \tanh(\pi\lambda) \left(\left(\frac{d-2}{2}+s\right)^2+\lambda^2\right) \prod_{j=1/2}^{\frac{d-4}{2}} (j^2+\lambda^2)$$

Zeta for Romans $F(4)$ Multiplets

SUSY does help in the higher-spin case too. For example, in AdS_6 for bosonic spin-s field we find

$$\zeta(0) = -\frac{(s+1)^2(7s(s+2)(s(s+2)(9s(s+2)+13)+2)-20)}{30240}$$

If we sum over the Romans spin-s $F(4)$ multiplet we find

$$\zeta_{\text{Romans},s}(0) = -\frac{3}{8}s^4$$

The regulated sum is $3/8$, which is then cancelled by the $L(8|8)$ multiplet.

We also showed that fermionic HS fields $s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$, give exactly zero in all dimensions, which is necessary to make SUSY HS consistent.

Results in Integer Dimensions

Focusing on the simplest case Type-A vs. free/critical boson the results (Giombi, Klebanov, Safdi, Tseytlin) are (in a number of integer dimensions):

$\zeta(0) = \zeta'(0) = 0$ for the non-minimal model (all integer spins)

for the minimal model (all even spins) $\zeta(0) = 0$, but $\zeta'(0)$ gives a anomaly of one scalar for d even

$$a^4_\phi = \frac{1}{90}\,, \qquad a^6_\phi = -\frac{1}{756}\,, \qquad a^8_\phi = \frac{23}{113400}$$

and sphere free energy of one scalar for d odd

$$\begin{aligned} F^3_\phi &= \frac{1}{16}\left(2\log 2 - \frac{3\zeta(3)}{\pi^2}\right) \\ F^5_\phi &= \frac{-1}{2^8}\left(2\log 2 + \frac{2\zeta(3)}{\pi^2} - \frac{15\zeta(5)}{\pi^4}\right) \end{aligned}$$

Generalized Sphere Free Energy

It has been a long quest to find some measure of degrees of freedom in general QFT: monotonic along RG and stationary at fixed points.

$d = 2$ Zamolodchikov c-theorem

$d = 4$ a-theorem (Cardy; Komargodski, Schwimmer)

$d = 3$ what to do in odd dimensions? Sphere Free Energy (Myers et al; Jafferis et al; Klebanov et al; Casini, Huerta):

$$F = -\log Z_{S^d}$$

all the cases above a particular cases of the Generalized Sphere Free Energy (Klebanov, Pufu, Safdi; Giombi, Klebanov)

$$\tilde{F} = (-)^{(d-1)/2} \log Z_{S^d} \to \sin(\tfrac{\pi d}{2}) \log Z_{S^d}$$

in particular even dimensions are covered as well:
$\tilde{F} = (-1)^{d/2} \pi a/2$ and $c = -3a$ in $2d$.

Fractional Dimensions

On the CFT the generalized sphere free energy for free scalar (Diaz, Dorn; Giombi, Klebanov) can be computed for all d

$$\tilde{F}^{\phi} = \frac{1}{\Gamma(d+1)} \int_0^1 u \sin(\pi u) \Gamma\left(\frac{d}{2} + u\right) \Gamma\left(\frac{d}{2} - u\right) du\,.$$

We managed to reproduce this result as one-loop effect in HS theory, which also completes the proof for all integer dimensions.

By changing boundary conditions for the scalar field from $\Delta = d - 2$ to $\Delta = 2$ the same computation gives the change in the sphere free energy due to a double-trace deformation: free energy for the large-N critical vector model.

Fractional Dimensions

Zeta-function for a single field of spin-s and weight Δ

$$\zeta(z) \sim \frac{\mathrm{vol}(\mathbb{H}^{d+1})}{\mathrm{vol}(S^d)} g(s) \int_0^\infty d\lambda \, \frac{\tilde{\mu}(\lambda)}{\left[\lambda^2 + \left(\Delta - \frac{d}{2}\right)^2\right]^z}$$

There is a representation of the spectral density that works in all dimensions (Camporesi, Higuchi)

$$\tilde{\mu}(\lambda) = \left(\left(\frac{d-2}{2} + s\right)^2 + \lambda^2\right) \left| \frac{\Gamma\left(\frac{d-2}{2} + i\lambda\right)}{\Gamma(i\lambda)} \right|^2$$

It is clear that odd and non-integer dimensions are on equal footing and only even d stand apart (density is polynomial).

Main Tools

Technicalities involve three main ingredients. Laplace transform

$$\frac{1}{(\lambda^2+\nu^2)^z}=\frac{\sqrt{\pi}}{\Gamma(z)}\int_0^\infty d\beta\, e^{-\beta\nu}\left(\frac{\beta}{2\lambda}\right)^{z-\frac{1}{2}} J_{z-\frac{1}{2}}(\lambda\beta)\,.$$

see also Bae, Joung, Lal. Modified regularization

$$\lim_{z\to 0}\frac{\beta^{z-\frac{1}{2}}J_{z-\frac{1}{2}}(\beta\lambda)}{(2\lambda)^{z-\frac{1}{2}}}=\frac{2\cos(\beta\lambda)}{\sqrt{\pi}\beta}+\mathcal{O}(z)\,.$$

which leads to some deficit for $\zeta'(0)$, which can be shown to vanish for Type-A Bae, Joung, Lal

Intermediate Form

As a result one arrives at the expected $0 = 0$, but still nontrivial equalities

$$\zeta_{non-min}(0) = \zeta'_{non-min}(0) = \zeta_{min}(0)$$

The interesting case is of the minimal Type-A where we arrive at the intermediate form

$$\tilde{\zeta}'^{A}_{\text{min}}(0) = -\int_0^\infty d\beta \frac{e^{-\beta(2-d)}(1+e^{2\beta})^2}{\beta(e^{2\beta}-1)^d}$$

Exactly this intermediate form arises in the computation of the generalized sphere energy in any d, (Giombi, Klebanov). It has been already observed that (Giombi, Klebanov, Tseytlin) that certain intermediate AdS results match those on the CFT side and the match is found whenever the same regularization is applied

Critical Vector Model

Minimal Type-A theory with the scalar quantized with $\Delta = 2$ should be dual to critical $O(N)$ model. At large N the change of the sphere free energy can be computed (Giombi, Klebanov)

$$\delta F = F_{UV} - F_{IR}$$
$$= -\frac{1}{\sin\left(\frac{\pi d}{2}\right)\Gamma(d+1)}\int_0^{2-d/2} u\sin(\pi u)\Gamma\left(\frac{d}{2}-u\right)\Gamma\left(\frac{d}{2}+u\right)du$$

The AdS computation (only scalar needs to be taken into account) gives the intermediate form

$$\tilde{\zeta}'^{A}_{\text{min}}(0) = \int_0^\infty \frac{(1+e^\beta)\left(e^{2\beta}-e^{\beta(d-2)}\right)}{\beta(e^\beta-1)^{d+1}}$$

that evaluates to ΔF above.

Summary

Free and Critical $O(N)$ vector models make sense in non-integer dimensions. Therefore, higher spin theories and higher spin AdS/CFT should work in fractional dimensions as well. It is difficult to compute anything at present...

Tests of AdS/CFT in fractional dimension are performed: generalized sphere free energy of free/critical (Wilson-Fisher) boson is reproduced. The earlier results on one-loop determinants are extended to all integer dimension as a by-product.

It would be interesting to reconsider the Type-B (dual of free/critical fermion) puzzle: F is nice for d odd, but does not match free fermion

Other results that support fractional AdS/CFT: extremality of ϕ^3 in AdS_4 is properly compensated by the zer of the coupling $g \sim (d-3)$, Bekaert, Erdmenger, Ponomarev, Sleight