Radiation from a moving mirror and growing loop corrections

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Introduction

Consider the massless, two-dimensional, real scalar field theory with the following action:

$$\mathrm{S} = \int \mathrm{d}^2\mathrm{x} \left[\frac{1}{2}(\partial_\mu \phi)^2 - \frac{\lambda}{4!}\phi^4\right], \ \mu = 0, \ 1$$

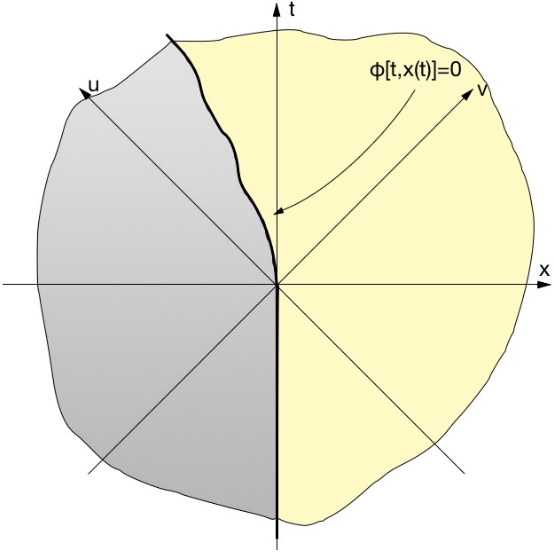
with the scalar field ϕ satisfying the null boundary condition:

$$\phi[\mathrm{t}, \ \mathrm{z}(\mathrm{t})] = 0,$$

where $\big(\mathrm{t}, \ \mathrm{z}(\mathrm{t})\big)$ is a time-like curve (world-line of the mirror).

QFT is considered in the region to the right hand side of the world-line.

Introduction



Introduction

The aim is to show that loop corrections to the Keldysh propagator grow with time in this theory, which can significantly change the vacuum expectation value of the energy-momentum tensor, calculated in the tree-level approximation.

Mode functions

Tree-level approximation ($\lambda = 0$).

The quantized field can be represented as follows:

$$\phi(\mathrm{u}, \mathrm{v}) = \int_0^\infty \frac{\mathrm{dk}}{2\pi} \left[\mathrm{h_k}(\mathrm{u}, \mathrm{v})\ \mathrm{a_k} + \mathrm{h.\,c.}\right],$$

where $\mathrm{a_k},\ \mathrm{a_k^\dagger}$ are creation and annihilation operators, satisfying the commutation relation $[\mathrm{a_k},\ \mathrm{a_{k'}^\dagger}] = 2\pi\delta(\mathrm{k} - \mathrm{k'})$.

Modes $\mathrm{h_k}(\mathrm{t}, \mathrm{x})$ solve Klein-Gordon equation and satisfy the boundary condition:

$$\mathrm{h_k}\big(\mathrm{t}, \mathrm{z}(\mathrm{t})\big) = 0,$$

where $\mathrm{z}(\mathrm{t})$ is a mirror world-line.

The canonical commutation relation

These mode functions can be calculated explicitly:

$$h_k(u,v) = \frac{i}{\sqrt{2k}}\left[e^{-ikv} - e^{-ik(2t_u - u)}\right],$$

where t_u is a solution of the equation: $t_u - z(t_u) = u$.

The normalization factor $i/\sqrt{2k}$ provides that the canonical commutation relation is satisfied:

$$[\phi(t,\ x), \partial_t \phi(t,y)] = i\delta(x-y).$$

Denote

$$2t_u - u \equiv p(u).$$

The energy flux (tree-level)

The calculation of the expectation value of $\mathrm{T}_{\mu\nu}$ is straightforward. The energy flux from the mirror is described by its tx-component:

$$\langle \mathrm{T}_{\mathrm{tx}} \rangle = \lim_{\varepsilon \to 0} \frac{1}{2} \langle \partial_{\mathrm{t}} \phi(\mathrm{t}, \mathrm{x}) \partial_{\mathrm{x}} \phi(\mathrm{t} + \mathrm{i}\varepsilon, \mathrm{x}) + \partial_{\mathrm{x}} \phi(\mathrm{t}, \mathrm{x}) \partial_{\mathrm{t}} \phi(\mathrm{t} + \mathrm{i}\varepsilon, \mathrm{x}) \rangle ,$$

Using this formula and the expression of ϕ through mode functions, we have:

$$\langle \mathrm{T}_{\mathrm{tx}} \rangle (\mathrm{u}) = \frac{1}{24\pi} \left[\frac{\mathrm{p}^{'''}}{\mathrm{p}'} - \frac{3}{2} \left(\frac{\mathrm{p}^{''}}{\mathrm{p}^{'}} \right)^2 \right],$$

where $\mathrm{p}(\mathrm{u}) = 2\mathrm{t}_{\mathrm{u}} - \mathrm{u}$ and $\mathrm{p}'(\mathrm{u}) = \dfrac{\mathrm{dp}}{\mathrm{du}}$.

The energy flux (tree-level)

The previous result can be expressed in terms of mirror velocity:

$$\langle \mathrm{T_{tx}} \rangle (\mathrm{u}) = \frac{1}{24\pi}\left[\frac{\mathrm{p}'''}{\mathrm{p}'} - \frac{3}{2}\left(\frac{\mathrm{p}''}{\mathrm{p}'}\right)^2\right] = -\frac{1}{12\pi}\,\frac{(1+\mathrm{v})^{1/2}}{(1-\mathrm{v})^{3/2}}\,\frac{\mathrm{d}}{\mathrm{dt}}\,\frac{\dot{\mathrm{v}}}{(1-\mathrm{v}^2)^{3/2}}\,\bigg|_{\mathrm{t}=\mathrm{t_u}}.$$

Notice that $\dfrac{\dot{\mathrm{v}}}{(1-\mathrm{v}^2)^{3/2}}$ is the mirror acceleration in its instantaneous rest frame.

Loop corrections

$$\lambda \neq 0$$

I will demonstrate the behaviour of loop corrections to the Keldysh propagator, which is the following quantity:

$$\mathrm{G}^{\mathrm{K}}_{\mathrm{xy}} = \frac{1}{2} \langle \{ \phi(\mathrm{x}), \phi(\mathrm{y}) \} \rangle \, .$$

It relates to the 01-component of the energy-momentum tensor in the following way:

$$\langle \mathrm{T}_{01} \rangle = \lim_{\mathrm{x} \to \mathrm{y}} \partial_{\mathrm{x}^0} \partial_{\mathrm{y}^1} \mathrm{G}^{\mathrm{K}}_{\mathrm{xy}} .$$

Loop corrections to the Keldysh propagator

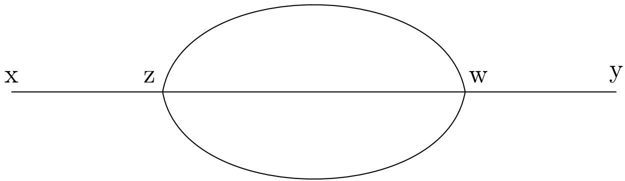
The exact Keldysh propagator can be represented in the following way, when $x^0 = y^0$:

$$G^K_{xy}\big|_{x^0=y^0} = G^{K(0)}_{xy}\big|_{x^0=y^0} + \\ + \iint \frac{dk}{2\pi}\frac{dk'}{2\pi}\left[n_{kk'}\bar{h}_k(x)h_{k'}(y) + \kappa_{kk'}h_k(x)h_{k'}(y) + \text{h. c.}\right],$$

where $G^{K(0)}$ is a tree-level Keldysh propagator, $n_{kk'} = \langle a_k^\dagger a_{k'}\rangle$ are occupation numbers, and $\kappa_{kk'} = \langle a_k a_{k'}\rangle$ is an anomalous quantum average.

Two-loop corrections

We consider corrections to $\mathrm{n}_{\mathrm{kk}'}$ and $\kappa_{\mathrm{kk}'}$, coming from the following two-loop diagram in the limit when $\frac{\mathrm{x}^0 + \mathrm{y}^0}{2} \equiv \mathrm{T} \to \infty$ while $|\mathrm{x}^0 - \mathrm{y}^0|$ is kept fixed:



The corrections depend on the mirror world-line.

Loop corrections for different world-lines

Consider corrections to $n_{kk'}$ for the following world-line:

$$x(t) = \begin{cases} 0 & t < 0 \\ -\beta t + a(1 - e^{-\beta t/a}) & t \geq 0, \end{cases}$$

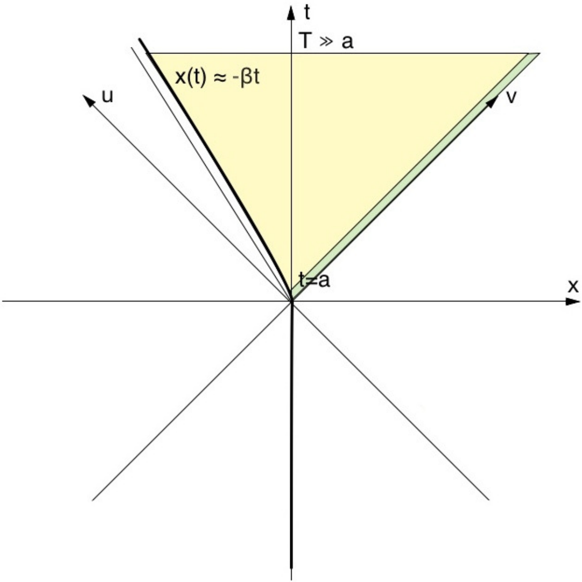
when the mirror eternally approaches to the line $(t, -\beta t + a)$ as t goes to ∞.

The correction to $n_{kk'}$ is as follows:

$$\Delta n_{kk'} \propto \lambda^2 T^2 \int \prod_{j=1}^{3} \frac{dp_j}{2\pi} \frac{1}{p_1 p_2 p_3} \frac{1}{2^4 \sqrt{kk'}} \left[\mathrm{v.p.} \frac{1}{k + p_1 + p_2 + p_3} \right] \times \left[\mathrm{v.p.} \frac{1}{k' + p_1 + p_2 + p_3} \right] + O(T).$$

The correction to $\kappa_{kk'}$ is also proportional to $\lambda^2 T^2$.

Loop corrections for different world-lines



The mirror, approaching the speed of light

Consider the world-line, which approaches the light-like line as t goes to infinity:

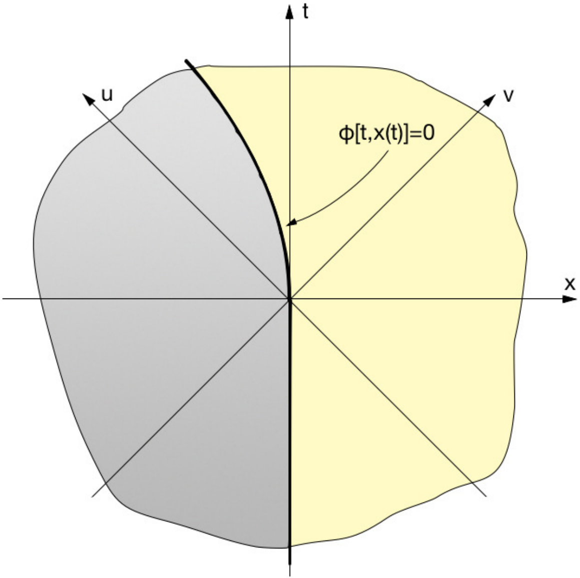
$$z(t) = \begin{cases} 0 & t < 0 \\ -t + a(1 - e^{-t/a}) & t \geq 0, \end{cases}$$

The correction to $n_{kk'}$ is proportional to $\lambda^2 T^4$:

$$n_{kk'} \propto \lambda^2 T^4 \, \frac{e^{-i(k-k')a}}{\sqrt{kk'}} \cdot \int \prod_{j=1}^{3} \frac{dp_j}{2\pi} \frac{1}{p_1 p_2 p_3} + O(T^3).$$

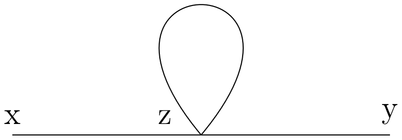
The correction to $\kappa_{kk'}$ is also proportional to $\lambda^2 T^4$.

The mirror, approaching the speed of light



One-loop corrections

Consider also one-loop corrections to $\mathrm{n}_{\mathrm{kk}'}$ and $\kappa_{\mathrm{kk}'}$, coming from the following diagram:



One-loop corrections

$$\Delta \mathrm{n_{kk'}} \approx 0.$$

For the first world-line:

$$\Delta \kappa_{\mathrm{kk'}} \propto -\lambda\ \mathrm{T} \int \frac{\mathrm{dp}}{2\pi} \frac{1}{\sqrt{2^4 \cdot \mathrm{kk'p^2}}} \cdot \Bigg[-\,\mathrm{v.\,p.}\,\frac{\mathrm{i}}{\mathrm{k}+\mathrm{k'}} + \\ + \frac{3-\beta}{1-\beta}\pi\delta(\mathrm{k}+\mathrm{k'}) \Bigg] + \mathrm{O}(1)$$

For the second world-line:

$$\Delta \kappa_{\mathrm{kk'}} \propto \lambda \mathrm{T}^2 \mathrm{e}^{\mathrm{i(k+k')a}} \int \frac{\mathrm{dp}}{2\pi} \frac{1}{\sqrt{2^4 \cdot \mathrm{kk'p^2}}} + \mathrm{O(T)}$$

Conclusions

In the case of $\lambda\phi^4$ theory, loop corrections to the Keldysh propagator, which is closely connected to the energy flux, are not suppressed given a sufficiently long time – in other words, perturbation theory breaks down.

However, in order to make a definitive conclusion about this effect it is necessary to consider the resummation of the leading corrections from all loops.

References

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