Local current interactions from nonlinear higher-spin equations in the one-form sector

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HS AdS/CFT correspondence

General idea of HS duality Sundborg (2001), Witten (2001)

AdS_4 HS theory is dual to $3d$ vectorial conformal models

Klebanov, Polyakov (2002), Petkou, Leigh (2005), Sezgin, Sundell (2005); Giombi and Yin (2009);

Maldacena, Zhiboedov (2011,2012); MV (2012); Koch, Jevicki, Jin, Rodrigues (2011-2014);

Giombi, Klebanov; Tseytlin (2013,2014) ...

AdS_3/CFT_2 correspondence Gaberdiel and Gopakumar (2010)

Analysis of HS holography helps to uncover the origin of AdS/CFT ?!

Despite significant progress in the construction of actions during last thirty years: A.Bengtsson, I.Bengtsson, Brink (1983); Berends, Burgers, van Dam (1984); Fradkin, MV (1987), ... Boulanger, Sundell (2012) ...

construction of the generating functional for correlators and entropies was lacking

Symmetries

The system is consistent because $\mathcal{B}$ commutes with itself and with all c and $\mathcal{L}$. The gauge transformations are

$$\delta\mathcal{W} = [\mathcal{W}\,,\varepsilon]_* \;, \qquad \delta\mathcal{B} = [\mathcal{B}\,,\varepsilon]_*\,, \qquad \varepsilon = \varepsilon(dx, x, dZ, \ldots)$$

$$\delta\mathcal{B} = \{\mathcal{W}\,,\xi\}\,, \qquad \delta\mathcal{W} = \xi^A \frac{\partial F(c,\mathcal{B})}{\partial \mathcal{B}^A}\,, \qquad \xi = \xi(dx, x, dZ, \ldots)$$

$$\delta\mathcal{L} = d\chi\,, \qquad \delta\mathcal{W} = \chi I\,, \qquad \chi(dx, x)$$

χ- transformation implies equivalence up to exact forms

allowing to choose canonical gauge $\mathcal{W}_I := \pi\mathcal{W} = 0$

π is the projection to I

$$\pi(f(Y,Z|x)) = f(0,0|x)\,, \qquad \pi(f \star g) \neq \pi(g \star f)$$

Gauge transformation preserving canonical gauge

$$\delta\mathcal{L} = \mathrm{d}\chi\,, \qquad \chi = -\pi\Big([\mathcal{W}\,,\varepsilon]_* + \xi^A \frac{\partial F(c,\mathcal{B})}{\partial \mathcal{B}^A}\Big)$$

$\mathcal{L}$ is on-shell closed and gauge invariant modulo exact forms

Higher derivatives in HS interactions

HS interactions contain higher derivatives Bengtsson, Bengtsson, Brink (1983)

Nonanaliticity in Λ via dimensionless combination $\Lambda^{-\frac{1}{2}}\frac{\partial}{\partial x}$ (Fradkin, MV 1987)

By a seemingly local field redefinition it is possible to get rid of currents from HS field equations including the stress tensor (Prokushkin, MV 1998)

$$\phi \to \phi' = \phi + \sum_n a_{nm}(\rho D)^n \phi\,(\rho D)^m \phi + \ldots\,,$$

ρ is the AdS radius, D is the space-time covariant derivative.

The problem: find restrictions on a_{nm} distinguishing between truly non-local and generalized local field redefinitions containing an infinite number of terms but a_{nm} decreasing fast enough with n and m.

Specific models and examples are helpful

Nonlinear HS equations

$$\mathcal{W}(Z;Y;k,\bar{k}|x) = (\mathsf{d} + W) + S\,, \qquad W = dx^n W_n\,, \qquad S = \theta^\alpha S_\alpha + \bar{\theta}^{\dot\alpha}\bar{S}_{\dot\alpha} \qquad \mathbf{1992}$$

$$\mathcal{W} \star \mathcal{W} = i(\theta^A\theta_A + \eta\theta^\alpha\theta_\alpha B \star k \star \kappa + \bar\eta\bar\theta^{\dot\alpha}\bar\theta_{\dot\alpha} B \star \bar{k} \star \bar\kappa)$$

$$\mathcal{W} \star B = B \star \mathcal{W}\ , \qquad B = B(Z;Y;k,\bar{k}|x)$$

HS star product

$$(f \star g)(Z;Y) = \frac{1}{(2\pi)^4}\int d^4U\, d^4V \exp\left[iU_A V^A\right] f(Z+U;Y+U)g(Z-V;Y+V)$$

$$\kappa = \exp iz_\alpha y^\alpha\,, \qquad \bar\kappa = \exp i\bar{z}_{\dot\alpha}\bar{y}^{\dot\alpha}$$

Massless fields

$$\mathcal{W}(Z;Y;k,\bar{k}|x) = \mathcal{W}(Z;Y;-k,-\bar{k}|x)\ , \qquad B(Z;Y;k,\bar{k}|x) = -B(Z;Y;-k,-\bar{k}|x)$$

Fields and Currents

Spin s is described by the 1-forms $\omega(y,\bar{y}|x)$ and 0-form $C(y,\bar{y}|x)$ obeying

$$\omega(\mu y,\mu\bar{y}\mid x)=\mu^{2(s-1)}\omega(y,\bar{y}\mid x)\,,\qquad C(\mu y,\mu^{-1}\bar{y}\mid x)=\mu^{\pm 2s}C(y,\bar{y}\mid x)$$

Generalized Weyl tensors $C(y,0|x)$ and $C(0,\bar{y}|x)$ describe gauge invariant combinations of derivatives of the gauge fields of spins $s\geq 1$ and matter fields of spins $s=0,1/2$

$C(y,0|x)$ and $C(0,\bar{y}|x)$ are primaries of the Weyl module formed by $C(y,\bar{y}|x)$

Higher powers in y and $\bar{y}$ for a given spin contain higher derivatives

Conserved currents $J(Y_1,Y_2|x)$ are associated with the bilinears of $C(Y|x)$

$$\mathcal{J}(Y_1,Y_2|x):=C(Y_1|x)\tilde{C}(Y_2|x)\,,\qquad \tilde{C}(y,\bar{y}|x)=C(-y,\bar{y}|x)\,.$$

As a consequence of the rank-one equation for $C(Y|x)$, the current $\mathcal{J}(Y_1,Y_2|x)$ obeys the rank-two equation Gelfond, MV (2003)

$$\tilde{D}_2\mathcal{J}(Y_1,Y_2|x)=0\,,\qquad \tilde{D}_2:=D^L-i\lambda h^{\alpha\dot{\beta}}\Big(y_{1\alpha}\bar{y}_{1\dot{\beta}}-y_{2\alpha}\bar{y}_{2\dot{\beta}}-\frac{\partial^2}{\partial y_1^{\alpha}\partial\bar{y}_1{}^{\dot{\beta}}}+\frac{\partial^2}{\partial y_2^{\alpha}\partial\bar{y}_2{}^{\dot{\beta}}}\Big)$$

Current deformation

Current deformation can be formulated as a linear system

$$D\omega + L(w,C) + \Gamma_{cur}(w,\mathcal{J}) = 0\,,$$

$$\tilde{D}C + \mathcal{H}_{cur}(w,\mathcal{J}) = 0\,, \qquad \tilde{D}_2\mathcal{J}(Y_1,Y_2|x) = 0$$

$$L(w,C) := i\Big(\eta\overline{H}^{\dot\alpha\dot\beta}\frac{\partial^2}{\partial\overline{y}^{\dot\alpha}\partial\overline{y}^{\dot\beta}}\;\bar{C}(0,\overline{y}|x) + \bar\eta H^{\alpha\beta}\frac{\partial^2}{\partial y^{\alpha}\partial y^{\beta}}\;C(y,0|x)\Big)$$

Linear functionals Γ and $\mathcal{H}$ should obey the compatibility conditions

The freedom in $\Gamma_{cur}(w,\mathcal{J})$ and $\mathcal{H}_{cur}(w,\mathcal{J})$ results from field redefinitions

$$\omega \to \omega' = \omega + \Omega(w,\mathcal{J})\,, \qquad C \to C' = C + \Phi(\mathcal{J})\,.$$

Nontrivial $\Gamma_{cur}(w,\mathcal{J})$ and $\mathcal{H}_{cur}(w,\mathcal{J})$ cannot be removed by a field redefinition. Usual current interactions are nontrivial.

Locality in the twistor variables

Technically, locality is due to the absence of integration over s and t.

$$\int \frac{dsdt}{(2\pi)^2} \exp i[s_\beta t^\beta] f(y+s,\bar{y}) g(y+t,\bar{y}) = f(y,\bar{y}) \exp[-i \overleftarrow{\partial}_\alpha \overrightarrow{\partial}_\beta \epsilon^{\alpha\beta}] g(y,\bar{y})$$

$$\int \frac{d\bar{s}d\bar{t}}{(2\pi)^2} \exp i[\bar{s}_{\dot\beta} \bar{t}^{\dot\beta}] f(y,\bar{y}+\bar{s}) g(y,\bar{y}+\bar{t}) = f(y,\bar{y}) \exp[-i \overleftarrow{\partial}_{\dot\alpha} \overrightarrow{\partial}_{\dot\beta} \epsilon^{\dot\alpha\dot\beta}] g(y,\bar{y})$$

For given helicities carried by g and f, only a single term in the sum contributes hence containing a finite number of derivatives.

When both integrations are present, the number of derivatives in y and $\bar{y}$ can be infinitely increased without affecting the helicities carried by g and f, implying appearance of infinite tails of derivatives and hence nonlocality.

Expressions like

$$X(\mathcal{J}) = \int d^3\bar{\tau} d^3\tau\, X(\tau,\bar{\tau}) \exp\left(\tau_3 \partial_{1\alpha} \partial_2{}^\alpha + \bar{\tau}_3 \bar{\partial}_{1\dot\alpha} \bar{\partial}_2{}^{\dot\alpha}\right) \mathcal{J}(\tau_1 y, -\tau_2 y; \bar{\tau}_1 \bar{y}, -\bar{\tau}_2 \bar{y}; K)$$

are local once they are $\tau_3, \bar{\tau}_3$-local being proportional to $\delta(\tau_3)$ and $\delta(\bar{\tau}_3)$ or their derivatives

Current deformation from nonlinear equations

In the 0-form sector the deformation is

$$D_0 C + [\omega\,, C]_* + \mathcal{H}(w, \mathcal{J}) = 0\,,$$

$$\mathcal{J}(y_1, y_2; \bar{y}_1, \bar{y}_2; K|x) = C(y_1, \bar{y}_1; k, \bar{k}|x) C(y_2, \bar{y}_2; k, \bar{k}|x)$$

A simple computation using the new technique Didenko, Misuna, MV 2015

$$\mathcal{H}(w, \mathcal{J}) = \mathcal{H}_\eta(w, \mathcal{J}) + \mathcal{H}_{\bar{\eta}}(w, \mathcal{J})\,,$$

$$\begin{aligned}\mathcal{H}_\eta(w, \mathcal{J}) = & -\frac{i}{2}\eta \int \frac{dSdT}{(2\pi)^4} \exp iS_A T^A \int_0^1 d\tau \\ & [h(s, \tau\bar{y} - (1-\tau)\bar{t})\mathcal{J}(\tau s, -(1-\tau)y + t; \bar{y} + \bar{s}, \bar{y} + \bar{t}; k, \bar{k}) \\ & -h(t, \tau\bar{y} - (1-\tau)\bar{s})\mathcal{J}((1-\tau)y + s, \tau t, \bar{y} + \bar{s}; \bar{y} + \bar{t}; k, \bar{k})] * k\end{aligned}$$

This deformation is not local, containing integrations overboth s, t and $\bar{s}, \bar{t}$.

The two terms result from the commutator $[\mathcal{W}, B]_*$

Field redefinition

Standard current interactions in the 0-form sector result from the field redefinition

$$C \to C'(Y;k,\bar{k}|x) = C(Y;k,\bar{k}|x) + \Phi(Y;k,\bar{k}|x)$$

$$\Phi_\eta(Y;k,\bar{k}|x) = \quad \eta \int \frac{dSdT}{(2\pi)^4} \exp iS_A T^A \int d\tau_i \prod_{i=1}^{3} \theta(\tau_i)\delta'\Big(1 - \sum_{i=1}^{3}\tau_i\Big)$$
$$\mathcal{J}(\tau_3 s + \tau_1 y, t - \tau_2 y; \bar{y} + \bar{s}, \bar{y} + \bar{t}; k, \bar{k}) * k\,,$$

which gives

$$\mathcal{H}_\eta(w,\mathcal{J}) = \mathcal{H}_{\eta\, cur}(w,\mathcal{J}) + D_0(\Phi_\eta)$$

$$\mathcal{H}_{\eta\, cur}(w,\mathcal{J}) = \frac{\eta}{4}\int \frac{d\bar{s}d\bar{t}}{(2\pi)^2} \exp i[\bar{s}_{\dot\beta}\bar{t}^{\dot\beta}] \int_0^1 d\tau h(y, \tau\bar{s} + (1-\tau)\bar{t})$$
$$\mathcal{J}(\tau y, (\tau - 1)y; \bar{y} + \bar{s}, \bar{y} + \bar{t}; k, \bar{k}) * k\,.$$

This expression is local since it contains only integration over $\bar{s}$ and $\bar{t}$

An important consequence of the analysis in the 0-form sector was that the current coupling constants are proportional to $\eta\bar{\eta}$.

One-form sector

Bilinear corrections to the field equations that follow from the nonlinear corrections contain terms $R^{\eta\eta}$, $R^{\bar\eta\bar\eta}$ and $R^{\eta\bar\eta}$ proportional to η^2, $\bar\eta^2$ and $\eta\bar\eta$, resp. As will be explained in the talk of Gelfond the terms $R^{\eta\eta}$ and $R^{\bar\eta\bar\eta}$ can be completely removed by a field redefinition. The remaining terms are proportional to $\eta\bar\eta$, having the form

$$
\begin{aligned}
R_1^{\eta\bar\eta} =& -\frac{i}{2^3}\eta\bar\eta\int dSdT \exp iS_A T^A \int d^3\bar\tau d^3\tau \prod_{i=1}^{3}\theta(\bar\tau_i)\theta(\tau_i)\delta\Big(1-\sum_{i=1}^{3}\tau_i\Big)\delta\Big(1-\sum_{i=1}^{3}\bar\tau_i\Big)\\
&\Big\{\delta(\tau_1)\delta(\tau_2)\mathbf{h}_\alpha{}^{\dot\alpha}\mathbf{h}^{\alpha\dot\beta}\frac{\partial^2}{\partial\bar{\mathbf{y}}^{\dot\alpha}\partial\bar{\mathbf{y}}^{\dot\beta}}\frac{\partial}{\partial\bar\tau_3} + \delta(\bar\tau_1)\delta(\bar\tau_2)\mathbf{h}^{\alpha}{}_{\dot\alpha}\mathbf{h}^{\beta\dot\alpha}\frac{\partial^2}{\partial\mathbf{y}^{\alpha}\partial\mathbf{y}^{\beta}}\frac{\partial}{\partial\tau_3}\Big\}\\
&\mathcal{J}(\tau_3 s+\tau_1 y, t-\tau_2 y; \bar\tau_3\bar s+\bar\tau_1\bar y, \bar t-\bar\tau_2\bar y; K)\ .
\end{aligned}
$$

The blue and green terms are linearly independent and obey the compatibility conditions separately!

The naive idea to bring each of them to the local form does not work however. To solve the problem it is important to have both of them with equal coefficients resulting from the nonlinear HS equations.

Field redefinition

Let

$$\omega \to \omega' = \omega + X(\mathcal{J})$$

$$X(\mathcal{J}) = \int d^3\bar{\tau} d^3\tau \, h^{\alpha\dot{\beta}} X_{\alpha\dot{\beta}} \exp\left(\tau_3 \partial_{1\alpha} \partial_2{}^{\alpha} + \bar{\tau}_3 \bar{\partial}_{1\dot{\alpha}} \bar{\partial}_2{}^{\dot{\alpha}}\right) \mathcal{J}(\tau_1 y, -\tau_2 y; \bar{\tau}_1 \bar{y}, -\bar{\tau}_2 \bar{y}; K)$$

where

$$X_{\alpha\dot{\beta}} = a(\tau,\bar{\tau}) y_\alpha \bar{y}_{\dot{\beta}} + y_\alpha \sum_i \bar{b}_i(\tau,\bar{\tau}) \bar{\partial}_{i\dot{\beta}} + \sum_i b_i(\tau,\bar{\tau}) \partial_{i\alpha} \bar{y}_{\dot{\beta}} + \sum_{i,j} g_{i\,j}(\tau,\bar{\tau}) \partial_{i\alpha} \bar{\partial}_{j\,\dot{\beta}}$$

with some coefficients $a(\tau,\bar{\tau}), b_i(\tau,\bar{\tau}), \bar{b}_i(\tau,\bar{\tau}), g_{i\,j}(\tau,\bar{\tau})$ to be determined from the condition that

$$R'^{\eta\bar{\eta}} = R^{\eta\bar{\eta}} + DX(\mathcal{J})$$

be local

Fierz-Schoutens identities

That antisymmetrization over any three two-component indices gives zero implies

$$y_\alpha \partial_{1\beta}\partial_2^\beta + \partial_{1\alpha}\partial_{2\beta}y^\beta + \partial_{2\alpha}y_\beta\partial_1^\beta = 0$$

This has a consequence

$$\left(iy_\alpha\frac{\partial}{\partial\tau_3} + \partial_{1\alpha}\frac{\partial}{\partial\tau_2} + \partial_{2\alpha}\frac{\partial}{\partial\tau_1}\right)\exp\left(\tau_3\partial_{1\alpha}\partial_2{}^\alpha\right)\mathcal{J}(\tau_1 y, -\tau_2 y; \bar\tau_1\bar y, -\bar\tau_2\bar y; K) = 0\,.$$

Analogously

$$\left(i\bar y_{\dot\alpha}\frac{\partial}{\partial\bar\tau_3} + \bar\partial_{1\dot\alpha}\frac{\partial}{\partial\bar\tau_2} + \bar\partial_{2\dot\alpha}\frac{\partial}{\partial\bar\tau_1}\right)\exp\left(\bar\tau_3\bar\partial_{1\dot\alpha}\bar\partial_2{}^{\dot\alpha}\right)\mathcal{J}(\tau_1 y, -\tau_2 y; \bar\tau_1\bar y, -\bar\tau_2\bar y; K) = 0\,.$$

These expressions can be added with arbitrary coefficients

$$\mathcal{F} = \frac{1}{2}h_\mu{}^{\dot\alpha}h^{\mu\dot\beta}\left(\alpha\bar y_{\dot\beta} + \beta_1\bar\partial_{1\dot\beta} + \beta_2\bar\partial_{2\dot\beta}\right)\left(i\bar y_{\dot\alpha}\frac{\partial}{\partial\bar\tau_3} + \bar\partial_{1\dot\alpha}\frac{\partial}{\partial\bar\tau_2} + \bar\partial_{2\dot\alpha}\frac{\partial}{\partial\bar\tau_1}\right)$$

Ansatz

In the sector of $\bar{H}^{\dot{\alpha}\dot{\beta}} = -h_\alpha{}^{\dot{\alpha}} h^{\alpha\dot{\beta}}$ this gives

$$
\begin{aligned}
\Big(D_{ad}X + \mathcal{F}\Big)\Big|_{\bar{H}} &= -\frac{1}{2}\bar{H}^{\dot{\alpha}\dot{\beta}} \int d^3\bar{\tau} d^3\tau \\
\Big\{ \Big[B\bar{y}_{\dot{\alpha}} + F_j \bar{\partial}_{j\dot{\alpha}}\Big] &\Big[(i\tau_1\bar{\tau}_1 - i\tau_2\bar{\tau}_2)\bar{y}_{\dot{\beta}} + (\bar{\tau}_1 + \tau_2\bar{\tau}_3)\bar{\partial}_{1\dot{\beta}} - (\bar{\tau}_2 + \tau_1\bar{\tau}_3)\bar{\partial}_{2\dot{\beta}}\Big] \\
&+ i\Big[A_1\bar{y}_{\dot{\alpha}} + G_{1j}\bar{\partial}_{j\dot{\alpha}}\Big]\Big[(\tau_1 + \tau_3\bar{\tau}_2)\bar{y}_{\dot{\beta}} + i(\tau_3\bar{\tau}_3 - 1)\bar{\partial}_{1\dot{\beta}}\Big] \\
&+ i\Big[A_2\bar{y}_{\dot{\alpha}} + G_{2j}\bar{\partial}_{j\dot{\alpha}}\Big]\Big[(\tau_2 + \tau_3\bar{\tau}_1)\bar{y}_{\dot{\beta}} + i(\tau_3\bar{\tau}_3 - 1)\bar{\partial}_{2\dot{\beta}}\Big] \\
&- \Big(i\bar{y}_{\dot{\alpha}}\bar{\nabla}_3 + \bar{\partial}_{1\dot{\alpha}}\bar{\nabla}_2 + \bar{\partial}_{2\dot{\alpha}}\bar{\nabla}_1\Big)\Big(\alpha\bar{y}_{\dot{\beta}} + \beta_1\bar{\partial}_{1\dot{\beta}} + \beta_2\bar{\partial}_{2\dot{\beta}}\Big)\Big\} \\
&\exp\Big(\tau_3\partial_{1\alpha}\partial_2{}^{\alpha} + \bar{\tau}_3\bar{\partial}_{1\dot{\alpha}}\bar{\partial}_2{}^{\dot{\alpha}}\Big)\mathcal{J}(\tau_1 y, -\tau_2 y; \bar{\tau}_1\bar{y}, -\bar{\tau}_2\bar{y}; K)\,,
\end{aligned}
$$

where

$$
\nabla_j := \frac{\partial}{\partial\tau_j}\,, \qquad \bar{\nabla}_j := \frac{\partial}{\partial\bar{\tau}_j}
$$

$$
A_j = i\nabla_j a + \nabla_3\tilde{b}_j\,, \qquad B = \nabla_1 b_1 - \nabla_2 b_2\,, \qquad \tilde{b}_2 = b_1\,, \qquad \tilde{b}_1 = b_2
$$

$$
G_{kj} = \nabla_3\tilde{g}_{kj} + i\nabla_k\bar{b}_j\,, \qquad F_j = \nabla_1 g_{1j} - \nabla_2 g_{2j}
$$

$$
\tilde{g}_{2j} = g_{1j}\,, \qquad \tilde{g}_{1j} = g_{2j}
$$

Solution

Using notations

$$x = \left(1 - \sum_{i=1}^{3} \tau_i\right), \qquad \bar{x} = \left(1 - \sum_{i=1}^{3} \bar{\tau}_i\right).$$

$$\theta^6(\tau) = \theta(\tau_1)\theta(\tau_2)\theta(\tau_3)\theta(\bar{\tau}_1)\theta(\bar{\tau}_2)\theta(\bar{\tau}_3)$$

an appropriate solution is

Fierz coefficients:

$$\begin{aligned}
\alpha &= i\frac{1}{2}[\bar{\tau}_1(\tau_1 + \tau_3\bar{\tau}_2)\delta(\tau_2) + \bar{\tau}_2(\tau_2 + \tau_3\bar{\tau}_1)\delta(\tau_1)]\delta'(x)\delta(\bar{x})\theta^6(\tau) \\
\beta_1 &= \frac{1}{2}[\bar{\tau}_1(\bar{\tau}_1 - \bar{\tau}_2)\delta(\tau_1)\delta(\tau_2)\delta(x)\delta(\bar{x}) + (1 - \bar{\tau}_3\tau_3)\bar{\tau}_1\delta(\tau_2)\delta'(x)\delta(\bar{x})]\theta^6(\tau) \\
\beta_2 &= \frac{1}{2}[\bar{\tau}_2(\bar{\tau}_2 - \bar{\tau}_1)\delta(\tau_1)\delta(\tau_2)\delta(x)\delta(\bar{x}) + (1 - \bar{\tau}_3\tau_3)\bar{\tau}_2\delta(\tau_1)\delta'(x)\delta(\bar{x})]\theta^6(\tau)
\end{aligned}$$

Field redefinition

$$
\begin{aligned}
a &= -\frac{1}{2}\Big\{\delta(\tau_3)\Big[(\bar\tau_1+\tau_2\bar\tau_3)\tau_1\delta(\bar\tau_2)+(\bar\tau_2+\tau_1\bar\tau_3)\tau_2\delta(\bar\tau_1)\Big] \\
&\qquad +\delta(\bar\tau_3)\Big[(\tau_1+\tau_3\bar\tau_2)\bar\tau_1\delta(\tau_2)+(\tau_2+\tau_3\bar\tau_1)\bar\tau_2\delta(\tau_1)\Big]\Big\}\delta(x)\delta(\bar x)\theta^6(\tau)\,, \\
b_1 &= i\frac{1}{2}(\tau_1+\tau_3\bar\tau_2)\delta(\tau_2)\delta(x)\delta'(\bar x)\theta^6(\tau)\,, \\
b_2 &= i\frac{1}{2}(\tau_2+\tau_3\bar\tau_1)\delta(\tau_1)\delta(x)\delta'(\bar x)\theta^6(\tau)\,, \\
\bar b_1 &= i\frac{1}{2}(\bar\tau_1+\bar\tau_3\tau_2)\delta(\bar\tau_2)\delta'(x)\delta(\bar x)\theta^6(\tau)\,, \\
\bar b_2 &= i\frac{1}{2}(\bar\tau_2+\bar\tau_3\tau_1)\delta(\bar\tau_1)\delta'(x)\delta(\bar x)\theta^6(\tau)\,, \\
g_{12} &= -(1-\tau_3\bar\tau_3)\delta(\tau_2)\delta(\bar\tau_1)\delta(x)\delta(\bar x)\theta^6(\tau)\,, \\
g_{21} &= -(1-\tau_3\bar\tau_3)\delta(\tau_1)\delta(\bar\tau_2)\delta(x)\delta(\bar x)\theta^6(\tau)\,, \\
g_{11} &= \frac{1}{2}(1-\tau_3\bar\tau_3)\Big[\delta(\bar\tau_2)\delta'(x)\delta(\bar x)+\delta(\tau_2)\delta(x)\delta'(\bar x)-\delta(\tau_2)\delta(\bar\tau_2)\delta(x)\delta(\bar x)\Big]\theta^6(\tau)\,, \\
g_{22} &= \frac{1}{2}(1-\tau_3\bar\tau_3)\Big[\delta(\bar\tau_1)\delta'(x)\delta(\bar x)+\delta(\tau_1)\delta(x)\delta'(\bar x)-\delta(\tau_1)\delta(\bar\tau_1)\delta(x)\delta(\bar x)\Big]\theta^6(\tau)\,.
\end{aligned}
$$

Final result

$$R'^{\eta\bar\eta}\Big|_{\bar H} = \frac{1}{2}\eta\bar\eta \bar H^{\dot\alpha\dot\beta}\int d^3\bar\tau d^3\tau \Big(U^{ij}(\tau,\bar\tau)\bar\partial_{i\dot\alpha}\bar\partial_{j\dot\beta} + V^i(\tau,\bar\tau)\bar\partial_{i\dot\alpha}\bar y_{\dot\beta} + P(\tau,\bar\tau)\bar y_{\dot\alpha}\bar y_{\dot\beta}\Big)$$
$$\exp\left(\tau_3\partial_{1\alpha}\partial_2{}^{\alpha} + \bar\tau_3\bar\partial_{1\dot\alpha}\bar\partial_2{}^{\dot\alpha}\right)\mathcal{J}(\tau_1 y, -\tau_2 y; \bar\tau_1\bar y, -\bar\tau_2\bar y; K)\,,$$

where

$$U^{11} = -\delta(\tau_3)\delta(\tau_1)\delta(\bar\tau_2)\delta(x)\delta(\bar x)\,, \qquad U^{22} = -\delta(\tau_3)\delta(\tau_2)\delta(\bar\tau_1)\delta(x)\delta(\bar x)$$

$$U^{12} = U^{21} = \frac{1}{2}\delta(\tau_3)\Big(\Big(\delta(\tau_1)+\delta(\tau_2)\Big)\delta(x)\delta'(\bar x) + \Big(\delta(\bar\tau_1)+\delta(\bar\tau_2)\Big)\delta'(x)\delta(\bar x)$$
$$-\Big(\delta(\tau_1)\delta(\bar\tau_1)+\delta(\tau_2)\delta(\bar\tau_2)\Big)\delta(x)\delta(\bar x)\Big)$$

$$V^1 = i\delta(\tau_3)\tau_2\Big(\delta(\tau_1)\delta(x)\delta'(\bar x) + \delta(\bar\tau_2)\delta'(x)\delta(\bar x)\Big)$$
$$-i\delta(\bar\tau_3)\Big[\bar\tau_1(\bar\tau_1-\bar\tau_2)\delta(\tau_1)\delta(\tau_2)\delta(x)\delta(\bar x) + \bar\tau_1\delta(\tau_2)\delta'(x)\delta(\bar x)\Big]$$

$$V^2 = i\delta(\tau_3)\tau_1\Big(\delta(\tau_2)\delta(x)\delta'(\bar x) + \delta(\bar\tau_1)\delta'(x)\delta(\bar x)\Big)$$
$$-i\delta(\bar\tau_3)\Big[\bar\tau_2(\bar\tau_2-\bar\tau_1)\delta(\tau_2)\delta(\tau_1)\delta(x)\delta(\bar x) + \bar\tau_2\delta(\tau_1)\delta'(x)\delta(\bar x)\Big]$$

$$P = \delta(\bar\tau_3)[\bar\tau_1(\tau_1+\tau_3\bar\tau_2)\delta(\tau_2)\theta(\tau_1) + \bar\tau_2(\tau_2+\tau_3\bar\tau_1)\delta(\tau_1)\theta(\tau_2)]\,.$$

The result is local since all terms contain either $\delta(\tau_3)$ or $\delta(\bar\tau_3)$

General features

The field redefinition in the one-form sector

- mixes left and right sectors of y and $\bar{y}$. E.g. the factor $(1-\tau_3\bar{\tau}_3)$. is invisible in the final local result at $\tau_3=0$ or $\bar{\tau}_3=0$ but is necessary in the nonlinear field redefinition.

- exhibits the gauge ambiguity $\delta\omega = D(\phi(J))$ that does not affect the final result raising the question on the most appropriate gauge choice

- admits a lot of different Fierz identities because of appearance of additional indices carried by $\overline{H}_{\dot{\alpha}\dot{\beta}}$ making the analysis in the 1-form sector much more involved than for 0-forms.

Conclusion

Nonlinear HS equations admit a distinguished field redefinition bringing the first nonlinear corrections to the local form both in the one-form and in the zero-form sector.

The coupling constant is independent of the phase of η depending only on $\eta\bar{\eta}$. Proper dependence on the phase parameter in the holographic duals of the AdS_4 HS theory is reproduced by the phase-independent vertex of the bulk theory HS theory via phase-dependent boundary conditions resulting from the condition that the boundary theory is indeed conformal.

Explicit form of the appropriate field redefinition suggests a proper form of generalized local field redefinitions

Green light for the analysis of HS field equations

Invariant functionals

HS holography

The phase φ of η should be related to the Chern-Simons coupling of the boundary vector model. Does this fit the conclusion that the HS cubic vertex is φ-independent?

$$C^{j\,1-j}(y,\bar{y}|\mathbf{x},\mathbf{z}) = \mathbf{z}\,\mathsf{exp}(y_\alpha \bar{y}^\alpha) T^{j\,1-j}(w,\bar{w}|\mathbf{x},\mathbf{z})\,, \qquad w^\alpha = \mathbf{z}^{1/2} y^\alpha \quad \bar{w}^\alpha = \mathbf{z}^{1/2}\bar{y}^\alpha$$

where $T^{j\,1-j}$ are associated with the boundary currents.

The contribution of HS connections at the boundary cannot be neglected except for the boundary conditions MV 2012

$$\bar{\eta} T_+^{j\,1-j}(y,\bar{y}|\mathbf{x},0) - \eta T_-^{1-j\,j}(i\bar{y},iy|\mathbf{x},0) = 0\,,$$

where T_+ and T_- are the positive and negative helicity parts of $T(y,\bar{y}|x)$.

In terms of remaining real boundary fields

$$j^j(y,\bar{y}|\mathbf{x}) := \frac{1}{2}\Big(\bar{\eta} T_+^{j\,1-j}(y,\bar{y}|\mathbf{x},0) + \eta T_-^{1-j\,j}(i\bar{y},iy|\mathbf{x},0)\Big) = \bar{\eta} T_+^{j\,1-j}(y,\bar{y}|\mathbf{x},0)$$

the final result matches the form of the deformation of the HS current algebra found by Maldacena and Zhiboedov

$$V = cos^2(\varphi) V_b + sin^2(\varphi) V_f + \frac{1}{2} sin(2\varphi) V_o$$

Phase dependence via boundary conditions

The contribution of HS connections at the boundary cannot be neglected except for the boundary conditions MV 2012

$$\bar{\eta} T_{+}^{j\,1-j}(y,\bar{y}|\mathbf{x},0) - \eta T_{-}^{1-j\,j}(i\bar{y},iy|\mathbf{x},0) = 0\,,$$

where T_+ and T_- are the positive and negative helicity parts of $T(y,\bar{y}|x)$.

Upon imposing boundary conditions, remaining real boundary fields are

$$j^j(y,\bar{y}|\mathbf{x}) := \frac{1}{2}\Big(\bar{\eta} T_{+}^{j\,1-j}(y,\bar{y}|\mathbf{x},0) + \eta T_{-}^{1-j\,j}(i\bar{y},iy|\mathbf{x},0)\Big) = \bar{\eta} T_{+}^{j\,1-j}(y,\bar{y}|\mathbf{x},0)\,.$$

Independence of the bulk HS vertex on φ implies that the boundary vertex has the structure

$$V = \sum_{i,j=1,2} (a_{ij} T_{+}^{i\,1-i} T_{+}^{j\,1-j} + b_{ij} T_{-}^{i\,1-i} T_{-}^{j\,1-j} + e_{ij} T_{-}^{i\,1-i} T_{+}^{j\,1-j})\,,$$

where a_{ij}, b_{ij} and e_{ij} are some φ-independent coefficients built from components of the boundary HS connections and background fields.

In terms of real φ-independent currents V reads

$$V = \frac{1}{\eta\bar{\eta}} \sum_{i,j=1,2} (\exp 2i\varphi\, a_{ij} j_+^{i\,1-i} j_+^{j\,1-j} + \exp -2i\varphi\, b_{ij} j_-^{i\,1-i} j_-^{j\,1-j} + e_{ij} j_-^{i\,1-i} j_+^{j\,1-j})\,.$$

Manifest dependence on φ identifies the parity even boson ($\varphi = 0$) vertex V_+ and fermion ($\varphi = \pi/2$) vertex V_-

$$V_\pm = \frac{1}{\eta\bar{\eta}} \sum_{i,j=1,2} (\pm a_{ij} j_+^{i\,1-i} j_+^{j\,1-j} \pm\, b_{ij} j_-^{i\,1-i} j_-^{j\,1-j} + e_{ij} j_-^{i\,1-i} j_+^{j\,1-j})\,,$$

Since parity transformation exchanges the positive and negative helicities, the remaining parity-odd vertex is

$$V_o = \frac{i}{\eta\bar{\eta}} \sum_{i,j=1,2} (a_{ij} j_+^{i\,1-i} j_+^{j\,1-j} -\, b_{ij} j_-^{i\,1-i} j_-^{j\,1-j})\,.$$

This gives the following formula matching the form of the deformation of the HS current algebra found by Maldacena and Zhiboedov

$$V = cos^2(\varphi) V_b + sin^2(\varphi) V_f + \frac{1}{2} sin(2\varphi) V_o\,,$$