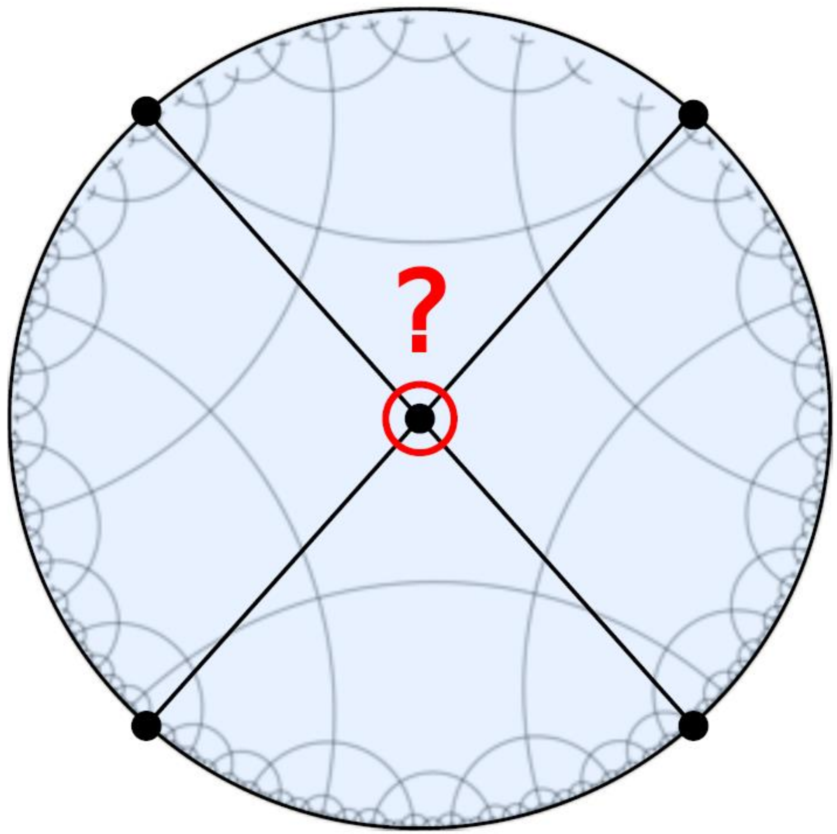
Higher-Spin Algebras, Holography & Flat Space

Massimo Taronna

Based on: arXiv:1603.00022 & arXiv:1609.00991 (with Charlotte Sleight) and arXiv:1602.08566

Can holography help us understand higher-spin Interactions?



What do we want to know?

- How singular is the flat limit of HS theories?
- Can AdS/CFT teach us about non-localities in HS context?
- **How to check** AdS/CFT dualities?

Conventional Approach: Noether

Take as starting point the Fronsdal Lagrangian [Fronsdal '78]

$$S^{(2)} = \sum_s \int \frac{1}{2}\, \varphi^{\mu_1 \dots \mu_s} \Box \varphi_{\mu_1 \dots \mu_s} + \dots$$

$$\delta^{(0)} \varphi_{\mu(s)} = \nabla_\mu \xi_{\mu(s-1)}$$

Consider a **weak field expansion** of a would be non-linear action and enforce gauge invariance:

$$S = S^{(2)} + S^{(3)} + S^{(4)} + \dots$$
$$\delta\varphi = \delta^{(0)}\varphi + \delta^{(1)}\varphi + \dots$$

$$\Longrightarrow$$

$$\delta^{(0)} S^{(2)} = 0$$
$$\delta^{(1)} S^{(2)} + \delta^{(0)} S^{(3)} = 0$$
$$\delta^{(2)} S^{(2)} + \delta^{(1)} S^{(3)} + \delta^{(0)} S^{(4)} = 0$$
$$\dots$$

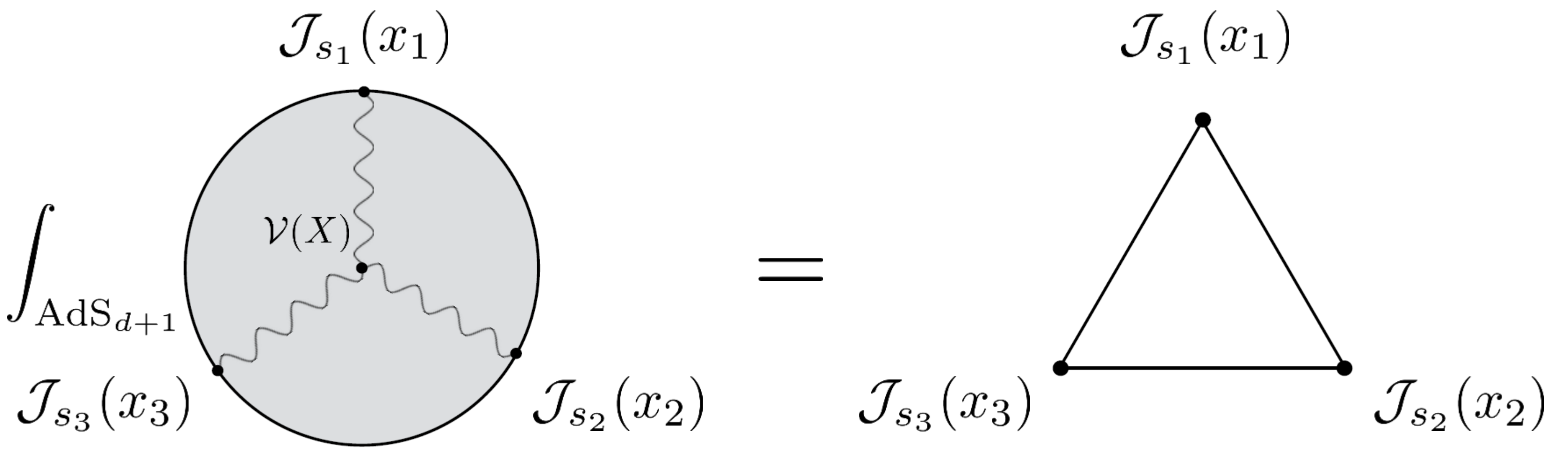
Becomes more and more **involved** beyond the cubic order (Locality?)

[Boulanger, Leclercq, Sundell 2008, M.T. 2011; Boulanger, Kessel, Skvortsov & M.T. 2015; Bekaert, Erdmenger, Ponomarev & Sleight 2015; M.T. 2016; ...]

Holographic Approach

Higher-spin theory on AdS_{d+1} ⟷ Free O(N) vector model

[Sezgin-Sundell, Klebanov-Polyakov, '02]



Solve the above equation for the bulk vertices $\mathcal{V}(X)$ and check that the CFT gives a solution to the Noether procedure

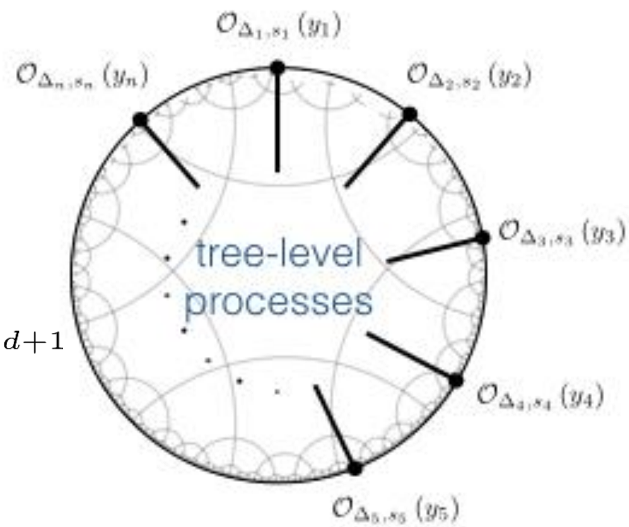
Petkou; Bekaert, Erdmenger, Ponomarev, Sleight; Skvortsov; Sleight & M.T.

Holographic Approach

Higher-spin theory on AdS$_{d+1}$ ⟷ Free O(N) vector model

[Sezgin-Sundell, Klebanov-Polyakov, '02]

$$\int_{\mathrm{AdS}_{d+1}}$$



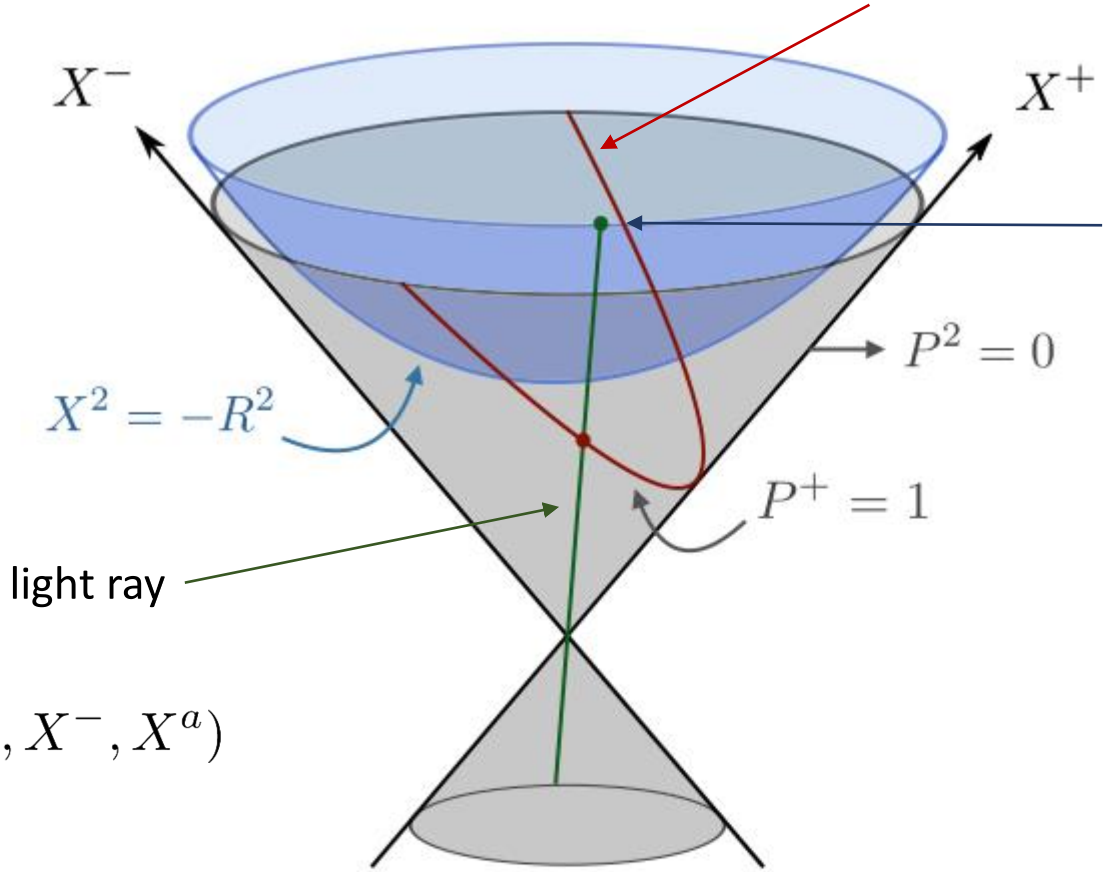
$$= \langle \mathcal{O}_{\Delta_1,s_1}(y_1)\ldots\mathcal{O}_{\Delta_n,s_n}(y_n)\rangle$$

$$\approx -\frac{1}{G}\prod_{i=1}^{n}\frac{\delta}{\delta\bar{\varphi}_{s_1}(y_1)}\,S_{AdS}[\varphi_i,\varphi_i|_{\partial AdS=\bar{\varphi}_i}]$$

Solve the above equation for the bulk vertices $\mathcal{V}(X)$ and check that the CFT gives a solution to the Noether procedure

Ambient Space Trick

[Dirac, Fronsdal, ...]



$$X^A = (X^+, X^-, X^a)$$

$$\nabla^M = \partial_X^M - \frac{1}{X^2}(X^M X \cdot \partial_X + \ldots)$$

Ambient Space Trick

$$\varphi_{\mu_1\ldots\mu_s}(x) \rightarrow \Phi_{A_1\ldots A_s}(X)$$

point intrinsic to AdS

Ambient point

To ensure same # d.o.f. & one-to-one correpondence:

$$X^A \varphi_{AA_2\ldots A_s}(X) = 0$$

$$(X \cdot \partial_X - \Delta)\varphi_{A_1\ldots A_s} = 0$$

Generating function notation:

$$\varphi_{A_1\ldots A_s}(X) \rightarrow \varphi(X,U) = \frac{1}{s!}\varphi_{A_1\ldots A_s}(X)U^{A_1}\cdots U^{A_s}$$

Bulk Cubic Couplings

Most general coupling (up to total deriv & redefs): sum of **building blocks**:

$$I^{n_1,n_2,n_3}_{s_1,s_2,s_3}(\Phi_i) = \eta^{M_1(n_3)M_2(n_3)}\eta^{M_2(n_1)M_3(n_1)}\eta^{M_3(n_2)M_1(n_2)}(\partial^{N_3(k_3)}\Phi_{M_1(n_2+n_3)N_1(k_1)}) \times (\partial^{N_1(k_1)}\Phi_{M_2(n_3+n_1)N_2(k_2)})(\partial^{N_2(k_2)}\Phi_{M_3(n_1+n_2)N_3(k_3)})$$

The ansatz for the bulk vertex reads:

$$\mathcal{V} = \sum_{s_i,n_i} g^{n_1,n_2,n_3}_{s_1,s_2,s_3}\, I^{n_1,n_2,n_3}_{s_1,s_2,s_3}$$

Bulk Cubic Couplings

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$$\times\ (\partial^{N_1(k_1)}\Phi_{M_2(n_3+n_1)N_2(k_2)})(\partial^{N_2(k_2)}\Phi_{M_3(n_1+n_2)N_3(k_3)})$$

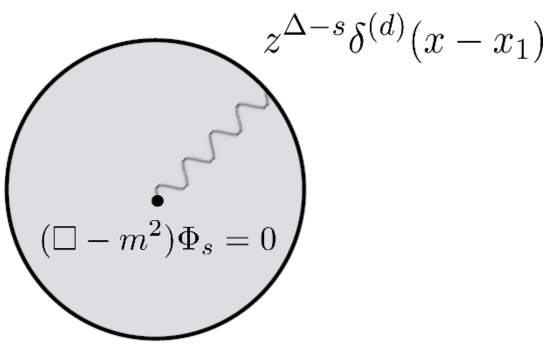
The ansatz for the bulk vertex reads:

$$\mathcal{V} = \sum_{s_i,n_i} g^{n_1,n_2,n_3}_{s_1,s_2,s_3} I^{n_1,n_2,n_3}_{s_1,s_2,s_3}$$

Need to solve for the relative coupling constants

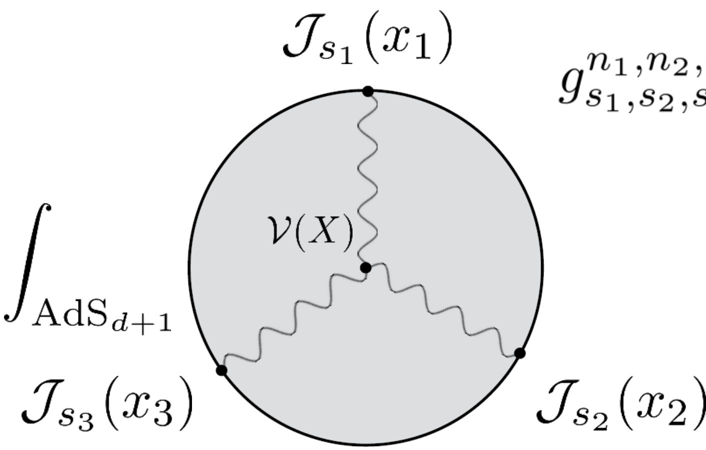
Plug boundary to bulk propagators and perform the **integral** over AdS:

$$\Phi_s \sim \frac{1}{(-2P(x)\cdot X)^{\Delta}}(\ldots)$$



Complete Higher-Spin Cubic Action

$$g^{n_1,n_2,n_3}_{s_1,s_2,s_3} = \mathcal{O}^{-1}[c^{n_1,n_2,n_3}_{s_1,s_2,s_3}]$$



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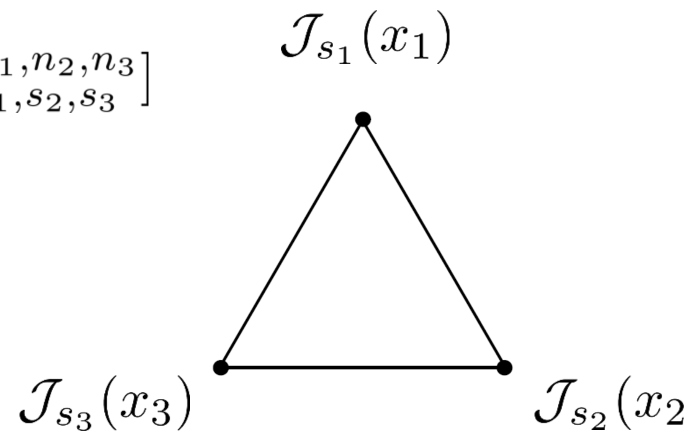

$$\mathcal{V} = \sum_{s_1,s_2,s_3} g_{s_1,s_2,s_3} I^{0,0,0}_{s_1,s_2,s_3} \qquad g_{s_1,s_2,s_3} = \frac{1}{\sqrt{N}} \frac{\pi^{\frac{d-3}{4}} 2^{\frac{3d-1+s_1+s_2+s_3}{2}}}{\Gamma(d+s_1+s_2+s_3-3)} \prod_{i=1}^{3} \sqrt{\frac{\Gamma(s_i+\frac{d-1}{2})}{\Gamma(s_i+1)}}$$

We obtain the **complete higher-spin cubic action**

$$I^{0,0,0}_{s_1,s_2,s_3}(\Phi_i) = (\partial^{N_3(k_3)}\Phi_{N_1(k_1)})(\partial^{N_1(k_1)}\Phi_{N_2(k_2)})(\partial^{N_2(k_2)}\Phi_{N_3(k_3)})$$

Complete Higher-Spin Cubic Action

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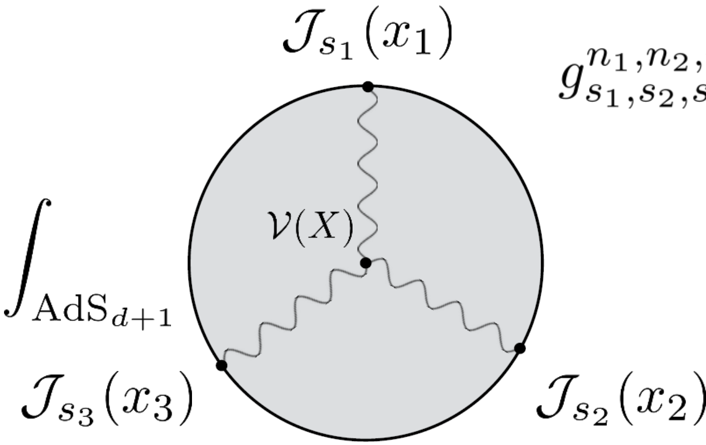

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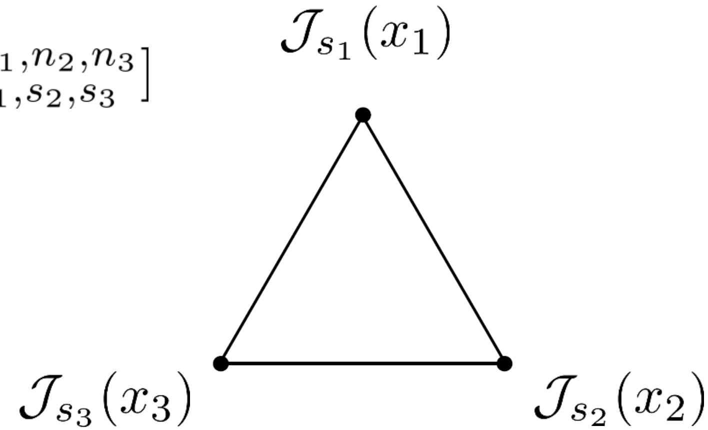


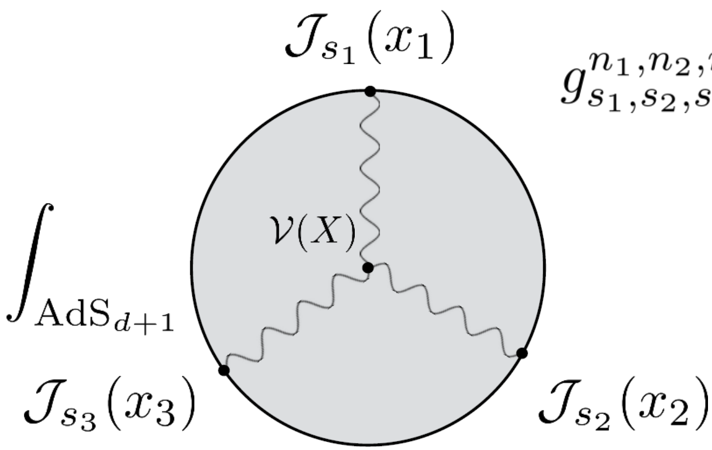
$$\mathcal{V} = \sum_{s_1,s_2,s_3} g_{s_1,s_2,s_3} I^{0,0,0}_{s_1,s_2,s_3} \qquad g_{s_1,s_2,s_3} = \frac{1}{\sqrt{N}} \frac{\pi^{\frac{d-3}{4}} 2^{\frac{3d-1+s_1+s_2+s_3}{2}}}{\Gamma(d+s_1+s_2+s_3-3)} \prod_{i=1}^{3} \sqrt{\frac{\Gamma(s_i+\frac{d-1}{2})}{\Gamma(s_i+1)}}$$

In generating functions terms

$$Y_1 = \partial_{U_1}\cdot\partial_{X_2} \qquad Y_2 = \partial_{U_2}\cdot\partial_{X_3} \qquad Y_3 = \partial_{U_3}\cdot\partial_{X_1}$$

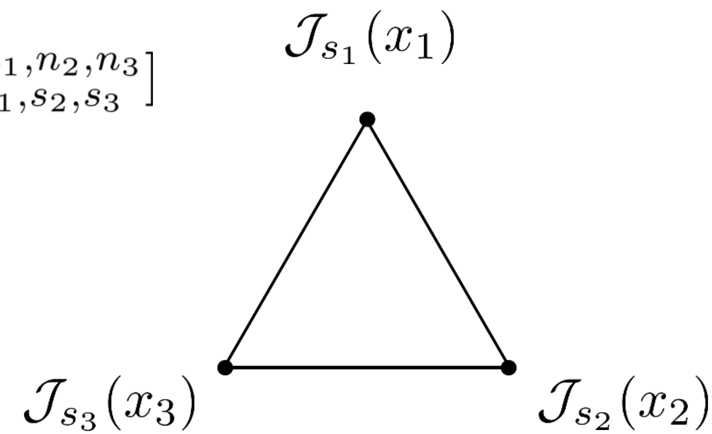
$$H_1 = \partial_{U_2}\cdot\partial_{U_3} \qquad H_2 = \partial_{U_3}\cdot\partial_{U_1} \qquad H_3 = \partial_{U_1}\cdot\partial_{U_2}$$

Complete Higher-Spin Cubic Action



$$g^{n_1,n_2,n_3}_{s_1,s_2,s_3} = \mathcal{O}^{-1}[c^{n_1,n_2,n_3}_{s_1,s_2,s_3}]$$

$=$



$$\mathcal{V} = \sum_{s_1,s_2,s_3} g_{s_1,s_2,s_3} I^{0,0,0}_{s_1,s_2,s_3} \qquad g_{s_1,s_2,s_3} = \frac{1}{\sqrt{N}} \frac{\pi^{\frac{d-3}{4}} 2^{\frac{3d-1+s_1+s_2+s_3}{2}}}{\Gamma(d+s_1+s_2+s_3-3)} \prod_{i=1}^{3} \sqrt{\frac{\Gamma(s_i+\frac{d-1}{2})}{\Gamma(s_i+1)}}$$

We get:

$$I^{0,0,0}_{s_1,s_2,s_3}(\Phi_i) = Y_1^{s_1} Y_2^{s_2} Y_3^{s_3}\, \Phi_1(X_1,U_1)\, \Phi_2(X_2,U_2)\, \Phi_3(X_3,U_3)\Big|_{X_i=X,U_i=0}$$

Checks of the Duality

The Holographically Reconstructed HS algebra

Cubic couplings induce deformations of gauge transformations and gauge symmetries

$$\int \left[(\delta^{(1)} \Phi) \Box \Phi + \delta^{(0)} \mathcal{V} \right] = 0$$

The first simple test is that it is possible to solve for the induced gauge transformations

The commutator of two gauge transformations closes to the lowest order automatically: extract gauge bracket (field independent)

$$\delta^{(0)}_{[\epsilon_1} \delta^{(1)}_{\epsilon_2]} \approx \delta^{(0)}_{[\![\epsilon_1, \epsilon_2]\!]^{(0)}}$$

Explicit classification (modulo field and parameter redefinitions) known in constant curvature backgrounds (Joung & MT ‘13)

The Classification

E. Joung & M.T.

$$P^{[n]}_{s_1 s_2 s_3} = e^{\lambda \mathcal{D}} Y_1^{s_1-n} Y_2^{s_2-n} Y_3^{s_3-n} G^n \qquad s_1 \geq s_2 \geq s_3$$

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flat-space coupling

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Lower derivative tail

flat-space coupling

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$$P^{[n]}_{s_1 s_2 s_3} = e^{\lambda \mathcal{D}} Y_1^{s_1-n} Y_2^{s_2-n} Y_3^{s_3-n} G^n \qquad s_1 \geq s_2 \geq s_3$$

Lower derivative tail

flat-space coupling

	n	$\#\partial$	$\delta^{(1)}_{E_1}$	$\delta^{(1)}_{E_2}$	$\delta^{(1)}_{E_3}$	$C^{(3)}$
	0	$s_1+s_2+s_3$	$=0$	$=0$	$=0$	
Class I	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\approx \tilde{K}(Y_\ell, H_{12}, H_{23}, H_{31})$
	$\frac{s_2+s_3-s_1}{2}$	$2\,s_1$	$=0$	$\vdots$	$\vdots$	$\ell = 2$ or 3
	$\vdots$	$\vdots$	$\neq 0$	$\vdots$	$\vdots$	
Class II	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\approx \tilde{K}(Y_1, H_{12}, H_{23}, H_{31})$
	$\frac{s_3+s_1-s_2}{2}$	$2\,s_2$	$\vdots$	$=0$	$=0$	
	$\vdots$	$\vdots$	$\vdots$	$\neq 0$	Λ	
Class III	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
	$\frac{s_1+s_2-s_3}{2}$	$2s_3$	$\vdots$	$\vdots$	Λ	
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\neq 0$	
Class IV	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
	s_3	$s_1+s_2-s_3$	$\neq 0$	$\neq 0$	$\neq 0$	

The Classification

E. Joung & M.T.

$$P^{[n]}_{s_1 s_2 s_3} = e^{\lambda \mathcal{D}} Y_1^{s_1-n} Y_2^{s_2-n} Y_3^{s_3-n} G^n \qquad s_1 \geq s_2 \geq s_3$$

Do not deform gauge transformations

Lower derivative tail

flat-space coupling

	n	$\#\partial$	$\delta^{(1)}_{E_1}$	$\delta^{(1)}_{E_2}$	$\delta^{(1)}_{E_3}$	$C^{(3)}$
	0	$s_1+s_2+s_3$	$=0$	$=0$	$=0$	
Class I	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\approx \tilde{K}(Y_\ell, H_{12}, H_{23}, H_{31})$
	$\frac{s_2+s_3-s_1}{2}$	$2\,s_1$	$=0$	$\vdots$	$\vdots$	$\ell = 2$ or 3
	$\vdots$	$\vdots$	$\neq 0$	$\vdots$	$\vdots$	
Class II	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\approx \tilde{K}(Y_1, H_{12}, H_{23}, H_{31})$
	$\frac{s_3+s_1-s_2}{2}$	$2\,s_2$	$\vdots$	$=0$	$=0$	
	$\vdots$	$\vdots$	$\vdots$	$\neq 0$	Λ	
Class III	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
	$\frac{s_1+s_2-s_3}{2}$	$2s_3$	$\vdots$	$\vdots$	Λ	
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\neq 0$	
Class IV	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
	s_3	$s_1+s_2-s_3$	$\neq 0$	$\neq 0$	$\neq 0$	

The Classification

E. Joung & M.T.

$$P^{[n]}_{s_1 s_2 s_3} = e^{\lambda \mathcal{D}} Y_1^{s_1 - n} Y_2^{s_2 - n} Y_3^{s_3 - n} G^n \qquad s_1 \geq s_2 \geq s_3$$

Lower derivative tail

flat-space coupling

Do not deform gauge transformations

Current Interactions

	n	$\#\partial$	$\delta^{(1)}_{E_1}$	$\delta^{(1)}_{E_2}$	$\delta^{(1)}_{E_3}$	$C^{(3)}$
	0	$s_1 + s_2 + s_3$	$= 0$	$= 0$	$= 0$	
Class I	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\approx \tilde{K}(Y_\ell, H_{12}, H_{23}, H_{31})$
	$\frac{s_2+s_3-s_1}{2}$	$2\, s_1$	$= 0$	$\vdots$	$\vdots$	$\ell = 2$ or 3
	$\vdots$	$\vdots$	$\neq 0$	$\vdots$	$\vdots$	
Class II	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\approx \tilde{K}(Y_1, H_{12}, H_{23}, H_{31})$
	$\frac{s_3+s_1-s_2}{2}$	$2\, s_2$	$\vdots$	$= 0$	$= 0$	
	$\vdots$	$\vdots$	$\vdots$	$\neq 0$	Λ	
Class III	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
	$\frac{s_1+s_2-s_3}{2}$	$2 s_3$	$\vdots$	$\vdots$	Λ	
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\neq 0$	
Class IV	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
	s_3	$s_1 + s_2 - s_3$	$\neq 0$	$\neq 0$	$\neq 0$	

The Classification

E. Joung & M.T.

$$P^{[n]}_{s_1 s_2 s_3} = e^{\lambda \mathcal{D}} Y_1^{s_1-n} Y_2^{s_2-n} Y_3^{s_3-n} G^n \qquad s_1 \geq s_2 \geq s_3$$

Lower derivative tail

flat-space coupling

Do not deform gauge transformations → Class I

Current Interactions → Class II

Quasi minimal couplings → Class III, Class IV

	n	$\#\partial$	$\delta^{(1)}_{E_1}$	$\delta^{(1)}_{E_2}$	$\delta^{(1)}_{E_3}$	$C^{(3)}$
Class I	0	$s_1+s_2+s_3$	$=0$	$=0$	$=0$	
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\approx \tilde{K}(Y_\ell, H_{12}, H_{23}, H_{31})$
	$\frac{s_2+s_3-s_1}{2}$	$2\,s_1$	$=0$	$\vdots$	$\vdots$	$\ell = 2$ or 3
Class II	$\vdots$	$\vdots$	$\neq 0$	$\vdots$	$\vdots$	
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\approx \tilde{K}(Y_1, H_{12}, H_{23}, H_{31})$
	$\frac{s_3+s_1-s_2}{2}$	$2\,s_2$	$\vdots$	$=0$	$=0$	
Class III	$\vdots$	$\vdots$	$\vdots$	$\neq 0$	Λ	
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
	$\frac{s_1+s_2-s_3}{2}$	$2s_3$	$\vdots$	$\vdots$	Λ	
Class IV	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\neq 0$	
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
	s_3	$s_1+s_2-s_3$	$\neq 0$	$\neq 0$	$\neq 0$	

The Classification

E. Joung & M.T.

$$P^{[n]}_{s_1 s_2 s_3} = e^{\lambda \mathcal{D}} Y_1^{s_1 - n} Y_2^{s_2 - n} Y_3^{s_3 - n} G^n \qquad s_1 \geq s_2 \geq s_3$$

Lower derivative tail

flat-space coupling

Do not deform gauge transformations

Current Interactions

Gravitational coupling

Quasi minimal couplings

	n	$\#\partial$	$\delta^{(1)}_{E_1}$	$\delta^{(1)}_{E_2}$	$\delta^{(1)}_{E_3}$	$C^{(3)}$
	0	$s_1 + s_2 + s_3$	$= 0$	$= 0$	$= 0$	
Class I	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\approx \tilde{K}(Y_\ell, H_{12}, H_{23}, H_{31})$
	$\frac{s_2+s_3-s_1}{2}$	$2\, s_1$	$= 0$	$\vdots$	$\vdots$	$\ell = 2$ or 3
	$\vdots$	$\vdots$	$\neq 0$	$\vdots$	$\vdots$	
Class II	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\approx \tilde{K}(Y_1, H_{12}, H_{23}, H_{31})$
	$\frac{s_3+s_1-s_2}{2}$	$2\, s_2$	$\vdots$	$= 0$	$= 0$	
	$\vdots$	$\vdots$	$\vdots$	$\neq 0$	Λ	
Class III	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
	$\frac{s_1+s_2-s_3}{2}$	$2 s_3$	$\vdots$	$\vdots$	Λ	
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\neq 0$	
Class IV	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
	s_3	$s_1 + s_2 - s_3$	$\neq 0$	$\neq 0$	$\neq 0$	

The Classification

E. Joung & M.T.

$$P^{[n]}_{s_1 s_2 s_3} = e^{\lambda \mathcal{D}} Y_1^{s_1-n} Y_2^{s_2-n} Y_3^{s_3-n} G^n \qquad s_1 \geq s_2 \geq s_3$$

Lower derivative tail

flat-space coupling

Do not deform gauge transformations

Current Interactions

Gravitational coupling

Quasi minimal couplings

	n	$\#\partial$	$\delta^{(1)}_{E_1}$	$\delta^{(1)}_{E_2}$	$\delta^{(1)}_{E_3}$	$C^{(3)}$
	0	$s_1+s_2+s_3$	$=0$	$=0$	$=0$	
Class I	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\approx \tilde{K}(Y_\ell, H_{12}, H_{23}, H_{31})$
	$\frac{s_2+s_3-s_1}{2}$	$2\,s_1$	$=0$	$\vdots$	$\vdots$	$\ell = 2$ or 3
	$\vdots$	$\vdots$	$\neq 0$	$\vdots$	$\vdots$	
Class II	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\approx \tilde{K}(Y_1, H_{12}, H_{23}, H_{31})$
	$\frac{s_3+s_1-s_2}{2}$	$2\,s_2$	$\vdots$	$=0$	$=0$	
	$\vdots$	$\vdots$	$\vdots$	$\neq 0$	Λ	Reason why flat limit is hard
Class III	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
	$\frac{s_1+s_2-s_3}{2}$	$2s_3$	$\vdots$	$\vdots$	Λ	
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\neq 0$	
Class IV	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
	s_3	$s_1+s_2-s_3$	$\neq 0$	$\neq 0$	$\neq 0$	

The Classification

E. Joung & M.T.

$$P^{[n]}_{s_1 s_2 s_3} = e^{\lambda \mathcal{D}} Y_1^{s_1-n} Y_2^{s_2-n} Y_3^{s_3-n} G^n \qquad s_1 \geq s_2 \geq s_3$$

Lower derivative tail

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Quasi minimal couplings

	n	$\#\partial$	$\delta^{(1)}_{E_1}$	$\delta^{(1)}_{E_2}$	$\delta^{(1)}_{E_3}$	$C^{(3)}$
	0	$s_1+s_2+s_3$	$=0$	$=0$	$=0$	
Class I	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\approx \tilde{K}(Y_\ell, H_{12}, H_{23}, H_{31})$
	$\frac{s_2+s_3-s_1}{2}$	$2\,s_1$	$=0$	$\vdots$	$\vdots$	$\ell = 2$ or 3
	$\vdots$	$\vdots$	$\neq 0$	$\vdots$	$\vdots$	
Class II	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\approx \tilde{K}(Y_1, H_{12}, H_{23}, H_{31})$
	$\frac{s_3+s_1-s_2}{2}$	$2\,s_2$	$\vdots$	$=0$	$=0$	
	$\vdots$	$\vdots$	$\vdots$	$\neq 0$	Λ	Reason why flat limit is hard
Class III	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
	$\frac{s_1+s_2-s_3}{2}$	$2s_3$	$\vdots$	$\vdots$	Λ	
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\neq 0$	
Class IV	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
	s_3	$s_1+s_2-s_3$	$\neq 0$	$\neq 0$	$\neq 0$	

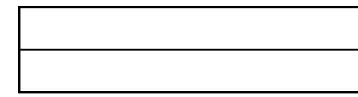
$$[E_1, E_2]^{(0)}_3 \neq 0 \quad \text{iff} \quad \delta^{(1)}_{E_1} \neq 0 \quad \& \quad \delta^{(1)}_{E_2} \neq 0$$

Quartic Consistency

At cubic order no condition is imposed on the deformations but at quartic

A key trick is to focus on Killing tensors (asymptotic charges)

$$\nabla_{\mu}\epsilon_{\mu(s-1)} = 0 \longrightarrow$$



Jacobi:

Fradking & Vasiliev; Boulanger, Ponomarev, Skvortsov & MT

$$\llbracket \epsilon_1, \llbracket \epsilon_2, \epsilon_3 \rrbracket^{(0)} \rrbracket^{(0)} + \text{cyclic} = 0$$

Admissibility:

Konstein & Vasiliev; Boulanger, Kessel, Skvortsov & MT

$$\delta^{(1)}_{[\epsilon_1}\delta^{(1)}_{\epsilon_2]} \approx \delta^{(1)}_{\llbracket \epsilon_1, \epsilon_2 \rrbracket^{(0)}}$$

Cubic covariance:

$$\delta^{(1)}_{\epsilon} S^{(3)} \approx 0$$

Completely fix $S^{(3)}$

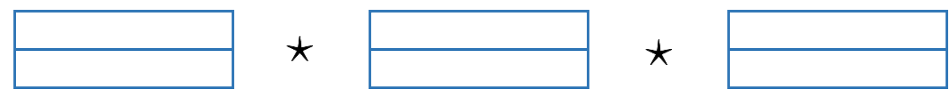
A test in this context goes backwords: we have the cubic action and we can test that it solves the above necessary conditions

[C.Sleight & M.T. 1609.00991]

The Holographically Reconstructed HS algebra

The deformation of the gauge algebra induced by the cubic couplings **matches** the structure constants of the HS algebras **in any D**

$$\left\langle \epsilon_3 \middle| [\![\epsilon_1, \epsilon_2]\!]^{(0)} \right\rangle \stackrel{?}{=} \mathrm{Tr}\left[\square \star \square \star \square \right]$$



[Eastwood, Vasiliev; Joung, Mkrtchyan ...]

The **reconstructed bracket** reproduces as expected the HS algebra structure constants with the following normalisation of the invariant bilinear:

$$\mathrm{Tr}(T_s \star T_s) = \frac{1}{(s-1)!^2} \frac{\pi^{\frac{d}{2}-1} s\, 2^{d-4s+7} \Gamma\left(\frac{d}{2}\right)}{\Gamma\left(\frac{d-3}{2}\right) \Gamma\left(\frac{d-5}{2}+s\right)}$$

[C.Sleight & M.T. 1609.00991]

Flat Limit

The Classification

$$P^{[n]}_{s_1 s_2 s_3} = e^{\lambda \mathcal{D}} Y_1^{s_1 - n} Y_2^{s_2 - n} Y_3^{s_3 - n} G^n \qquad s_1 \geq s_2 \geq s_3$$

	n	$\#\partial$	$\delta^{(1)}_{E_1}$	$\delta^{(1)}_{E_2}$	$\delta^{(1)}_{E_3}$	$C^{(3)}$
	0	$s_1 + s_2 + s_3$	$= 0$	$= 0$	$= 0$	
Class I	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\approx \tilde{K}(Y_\ell, H_{12}, H_{23}, H_{31})$
	$\frac{s_2+s_3-s_1}{2}$	$2\,s_1$	$= 0$	$\vdots$	$\vdots$	$\ell = 2$ or 3
	$\vdots$	$\vdots$	$\neq 0$	$\vdots$	$\vdots$	
Class II	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\approx \tilde{K}(Y_1, H_{12}, H_{23}, H_{31})$
	$\frac{s_3+s_1-s_2}{2}$	$2\,s_2$	$\vdots$	$= 0$	$= 0$	
	$\vdots$	$\vdots$	$\vdots$	$\neq 0$	Λ	
Class III	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	Reason why flat limit is hard
	$\frac{s_1+s_2-s_3}{2}$	$2 s_3$	$\vdots$	$\vdots$	Λ	
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\neq 0$	
Class IV	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
	s_3	$s_1 + s_2 - s_3$	$\neq 0$	$\neq 0$	$\neq 0$	

Gravitational coupling → Class III

The Classification

$$P^{[n]}_{s_1 s_2 s_3} = e^{\lambda \mathcal{D}} Y_1^{s_1-n} Y_2^{s_2-n} Y_3^{s_3-n} G^n \qquad s_1 \geq s_2 \geq s_3$$

	n	$\#\partial$	$\delta^{(1)}_{E_1}$	$\delta^{(1)}_{E_2}$	$\delta^{(1)}_{E_3}$	$C^{(3)}$
Class I	0	$s_1+s_2+s_3$	$=0$	$=0$	$=0$	$\approx \tilde{K}(Y_\ell, H_{12}, H_{23}, H_{31})$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
	$\frac{s_2+s_3-s_1}{2}$	$2\,s_1$	$=0$	$\vdots$	$\vdots$	$\ell = 2$ or 3
Class II	$\vdots$	$\vdots$	$\neq 0$	$\vdots$	$\vdots$	$\approx \tilde{K}(Y_1, H_{12}, H_{23}, H_{31})$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
	$\frac{s_3+s_1-s_2}{2}$	$2\,s_2$	$\vdots$	$=0$	$=0$	
Class III	$\vdots$	$\vdots$	$\vdots$	$\neq 0$	Λ	Reason why flat limit is hard
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
	$\frac{s_1+s_2-s_3}{2}$	$2s_3$	$\vdots$	$\vdots$	Λ	
Class IV	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\neq 0$	
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
	s_3	$s_1+s_2-s_3$	$\neq 0$	$\neq 0$	$\neq 0$	

Gravitational coupling

$$[T_s, T_s] \sim T_2$$

$$[T_2, T_s] \sim \Lambda T_s \to 0$$

E. Joung & M.T.

Ambient Space Interpretation

A flat space total derivative gives a non-vanishing AdS coupling

$$\int_{\mathbb{R}^{d+2}} d^{d+2}X\,\delta(\sqrt{-X^2}+1)\,\partial_X^A f_A(X) \neq 0$$

AdS couplings can be written exactly as flat space ones but with appropriate choice of boundary terms

$$Y_1^{s_1} Y_2^{s_2} Y_3^{s_3} \Phi_1 \Phi_2 \Phi_3 \longrightarrow \Lambda^{s_1+s_2+s_3-2} \nabla^2 \Phi^3$$

In principle we can rewrite also the above coupling as a total derivative...

...flat limit ambiguous (non-abelian structure appear as total derivative)

[Joung, M.T.; Conde, Joung, Mkrtchyan]

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[Joung, M.T.; Conde, Joung, Mkrtchyan]

But the non-abelian structure should be non-vanishing!!

Metsaev's light-cone couplings?

Metsaev fixed **all cubic coupling** in flat space by requiring **Poincaré invariance** up to the quartic order:

$$\varphi^{+\cdots} = 0$$

$$\mathcal{V} = \sum_{|s_i|=0}^{\infty} \frac{(il)^{s_1+s_2+s_3}}{\Gamma(s_1+s_2+s_3)} \underbrace{\left[\partial_{x_1}(\partial_2^+ - \partial_3^+) + \text{cyclic}\right]}^{s_1+s_2+s_3} \frac{\varphi_{s_1}}{(\partial_{x_1}^+)^{s_1}} \frac{\varphi_{s_2}}{(\partial_{x_2}^+)^{s_2}} \frac{\varphi_{s_3}}{(\partial_{x_3}^+)^{s_3}} + h.c.$$

holomorphic-light cone momentum P $(\bar{P})$

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Lower derivative structure non-vanishing

The overall coupling constant is the same as in AdS_4

Covariantisation problematic (?) (non-local field frame?)

$$\frac{P}{\bar{P}}$$ **Well defined functional class**

$$f^{(k)}_{s_1,s_2,s_3} \equiv Y_1^{s_1} Y_2^{s_2} Y_3^{s_3} \left(\frac{G}{Y_1\, Y_2\, Y_3}\right)^k \phi_1\, \phi_2\, \phi_3$$

In generic d we must impose $k \leq s_{min}$ but in 4d the light-cone gauge fixing is non singular for $s_1+s_2+s_3-2k \geq 0$ (Exotic couplings!!)

$$\mathcal{V} = \mathcal{V}_{\text{standard}} + \#\mathcal{V}_{\text{exotic}}$$

Non-vanishing non abelian structure

Metsaev's theory has HS symmetry?

In this way we obtain the following formal (non-local field frame) covariantisation:

$$\mathcal{V}^{(M)} = \sum_{s_i=0}^{\infty} \left[\sum_k \frac{(il)^{s_1+s_2+s_3-2k}}{\Gamma(s_1+s_2+s_3-2k)}\, f^{(k)}_{s_1,s_2,s_3} \right]$$

The above covariantisation includes non-localities... (auxiliary fields needed??)

$$\mathcal{V} = \text{standard vertex} + (\partial_u)^{-1}$$

$$\left.\frac{(il)^{2s+2-2k}}{\Gamma(2s+2-2k)}\right|_{k=s} = (il)^2$$

Equivalence principle in flat space!

...but is enough to extract structure constants of the would be underlying HS algebra:

$$\Big\langle \epsilon_3^L \big| [\![\epsilon_1^L, \epsilon_2^L]\!]^{(0)} \Big\rangle_{\text{Metsaev}} \overset{!}{=} \mathrm{Tr}\Big[$$

$$\Big]$$

[C.Sleight & M.T. 1609.00991]

Eastwood-Vasiliev 4d HS Lorentz subalgebra

Summary

- Holographic reconstruction very powerful: allows to reconstruct HS action in AdS
- The coupling reconstructed are not only gauge invariant but solve the Noether procedure up to the quartic order
- First test of the duality in d>4
- Flat limit may be well defined (?)

$$\mathcal{V} = \sum_{s_1,s_2,s_3} g_{s_1,s_2,s_3} I^{0,0,0}_{s_1,s_2,s_3}$$

$$g_{s_1,s_2,s_3} = \frac{1}{\sqrt{N}} \frac{\pi^{\frac{d-3}{4}} 2^{\frac{3d-1+s_1+s_2+s_3}{2}}}{\Gamma(d+s_1+s_2+s_3-3)} \prod_{i=1}^{3} \sqrt{\frac{\Gamma(s_i+\frac{d-1}{2})}{\Gamma(s_i+1)}}$$

A quartic functional class

Locality of quartic scalar interactions can be studied with a trick in a theory that couples the scalar to HS:

$$C_{\alpha(k)\dot{\alpha}(k)} \sim \nabla^s \Phi$$

$$j_s^{(l)} \sim \Box^l \Phi \nabla^s \Phi$$

$$\mathcal{F}_{\alpha(s)\dot{\alpha}(s)}(\phi_s) = j^{(0)}_{\alpha(s)\dot{\alpha}(s)}$$

$$(\Box + 2)\Phi(x) = \sum_{s=0}^{\infty} \phi_{\alpha(s)\dot{\alpha}(s)} C^{\alpha(s)\dot{\alpha}(s)} + \sum_{s,l} \alpha_{s,l}\, j^{(l)}_{\alpha(s)\dot{\alpha}(s)}\, C^{\alpha(s)\dot{\alpha}(s)}$$

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A redefinition of the HS field of spin s can remove all couplings involving a term with s derivatives of the scalar (!)

$$\mathcal{F}_{\alpha(s)\dot\alpha(s)}(\phi_s) = \tilde{j}_{\alpha(s)\dot\alpha(s)}(\Phi, \Phi)$$

$$(\Box + 2)\Phi(x) = \sum_{s=0}^{\infty} \phi_{\alpha(s)\dot\alpha(s)} C^{\alpha(s)\dot\alpha(s)}$$

$$\tilde{j}_{\alpha(s)\dot\alpha(s)}(\Phi, \Phi) \equiv j^{(0)}_{\alpha(s)\dot\alpha(s)}(\Phi, \Phi) - \sum_{l=0}^{\infty} \alpha_{s,l}\, \mathcal{F}_{\alpha(s)\dot\alpha(s)} \left(j^{(l)}_s(\Phi, \Phi) \right)$$

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$$C_{\alpha(k)\alpha(\dot k)} \sim \nabla^s \Phi$$

$$\tilde{j}_{\alpha(s)\dot\alpha(s)}(\Phi,\Phi) \equiv j^{(0)}_{\alpha(s)\dot\alpha(s)}(\Phi,\Phi) - \sum_{l=0}^{\infty} \alpha_{s,l}\, \mathcal{F}_{\alpha(s)\dot\alpha(s)}\left(j_s^{(l)}(\Phi,\Phi)\right)$$

$$j_s^{(l)} \sim \Box^l \Phi \nabla^s \Phi$$

This simply amounts to a change of the source to the HS equations by an ``Improvement''

We can now use the cubic functional class to distinguish local couplings from non-local ones

$$\sum_{l=0}^{\infty} \tilde{\alpha}_l^{(s)} C_l^{(s)} = 1$$

$$\mathcal{I}_s^{(l)} = j_s^{(l-1)} - C_l^{(s)}\, j_s^{(0)}$$

$$\tilde{\alpha}_l^{(s)} = \delta_{l,0} - \frac{1}{4(s-1)}\left[l^2\alpha_{s,l-2} + \alpha_{s,l-1}\left(2ls + 2(l+1)^2 + s^2\right) + \alpha_{s,l}(l+s+2)^2\right]$$

- Local quartic couplings always satisfy the above condition trivially
- Even a convergent quartic interaction can be non-local (!)