Exceptional $F(4)$ Higher-Spin Theory at One-Loop and Other Tests of Duality

HSTH, higher spins and friends

based on arXiv:1608.07582 with M.Gunaydin and T.Tran,
see also Giombi, Klebanov, Tan and Pang, Zhu, Sezgin

Evgeny Skvortsov

LMU, Muenchen and Lebedev Institute, Moscow

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Main Messages

Higher-Spin Theories should be determined by their duals, which are free CFT's or SCFT's generically.

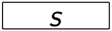
We bridged the gap in SUSY HS: there should exist a unique AdS_6 SUSY HS theory based on $F(4)$. The spectrum contains that of the Romans super-gravity.

The complete Lagrangian of HS theories is not known at present, including the $F(4)$. What can still be done? There are some global quantities — one-loop determinants on various backgrounds that require very few ingredients — spectrum! Many of such tests have been already done and we perform more. There is one puzzle left...

Higher-Spin Theories, to be constructed...

Type-A is the dual of the free scalar $\Box\phi = 0$:

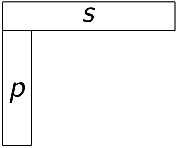
$$\delta\Phi_{\underline{m}(s)} = \nabla_{\underline{m}}\xi_{\underline{m}(s-1)}\,, \qquad s = 0, (1), 2, (3), ...$$



For another choice of b.c. should be dual to $O(N)$-models (Klebanov, Polyakov; Fei, Giombi, Klebanov).

Type-B is the dual of free fermion $\partial\!\!\!/\psi = 0$ (mixed-symmetry)

$$\Phi_{\underline{m}(s),\underline{a}[p]}\,, \qquad s = 0, (1), 2, (3), ...; p = 0, 1, ...$$



In $D = 4$ the dual is Gross-Neveu (Leigh, Petkou; Sezgin, Sundell).

Higher-Spin Theories, to be constructed...

Type-A_k are the duals of the 'higher-order' scalar $\Box^k\phi = 0$:

$$\delta\Phi_{\underline{m}(s)} = \nabla_{\underline{m}}...\nabla_{\underline{m}}\xi_{\underline{m}(s-t)} \qquad \boxed{\quad s \quad}$$

A_k makes sense since the HS algebras are related to sl-series A_k via Howe dual $hs(\lambda)$ (Alkalaev, Grigoriev, E.S; Bekaert, Grigoriev; Bekaert, Basile, Boulanger; Brust, Hinterbichler). They give fields of odd depths. Fields of even depths should appear as off-diagonal blocks in more general HS algebras.

Type-$C, D, E, ..., Z$ are the duals of doubletons or higher-spin singletons. For $j > 1$ there is no local stress-tensor and therefore they are not CFT's. The dual problem is that the graviton must be non-dynamical. (Talk by Euihun)

SUSY HS Theories

The duals of a number of fermion+boson+(Maxwell/Tensor) are SUSY HS theories:

$$\begin{pmatrix} \text{Type-A} \sim \phi\phi & \phi \times \psi \\ \psi \times \phi & \text{Type-B} \sim \psi\psi \end{pmatrix} = \sum \begin{pmatrix} \Phi^{\underline{a}(s)} & \Psi^{\underline{a}(s-\frac{1}{2});\underline{\alpha}} \\ \Psi^{\underline{a}(s-\frac{1}{2});\underline{\alpha}} & \Phi^{\underline{a}(s),\underline{m}[p]} \end{pmatrix}$$

- $\mathcal{N} \leq 8$ restriction does not formally apply to HS theories (Fradkin, Vasiliev);
- HS theories can extend spectrum of SUGRAs with HS fields;
- $\mathcal{N} > 8$ HS theories may not have usual SUSY (higher dimensions or Konstein, Vasiliev);
- AdS/CFT rolls back SUSY HS theories to $\mathcal{N} \leq 8$ via boundary conditions for interacting CFT's, Henneaux et al; Chang et al.

Usual HS SUSY

HS algebras admit nice oscillator realizations in lower dimension Gunaydin, Vasiliev, Konstein, Fradkin, Linetsky, Sezgin, Sundell:

$$AdS_4: \qquad [Y^A, Y^B] = C^{AB} \qquad \{\xi^a, \xi^b\} = \delta^{ab}$$

which is relevant for $osp(N|4)$;

$$AdS_5: \qquad [a_A, b^B] = \delta_A^B \qquad \{\xi_{\mathbf{a}}, \xi^{\mathbf{b}}\} = \delta_{\mathbf{a}}^{\mathbf{b}}$$

which is relevant for $(p)su(2,2|N)$;

$$AdS_7: \quad [c_i, c^j] = [d_i, d^j] = \delta_i^j \quad \{\alpha_r, \alpha^s\} = \{\beta_r, \beta^s\} = \delta_r^s$$

which is relevant for $osp(8^*|2N)$.

Classical (super)-algebras are generated by appropriate bilinears. HS algebras are generated as envelopings*/some discrete relations. This does not work in AdS_6!

Exceptional $F(4)$ Algebra

The exceptional Lie superalgebra $F(4)$ has 24 even and 16 odd generators with $SO(5,2)\oplus SU(2)$ as its even subalgebra:

$$\begin{aligned}
[M_{AB}\,,\,M_{CD}] &= i\,(\eta_{BC}M_{AD}-\eta_{AC}M_{BD}-\eta_{BD}M_{AC}+\eta_{AD}M_{BC})\;,\\
[T_{\rm a},T_{\rm b}] &= i\epsilon_{\rm abc}T_{\rm c}\,.\\
[M_{AB}\,,\,\Xi^r_\alpha] &= -\,(\Sigma_{AB})_{\alpha\beta}\,\Xi^r_\beta\,,\\
\left\{\Xi^r_\alpha\,,\,\Xi^s_\beta\right\} &= i\epsilon^{rs}\,M_{AB}\left(\Sigma^{AB}\mathcal{C}_7\right)_{\alpha\beta}+3i\,\left(\mathcal{C}_7\right)_{\alpha\beta}\left(i\sigma_2\sigma^{\rm a}\right)^{rs}\,T_{\rm a}\,,
\end{aligned}$$

It is a unique simple superconformal algebra in $5d$.

Gunaydin, Sudarshan constructed the $F(4)$ super-singleton, which consists of $2\times$ scalars $\oplus$ fermion. The quasi-conformal realization is nonlinear! Scalars are in $su(2)_R$ doublet:

$$F(4)-\text{singleton}=2\mathrm{Rac}\oplus\mathrm{Di}$$

Exceptional $F(4)$ SUGRA

The $AdS_{4,5,\not{6},7}$ gauged SUGRAs are covered by $su(n|m)$ and $osp(n|m)$. The gap of AdS_6 was bridged by Romans with the help of $F(4)$ only later.

Romans multiplet is the first in the tensor square of two $F(4)$-supersingletons:

l.w.s.	Romans Field
$\|\Omega\rangle$	scalar
$\bar{Q}^i_I\|\Omega\rangle$	complex spinor in a doublet of $su(2)_R$
$(\bar{Q}^i_I\bar{Q}^j_J)_A\|\Omega\rangle$	two-form $B_{\underline{mn}}$ and $su(2)_R$ triplet of vectors $A^i_{\underline{m}}$
$(\bar{Q}^i_I\bar{Q}^j_J\bar{Q}^k_K)_A\|\Omega\rangle$	complex gravitinos in an a doublet of $su(2)_R$
$(\bar{Q}^i_I\bar{Q}^j_J\bar{Q}^k_K\bar{Q}^l_L)_A\|\Omega\rangle$	graviton

Exceptional $F(4)$ HS Theory

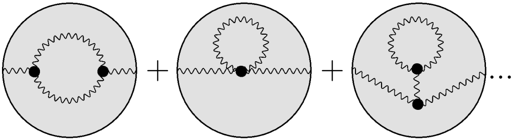
The full tensor square of two $F(4)$-supersingletons contains higher-spin states, which can be arranged into $F(4)$-multiplets:

Scalar tower:	$D(3+s;(s,0)_D)$	$\boxed{\quad s \quad}$
Spinor tower:	$D^r(7/2+s;(s,1)_D)$	$\boxed{\quad s \quad}_{\frac{1}{2}}$
Tensor field tower:	$D(4+s;(s,2)_D)$	$\begin{array}{l}\boxed{\quad s+1 \quad}\\ \boxed{}\end{array}$
Vector field tower:	$D^a(4+s;(s+1,0)_D)$	$\boxed{\quad s+1 \quad}$
Gravitino tower:	$D^r(9/2+s;(s+1,1)_D)$	$\boxed{\quad s+1 \quad}_{\frac{1}{2}}$
Graviton tower:	$D(5+s;(s+2,0)_D)$	$\boxed{\quad s+2 \quad}$

plus a short $L(8|8) = 0\times 3 + 0\times 1 + \frac{1}{2}\times 2 + 1\times 1$ multiplet that is found in $5d$ conformal supergravities (Bergshoeff et al).

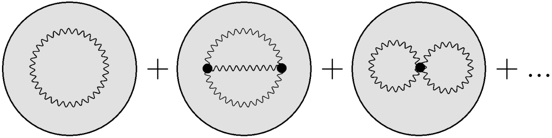
One-Loop

The part of the action known at present does not allow to compute legged diagrams:



Neither do we know the classical action on *AdS*

S_{cl} +



But the one-loop determinant can still be computed! (Giombi, Klebanov, Safdi, Tseytlin, Beccaria, Joung, Lal, Bekaert, Boulanger, ...)

Free Energy AdS/CFT Expansion

On both sides of the duality we expect

$$F_{AdS} = \frac{1}{G} F^0_{AdS} + F^1_{AdS} + \ldots$$
$$F_{CFT} = N F^0_{CFT} + F^1_{CFT} + \ldots$$

Free Energy AdS/CFT Expansion

On both sides of the duality we expect

$$F_{AdS} = \frac{1}{G} F^0_{AdS} + F^1_{AdS} + \dots$$
$$F_{CFT} = N F^0_{CFT}$$

- On general grounds $G^{-1} \sim N$;
- The first term is not available in AdS;
- In free CFT's the second and others vanish identically (!?);
- Generally we would expect to find boring $0 = 0$

$$F^1_{AdS} \sim \sum_s \log\det[\Box + m_s^2] = 0$$

- On contrary, in many cases $F^1_{AdS} \neq 0$ and is an integer multiplet of F^0_{CFT}, which naturally leads to (Giombi, Klebanov)

$$G^{-1} = a(N + \text{integer})$$

What Determinant Can Tell

Depending on the background, various information can be extracted.

Sphere vs. Euclidian AdS: in d odd F is sphere free energy, while in d even F is a-anomaly;

$$F = a \log \Lambda l \qquad\qquad F = \text{number}$$

$S^{d-1} \times R^1$ vs. Global AdS: Casimir Energy and a-anomaly:

$$F = a \log \Lambda l + \beta E_c$$

$S^{d-1} \times S^1$ vs. Thermal AdS: the same as before plus one more — thermal partition function:

$$F = \beta a \log \Lambda l + \beta E_c + F_\beta$$

Sum Rules

Instead of something like supermultiplet sum rules:

$$\sum_s (-)^{2s} d(s) s^p = 0$$

we have infinite sums that may run over bosonic fields only, so no usual SUSY cancellation is possible.

The recipe that was shown to work in many examples is to use (Hurwitz) zeta-function. For example, from Giombi, Klebanov:

$$\frac{1}{360} + \sum_s \left(\frac{1}{180} - \frac{s^2}{24} + \frac{5s^4}{24}\right) = 0$$

It is still unclear if the regularization respects higher-spin symmetries (well-defined question, in principle)!

Casimir Energy Tests

The Casimir Energy is easy to compute as

$$2E_c = (-)^F \zeta(-1) = \sum d_n \lambda_n$$

where d_n and λ_n can be extracted from the character of the representation that corresponds to a given field.

Almost all fields: bosons (Type-A); fermions (SUSY HS); Type-B, C mixed-symmetry fields; partially-massless fields pass the test effortlessly (Giombi, Klebanov, Safdi, Tseytlin, Beccaria, Joung, Lal, Bekaert, Boulager, Basile, ...)

For example, the contribution of super-currents/fermionic HS fields in AdS_4/CFT^3

$$\sum_m \frac{-240m^4 - 480m^3 - 200m^2 + 40m + 17}{1920} = 0$$

Zeta, Free Energy and a-anomaly

One-loop determinant can be computed as

$$F = -\zeta(0)\log \Lambda l - \frac{1}{2}\zeta'(0)$$

Whenever the first term is non-zero, the finite part is ill-defined.

Let's restrict to Euclidian AdS vs. free energy F on a sphere:

$$F^{even} = a\log R \qquad\qquad F^{odd} = \text{number}$$

In AdS there is also a volume divergence:

$$\mathrm{vol}\, AdS_{2n+1} \sim \log R \qquad\qquad \mathrm{vol}\, AdS_{2n+2} \sim \text{const}$$

Algorithm: compute $\zeta(0)$ for each field, then sum up; if(result$==$ 0) then compute $\zeta'(0)$ and keep fingers crossed.

Spectral Zeta Function

Thanks to Camporesi and Higuchi, we can derive the zeta-function for arbitrary-spin field in Euclidian AdS space:

$$\zeta = \frac{\mathrm{vol}(\mathbb{H}^{d+1})}{\mathrm{vol}(S^d)} v_d g(s) \int_0^\infty d\lambda \, \frac{\mu(\lambda)}{\left[\frac{1}{4}(d-2\Delta)^2+\lambda^2\right]^z}$$

where $g(s)$ counts the number of field components and $\mu(\lambda)$ is a spectral density. In flat space $\mu \sim p^{d-1}$, but in AdS_{2k+1}:

$$\text{bosons}: \quad \mu^B(\lambda) = w_d \left(\left(\frac{d-2}{2}+s\right)^2+\lambda^2\right)\prod_{j=0}^{\frac{d-4}{2}}(j^2+\lambda^2)$$

$$\text{fermions}: \quad \mu^F(\lambda) = w_d \left(\left(\frac{d-1}{2}+m\right)^2+\lambda^2\right)\prod_{j=0}^{\frac{d-4}{2}}\left(\left(j+\frac{1}{2}\right)^2+\lambda^2\right)$$

$$\text{hooks}: \quad \mu^H(\lambda) = w_d \frac{\left(\left(\frac{d-2}{2}+s\right)^2+\lambda^2\right)}{\left(\lambda^2+\left(\frac{d}{2}-p-1\right)^2\right)}\prod_{j=0}^{\frac{d-2}{2}}(j^2+\lambda^2)$$

Zeta for Romans $F(4)$ Multiplets

SUSY does help in the higher-spin case too. For example, in AdS_6 for bosonic spin-s field we find

$$\zeta(0) = -\frac{(s+1)^2(7s(s+2)(s(s+2)(9s(s+2)+13)+2)-20)}{30240}$$

If we sum over the Romans spin-s $F(4)$ multiplet we find

$$\zeta_{\text{Romans},s}(0) = -\frac{3}{8}s^4$$

The regulated sum is $3/8$, which is then cancelled by the $L(8|8)$ multiplet.

We also showed that fermionic HS fields $s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$, give exactly zero in all dimensions, which is necessary to make SUSY HS consistent.

a-anomaly can be extracted as $\log R$ term in

$$F_{AdS} = -\zeta(0)\log \Lambda l - \frac{1}{2}\zeta'(0)$$

where $\log R$ comes from volume of AdS_{2k+1}. Giombi, Klebanov, Safdi, Tseytlin, Beccaria showed on many examples that for Type-A (symmetric HS fields)

$$\begin{aligned} \zeta'_{HS,\text{non-min.}}(0) &= 0 \\ \zeta'_{HS,\text{min.}}(0) &= -2a_\phi \log R \end{aligned}$$

therefore $G^{-1} = N - 1$. Here a^d_ϕ is the Weyl-anomaly coefficient of the free scalar field in CFT^d:

$$a^4_\phi = \frac{1}{90}\,, \qquad a^6_\phi = -\frac{1}{756}\,, \qquad a^8_\phi = \frac{23}{113400}$$

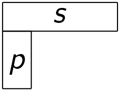
a-anomaly, AdS_{2k+1}/CFT^{2k}

a-anomaly can be extracted as $\log R$ term in

$$F_{AdS} = -\zeta(0)\log \Lambda l - \frac{1}{2}\zeta'(0)$$

where $\log R$ comes from volume of AdS_{2k+1}. We looked at the Type-B that contains mixed-symmetry fields

$$\begin{aligned} \zeta'_{HS,\text{non-min.}}(0) &= 0 \\ \zeta'_{HS,\text{min.}}(0) &= -2a_\psi \log R \end{aligned}$$



where a_ψ^d is the Weyl-anomaly coefficient of the free fermion in CFT^d:

$$a_\psi^4 = \frac{11}{180}\,, \qquad a_\psi^6 = -\frac{191}{7560} \qquad a_\psi^8 = \frac{2497}{226800}$$

a-anomaly, AdS_{2k+1}/CFT^{2k}

AdS zeta-function can be used (Giombi, Klebanov, Pufu, Safdi, Tarnopolsky; Beccaria, Tseytlin) to compute Weyl a-anomaly directly from the AdS side without having to deal with any infinities: the ratio of partition functions with Dirichlet and Neumann boundary conditions = det boundary operator.

With the help of ζ one can obtain a closed form for

$$a'(\Delta) = \frac{1}{\log R} \frac{1}{2\Delta - d} \frac{\partial}{\partial \Delta} \zeta'_\Delta(0)\,,$$

which is then integrated to a-coefficient

$$a(\Delta) = \frac{1}{\log R} \zeta'_\Delta(0) = \int_{d/2}^{\Delta} dx\, (2x - d) a'(x)\,.$$

$\zeta(z)$ is more complicated. In addition $\zeta(0)$ does not vanish for each field, only for specific spectra. Giombi, Klebanov, Safdi showed on many examples

$$\zeta_{HS,non-min}(0) = 0 \qquad \zeta_{HS,min.}(0) = 0$$
$$\zeta'_{HS,non-min}(0) = 0 \qquad \zeta'_{HS,min.}(0) = F^{\phi}_{d}$$

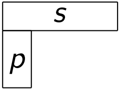
where F is sphere free energy of one scalar field

$$F^3_\phi = \frac{1}{16}(2\log 2 - \frac{3\zeta(3)}{\pi^2})$$
$$F^5_\phi = \frac{-1}{2^8}(2\log 2 + \frac{2\zeta(3)}{\pi^2} - \frac{15\zeta(5)}{\pi^4})$$

F-energy, Type-B, AdS_{2k+2}/CFT^{2k+1}

The Type-B theory that is dual to the singlet sector of $\partial\!\!\!/\psi = 0$ leads to some puzzles. For example,

$$AdS_4: \quad -\frac{1}{2}\zeta'(0) = -\frac{\zeta(3)}{8\pi^2}$$
$$AdS_6: \quad -\frac{1}{2}\zeta'(0) = -\frac{\pi^2\zeta(3)+3\zeta(5)}{96\pi^4}$$



These numbers look a bit different from F-energy of free fermion field

$$F^3_\psi = \frac{1}{16}\left(2\log 2 + \frac{3\zeta(3)}{\pi^2}\right)$$
$$F^5_\psi = \frac{-1}{2^8}\left(6\log 2 + \frac{10\zeta(3)}{\pi^2} + \frac{15\zeta(5)}{\pi^4}\right)$$

but they are not random — many strange numbers like Gleisher-Kinkelin constant are gone.

The Type-B theory that is dual to the singlet sector of $\partial\!\!\!/\psi = 0$ leads to some puzzles. The change in F due to double-trace deformation (Giombi, Klebanov):

$$\delta F^{\psi}_{\Delta} = \frac{2}{\Gamma(d+1)} \int_0^{\Delta - d/2} \cos(\pi u)\Gamma\left(\frac{d+1}{2} + u\right)\Gamma\left(\frac{d+1}{2} - u\right) du\,.$$

The free energy can computed as

$$F^{\psi}_{d} = (-)^{\frac{d-1}{2}} \delta F^{\psi}_{\Delta = \frac{d-1}{2}}\,.$$

The numbers that resulted from the tedious computations in AdS_{2n+2} arrange themselves into:

$$-\frac{1}{2}\zeta'_{HS}(0) = -\frac{1}{4}\delta F^{\psi}_{\Delta = \frac{d-2}{2}}$$

The same as in Giombi, Klebanov, Tan.

F-energy, Type-B, AdS_{2k+2}/CFT^{2k+1}

The Type-B theory that is dual to the singlet sector of $\partial\!\!\!/\psi = 0$ leads to some puzzles. The free energies do not match

$$F_d^\psi = (-)^{\frac{d-1}{2}} \delta F^\psi_{\Delta=\frac{d-1}{2}} \qquad -\frac{1}{2}\zeta'_{HS}(0) = -\frac{1}{4}\delta F^\psi_{\Delta=\frac{d-2}{2}}$$

Before concluding that the duality does not work

- it is not known how to impose the singlet constraint in $d>3$, so we may need to compare with something else that works homogeneously in all d!
- Assuming that in $d=3$ Chern-Simons is the right way to impose the singlet constraint we should remember that

$$F = N^2 \times \text{CS} + N \times \text{vector-model} + N^0 \times \text{one-loop} + \dots$$

  so we compute the second order correction! It unclear how to subtract the CS contribution.

Conclusions

The square of the $F(4)$ supersingleton gives the spectrum of a SUSY higher-spin theory in AdS_6 with the lowest multiplet corresponding to the Romans one. This bridges the gap of AdS_6 in SUSY HS.

Despite the lack of information about the action of higher-spin theories some quantum tests of the HS AdS/CFT duality can be done. Many HS spectra pass the tests. SUSY HS is safe.

Mixed-symmetry fields of Type-B that is dual to free fermion do pass the tests in AdS_{2n+1}/CFT^{2n}, but the F-energy is naively different for AdS_{2n+2}/CFT^{2n+1} — the numbers are not random and can be explained by δF due to double-trace deformations.

Using AdS/CFT one can compute a-anomaly for any CFT field.