Higher-spin currents in (Chern-Simons) vector-models and AdS/CFT

(HSTH) higher-spins and friends

Based on 1610.08472 (S.Giombi, V.Gurucharan, V.Kirilin, S.Prakash, E.S.) and 1610.06938 (A.N.Manashov, E.S.), also 1512.05994 (1601.01310)

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Intro Comments

Chern-Simons vector-models is a rich class of three-dimensional conformal (and not only) field theories, which capture some physics too;

According to (Klebanov, Polyakov; Sezgin,Sundell; Leigh, Petkou; Giombi et al) CS-matter CFT's should be dual to higher-spin theories in AdS_4 for different choice of boundary conditions and/or values of the additional coupling constant;

This naturally leads to three-dimensional bosonization when CS-boson and CS-fermion stay on the opposite sides of CS-coupling Leigh, Petkou; Maldacena, Zhiboedov; Giombi et al;...;

We would like to study these theories and make some tests of the bosonisation conjecture too. The results can also be phrased as predictions for quantum higher-spin theories.

Matter without Chern-Simons

Free Boson. $S = \frac{1}{2}\int \partial\phi\partial\phi$. Has exact HS symmetry in the form of higher-spin conserved tensors:

$$J_0 = \phi^2 \qquad J_s = \phi\partial^s\phi + ... \qquad \Delta = s+1$$

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Critical Boson. $S = \frac{1}{2}\int \partial\phi\partial\phi + \frac{1}{N}(\phi^2)\sigma$. Has exact HS symmetry at $N=\infty$, which is then broken by loops

$$J_0 = \sigma \qquad J_s = \phi\partial^s\phi + \dots \qquad \Delta = s+1+\delta_{s,0}+\frac{1}{N}$$

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Chern-Simons without Matter

Action for $U(N)$ Chern-Simons at level k is

$$S = \frac{ik}{4\pi} S_{\rm CS}$$
$$S_{\rm CS} = \int d^3x \epsilon^{\mu\nu\rho} \mathrm{Tr}(A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho)\,.$$

- this is a topological field theory;
- k does not renormalize;
- breaks parity;
- is a building block of many other theories;
- level-rank duality: in large-N $Z(N,k)$ is the same as $Z(k,N)$;

CS Boson.

$$S=\frac{ik}{4\pi}S_{\rm CS}+\int d^3x\left(D_\mu\bar{\phi}D^\mu\phi+\frac{\lambda_6}{N^2}(\bar{\phi}\phi)^3\right)$$

Critical CS-Boson.

$$S_{\rm crit}=\frac{ik}{4\pi}S_{\rm CS}+\int d^3x\left(D_\mu\bar{\phi}D^\mu\phi+\frac{1}{N}\sigma_b\bar{\phi}\phi\right)$$

CS Fermion.

$$S=\frac{ik}{4\pi}S_{\rm CS}+\int d^3x\bar{\psi}\not{D}\psi$$

Critical CS Fermion.

$$S_{\rm crit}=\frac{ik}{4\pi}S_{\rm CS}+\int d^3x\left(\bar{\psi}\not{D}\psi+\frac{1}{N}\sigma_f\bar{\psi}\psi+g_6\sigma_f^3\right)\,,$$

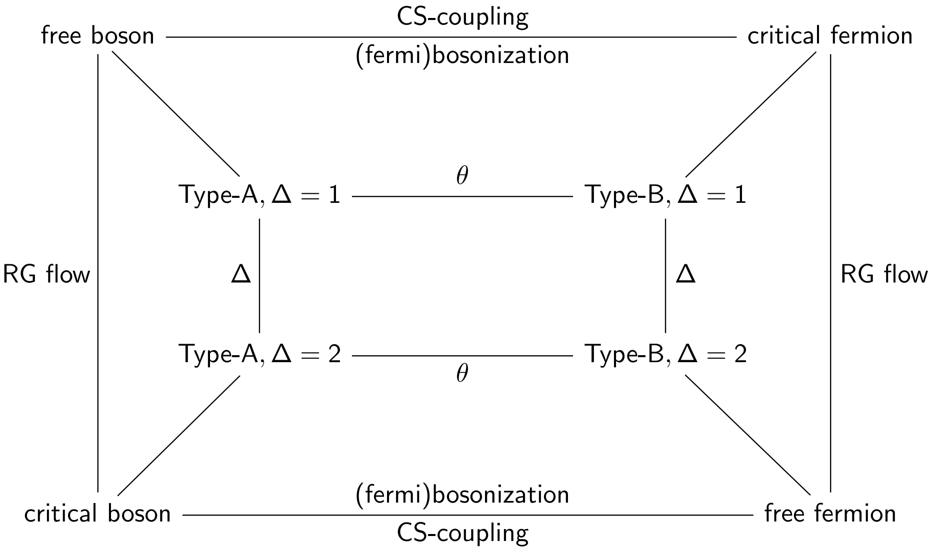
Basic Properties of CS-Matter

- have large-N limit for $\lambda = N/k$ fixed;
- non-SUSY CFT's with a line of fixed-points;
- solvable for any λ at large N;
- the spectrum of single trace operators is the same:

$$J_0: \quad \Delta_0 = 1(2) + O(\frac{1}{N}) \quad J_s: \quad \Delta_s = s + 1 + O(\frac{1}{N})$$

- therefore have an approximate higher-spin symmetry;
- parity is broken in general;
- describe some physics sometimes;
- exhibit a phenomenon of three-dimensional bosonization: CS-boson goes over into CS-fermion;
- should be dual to higher-spin theories in AdS_4 with $g \sim \frac{1}{N}$ and a parameter θ responsible for the violation of parity.

Web of Dualities and Bosonization



Higher-Spin Currents

The conserved currents of **free boson** are produced by Gegenbauer polynomials. More precisely $\phi\partial^s\phi$ is (Todorov et al)

$$J=(\hat{\partial}_1+\hat{\partial}_2)^s C_s^{\frac{d-3}{2}}\left(\frac{\hat{\partial}_1-\hat{\partial}_2}{\hat{\partial}_1+\hat{\partial}_2}\right)\phi(x_1)\phi(x_2)\Big|_{x_1=x_2=x}$$

where $\hat{\partial}_i=\xi\cdot\partial_i$ and $\xi\cdot\xi=0$ makes them traceless. In $3d$:

$$\begin{aligned} J&=f(\hat{\partial}_1,\hat{\partial}_2)\bar{\phi}(x_1)\phi(x_2)\\ f(u,v)&=e^{u-v}\cos(2\sqrt{uv}) \end{aligned}$$

In **CS-boson** we need to replace ∂ with $D=\partial\pm iA$:

$$J=f(\hat{\partial}_1,\hat{\partial}_2)\bar{\phi}(x_1)\phi(x_2)+ig(\hat{\partial}_1,\hat{\partial}_2,\hat{\partial}_3)\bar{\phi}(x_1)\hat{A}(x_3)\phi(x_2)+\ldots$$

Analogously in the CS-fermion case.

Higher-Spin Currents

We will study higher-spin currents at LO in $1/N$ but to all orders in $\lambda = N/k$. HS currents are responsible for their own non-conservation:

$$\partial \cdot J_s = \sum_{s_1,s_2} C_{s,s_1,s_2}(\lambda)\frac{1}{N}J_{s_1}J_{s_2} + F(\lambda)\frac{1}{N^2}JJJ$$

which is an exact non-perturbative quantum equation.

We will use the results of Maldacena-Zhiboedov and explicit computations of the two-point functions in order to recover the λ-dependence in $C(\lambda)$.

Then we will work out the spin-dependence using the equations of motion of CS-matter theories and compute anomalous dimensions with the help of Anselmi trick:

$$\frac{\langle \partial\cdot J\ \partial\cdot J\rangle}{\langle JJ\rangle} \sim \gamma$$

Slightly Broken HS Symmetry

In $3d$ any 3-point function of HS currents can be decomposed

$$\langle JJJ\rangle = \langle JJJ\rangle_b + \langle JJJ\rangle_f + \langle JJJ\rangle_o$$

into structures built from free boson, fermion and an odd one.

Maldacena, Zhiboedov found out that there can be two coupling constants only $\tilde{\lambda}$, $\tilde{N}$ ($\cos^2\theta = 1/(1+\tilde{\lambda}^2)$):

$$\langle J_sJ_sJ_s\rangle = \tilde{N}\left(\cos^2\theta\langle J_sJ_sJ_s\rangle_b + \sin^2\theta\langle J_sJ_sJ_s\rangle_f + \cos\theta\sin\theta\langle J_sJ_sJ_s\rangle_o\right)$$

where $\langle TT\rangle \sim \tilde{N}$ counts effective degrees of freedom and $\tilde{\lambda}$ is a measure of the HS symmetry violation:

$$\partial\cdot J_4 = \tilde{\lambda}\left(J_2\partial J_0 - \frac{2}{5}\partial J_2 J_0\right)$$

First Tests of the Bosonization

It is convenient to rewrite the correlators in terms of macroscopical parameters (Aharony et al; Gur-Ari et al):

$$\tilde{N} = 2N\frac{\sin(\pi\lambda)}{\pi\lambda}\,, \qquad \tilde{\lambda} = \tan(\frac{\pi\lambda}{2})$$

Maldacena, Zhiboedov fixed the structure of $\langle J_s J_s J_s \rangle$ in CS-boson and CS-fermion to be:

$$\langle J_s J_s J_s \rangle_b = \tilde{N}_b \left(\frac{1}{1+\tilde{\lambda}_b^2} \langle J_s J_s J_s \rangle_b + \frac{\tilde{\lambda}_b^2}{1+\tilde{\lambda}_b^2} \langle J_s J_s J_s \rangle_f + \frac{\tilde{\lambda}_b}{1+\tilde{\lambda}_b^2} \langle J_s J_s J_s \rangle_o \right)$$

$$\langle J_s J_s J_s \rangle_f = \tilde{N}_f \left(\frac{\lambda_f^2}{1+\tilde{\lambda}_f^2} \langle J_s J_s J_s \rangle_b + \frac{1}{1+\tilde{\lambda}_f^2} \langle J_s J_s J_s \rangle_f + \frac{\tilde{\lambda}_f}{1+\tilde{\lambda}_f^2} \langle J_s J_s J_s \rangle_o \right)$$

The duality map is $\tilde{N}_b = \tilde{N}_f$, $\tilde{\lambda}_b^2 = 1/\tilde{\lambda}_f^2$. Weak coupling: $\tilde{\lambda} \to 0$ approach the starting point. Strong coupling: $\tilde{\lambda} \to \infty$ approaches the dual model at weak coupling.

Fixing λ Dependence

Effectively the non-conservation operator should look like:

$$\partial \cdot J_s = \sum_{s_1,s_2} C_{s,s_1,s_2}(\lambda)\frac{1}{N} J_{s_1} J_{s_2} + \sum_{s_1,s_2} C_{s,s_1,0}(\lambda)\frac{1}{N} J_{s_1} J_0$$

We can compute the three-point function in two different ways at large-N:

$$\langle j_{s_1} j_{s_2} \partial \cdot j_s \rangle \quad \sim \quad \frac{1}{\tilde{N}} C_{s_1,s_2,s}(\tilde{\lambda}) \langle j_{s_1} j_{s_1} \rangle \langle j_{s_2} j_{s_2} \rangle \,.$$

Another way is provided by the Maldacena, Zhiboedov result:

$$\langle j_{s_1} j_{s_2} \partial \cdot j_s \rangle \sim \tilde{N} \frac{\tilde{\lambda}}{1+\tilde{\lambda}^2}\,, \qquad \langle j_{s_1} j_0 \partial \cdot j_s \rangle \sim \tilde{N}\tilde{\lambda}\,,$$

which gives

$$C_{s_1,s_2,s}(\tilde{\lambda}) \sim \frac{\tilde{\lambda}}{1+\tilde{\lambda}^2} = \left(\tilde{\lambda} + \frac{1}{\tilde{\lambda}}\right)^{-1}$$

Non-conservation Operator in Critical Boson

From the action we can derive the equations of motion:

$$S = \int \frac{1}{2}\partial\phi^2 + \frac{1}{2N}(\phi^2)\sigma \qquad \rightarrow \qquad \Box\phi^i = \phi^i\sigma$$

Using $\Box\phi = \phi\sigma$ we get for $\partial \cdot J \sim \phi\Box\phi$:

$$\partial \cdot J = K(\hat{\partial}_1, \hat{\partial}_2, \hat{\partial}_3)\, \phi(x_1)\phi(x_2)\sigma(x_3)\Big|_{x_i=x}$$

with similar point-splitting arguments.

We can decompose it into irreducibles to make JJ manifest:

$$\partial\cdot J_s = \sum_{a+c<s} C^s_{a,c}\, \partial^a J_{s-1-a-c}\, \partial^c \sigma\,.$$

For example, (Maldacena, Zhiboedov; Giombi et al),
$\partial\cdot J_4 \sim J_2\,\partial\sigma - \frac{2}{5}\partial J_2\,\sigma$

Analogous, but much more complicated expressions, can be found for CS-matter theories.

Anomalous dimensions: Anselmi's trick

Trivial identity — check of non-conservation

$$\langle \partial\cdot J_s(x_1)\partial\cdot J_s(x_2)\rangle = \partial\cdot\partial\cdot\langle J_s(x_1)J_s(x_2)\rangle$$

can give important information provided the two sides can be computed independently. Let J be anomalous

$$\langle J_s(x_1,\eta_1)J_s(x_2,\eta_2)\rangle = \frac{C_s}{\mu^{2\gamma}(x_{12}^2)^{d+s-2+\gamma}}(P_{12})^s$$

and the non-conservation be via double-trace operators

$$K = \partial\cdot J = g_\star JJ \qquad\qquad g_\star \sim \frac{1}{N}\,,\epsilon$$

The ratio gains $g_\star^2$ on the left and γ on the right

$$g_\star^2\frac{\langle \partial\cdot J\ \partial\cdot J\rangle}{\langle JJ\rangle} \sim \gamma$$

Anomalous Dimensions

Combining everything together we find for $\Delta = s + 1 + \gamma_s$:

$$\gamma_s = \frac{1}{\tilde{N}}\left(a_s\frac{\tilde{\lambda}^2}{1+\tilde{\lambda}^2} + b_s\frac{\tilde{\lambda}^2}{(1+\tilde{\lambda}^2)^2}\right) + O(\frac{1}{N^2})$$

where

$$a_s = \begin{cases} \frac{16}{3\pi^2}\frac{s-2}{2s-1}\,, & \text{for even } s\,, \\ \frac{32}{3\pi^2}\frac{s^2-1}{4s^2-1}\,, & \text{for odd } s\,, \end{cases}$$

$$b_s = \begin{cases} \frac{2}{3\pi^2}\left(3g(s) + \frac{-38s^4+24s^3+34s^2-24s-32}{4s^4-5s^2+1}\right)\,, & \text{for even } s\,, \\ \frac{2}{3\pi^2}\left(3g(s) + \frac{20-38s^2}{4s^2-1}\right)\,, & \text{for odd } s\,, \end{cases}$$

with

$$g(s) = \sum_{n=1}^{s}\frac{1}{n-1/2} = \gamma - \psi(s) + 2\psi(2s) = H_{s-1/2} + 2\log(2)\,,$$

Important Features

$$\gamma_s = \frac{1}{\tilde{N}}\left(a_s \frac{\tilde{\lambda}^2}{1+\tilde{\lambda}^2} + b_s \frac{\tilde{\lambda}^2}{(1+\tilde{\lambda}^2)^2}\right) + O(\frac{1}{N^2})$$

- independent computations for bosons and fermions give the same answer! Therefore the bosonization is confirmed;
- $b_s \sim J_{s_1} J_{s_2}$ the computations are identical in free/critical cases — σ-lines are suppressed;
- more non-trivially, a_s are the same in the dual theories;
- even strongly, the full non-conservation operators $\partial \cdot J = JJ + ...$ can be mapped into each other;
- there is $\gamma_s \sim \log s$ behaviour, which is expected for gauge theories in general (Alday, Maldacena);

Important Features

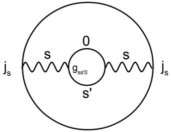
The answer for the critical cases is similar:

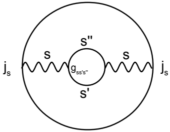
$$\gamma_s = \frac{1}{\tilde{N}}\left(a_s \frac{1}{1+\tilde{\lambda}^2} + b_s \frac{\tilde{\lambda}^2}{(1+\tilde{\lambda}^2)^2}\right) + O(\frac{1}{N^2})$$

At weak coupling $\tilde{\lambda} \rightarrow$ we recover the Wilson-Fisher and Gross-Neveu anomalous dimensions:

$$\gamma_s^{\text{W.F.}} = \gamma_s^{\text{GN}} = \frac{1}{2N} a_s = \begin{cases} \frac{8}{3N\pi^2}\frac{s-2}{2s-1}\,, & \text{for even } s\,, \\ \frac{16}{3N\pi^2}\frac{s^2-1}{4s^2-1}\,, & \text{for odd } s\,. \end{cases}$$

which is the same as the strong limit $\tilde{\lambda} \rightarrow \infty$ of the regular CS-matter theories.





This should correspond to one-loop corrections to the massess of higher-spin fields on the AdS side.

$O(N)$ versus $U(N)$ CS-matter

In $O(N)$ CS-matter theories there are additional diagrams to be evaluated as compared to the $U(N)$ CS-matter.

Fortunately, knowing the non-conservation operator $\partial \cdot J$ allows one to compute the anomalous dimensions in the $O(N)$ case by dropping all odd spins from the sums:

$$\gamma_s = \frac{1}{\tilde{N}}\left(a_s \frac{\tilde{\lambda}^2}{1+\tilde{\lambda}^2} + b_s \frac{\tilde{\lambda}^2}{(1+\tilde{\lambda}^2)^2}\right) + O(\frac{1}{N^2})$$

$$a_s^{O(N)} = \frac{16(s-2)}{3\pi^2(2s-1)}$$

$$b_s^{O(N)} = \frac{4}{\pi^2}\left(-2\sum_{n=1}^{\frac{s}{2}-1}\frac{1}{n-\frac{1}{2}} + \frac{9}{4}\sum_{n=1}^{s-1}\frac{1}{n-\frac{1}{2}} - \frac{59s^4+18s^3-4s^2+54s+35}{6(4s^2-1)(s^2-1)}\right)$$

where b_s vanishes for $s=2$ and $s=4$ due to $\partial \cdot J_4 \sim J_0 J_2$.

Gross-Neveu is a fermionic analog of the critical vector model and has Wilson-Fisher fixed point at large-N for any d.

$$S = -\int d^d x \left[\bar{q}\partial\!\!\!/ q + \frac{g}{2N}(\bar{q}q)^2\right].$$

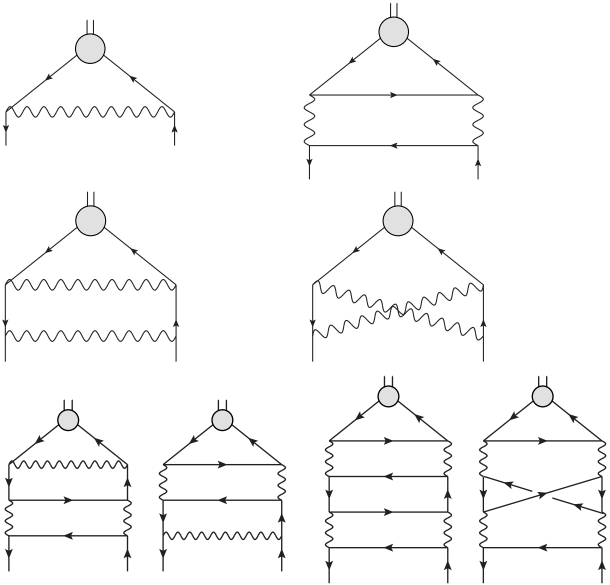
Large-N expansion is a non-perturbative method, which shows better convergence/approximation to the reality properties

Large-N is the kind of expansion scheme that applies to AdS/CFT

Large-N computations are in general much harder than $d \pm \epsilon$ due to a rapid proliferation of graphs and loops, and also due to non-integer indices. Results beyond $1/N$ are rare, especially for composite operators.

Describes the chiral phase transition;

Some Feynman Diagrams



Common point of view

We computed $1/n^2$ anomalous dimensions of the higher-spin currents, both singlets and non-singlets in any d. This requires diagrams up to four loops.

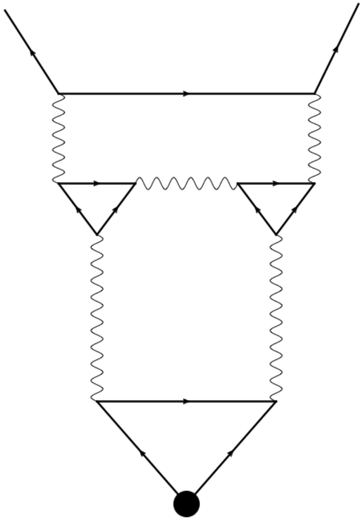
For example, the two-loop correction to the masses of higher-spin fields in Type-B should be:

$$\delta m_s^2 = \frac{2}{n}\eta_1(s-2) + \frac{\eta_1^2}{n^2}\frac{1}{s(1+2s)}\left\{\frac{9}{4}\pi\left(2s^2-1\right) + \frac{s\left(224s^3-244s^2+88s-317\right)}{3(2s-1)}\right.$$
$$+\frac{9}{4}\left(2s^2-1\right)\left[S_1\left(\tfrac{s}{2}-\tfrac{3}{4}\right)-S_1\left(\tfrac{s}{2}-\tfrac{1}{4}\right)\right]+9s\left[S_1\left(\tfrac{s-1}{2}\right)-S_1\left(\tfrac{s-2}{2}\right)\right]+$$
$$\left.+6s(8s+3)S_1(s-1)-6s(7s+5)S_1\left(s-\tfrac{3}{2}\right)-42s(2s+1)\log(2)\right\}.$$

It is very close the linear one:

$$\delta^{(2)}m_s^2 \sim 2\eta_1^2(s-2)\left(\frac{28}{3}+\frac{3}{2}\frac{\log s}{s}+\ldots\right)$$

Bikini problem



If we compare critical boson and critical fermion at large-N we find that there are many graphs that are absent in the latter but are present in the former. There is a one-to-one dictionary between the totality of Feynman and Witten-Feynman graphs in AdS/CFT duality Hartman, Rastelli; Giombi, Yin; Bekaert, Joung, Mourad. This observation shows that there are many that should annihilate for $\Delta = 1, \theta = \frac{\pi}{2}$ but are present for $\Delta = 2, \theta = 0$ in d slightly not equal to 3.

Conclusions

Anomalous dimensions of higher-spin currents in (Chern-Simons) vector-models have been computed using various methods: standard QFT and the non-conservation equation;

Order $1/N^2$ anomalous dimensions were computed in the Gross-Neveu model $\rightarrow$ two-loop masses in higher-spin theories in AdS_d, $3 \leq d \leq 5$;

The structure of the higher-spin symmetry breaking non-conservation equations is worked out;

The conjecture of three-dimensional bosonisation is confirmed at the level of the leading anomalous dimensions of higher-spin currents $\rightarrow$ one-loop masses in parity violating higher-spin theories;

Some other things: structure of the parity-violating correlation functions, subleading terms in γ_s