On mixed-symmetry fields: From Minkowski to (A)dS and back

Karapet Mkrtchyan
Albert-Einstein-Institut, Potsdam, Germany

“Higher Spin Theory and Holography”
30 Nov – 2 Dec 2016, Moscow, Russia

Based on arXiv:1604.05330 in collaboration with Euihun Joung

Motivation

- Particles are unitary irreps of isometry algebra.
- 4d “real world” particles: symmetric tensors.
- Higher dimensional “particles”: generically mixed-symmetry (MS) fields.
- Vasiliev theory (VT) and String theory (ST): two consistent theories of quantum gravity (fingers crossed).
- Missing link between VT and ST may pass through generalized HS theories with MS spectra.

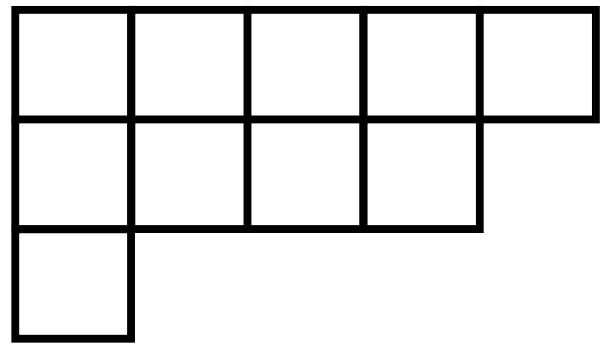
Mixed-Symmetry fields are weird!

Massless MS fields in flat space – Labastida.

Massless MS fields in (A)dS – What's that?

Labastida '87, Metsaev '95, Brink, Metsaev, Vasiliev '00

An example of MS field



Three different “massless” fields in (A)dS with different spectra, none of them same as flat.

The unitary one is unique!
But different in dS and AdS...

Metsaev '99, Zinoviev '02, Boulanger, Iazeolla, Sundell '08, Alkalaev, Grigoriev '09

Hook field in flat space

$$\phi_{\mu\nu,\rho} = -\phi_{\nu\mu,\rho}\,, \qquad \phi_{\mu\nu,\rho} + \phi_{\nu\rho,\mu} + \phi_{\rho\mu,\nu} = 0$$

$$\mathcal{S}_E[\phi] = \frac{1}{2}\int d^dx\sqrt{|g|}\,\phi^{\mu\nu\,,}{}_{\rho}(x)\,\mathcal{G}_{\mu\nu\,,}{}^{\rho}(\phi)$$

$$(\mathcal{G}\,\phi)_{\mu\nu,}{}^{\rho} = -\frac{1}{2}\,\delta^{\rho\lambda\gamma\delta}_{\mu\nu\alpha\beta}\,\partial^{\alpha}\,\partial_{\lambda}\,\phi_{\gamma\delta,}{}^{\beta}$$

Propagating degrees of freedom: hook on light cone.

Hook field in flat space

Gauge symmetries of the flat space action

$$\delta_\epsilon \, \phi_{\mu\nu,\rho} = 2 \, \partial_{[\mu} \, \epsilon_{\nu]\rho}, \qquad \delta_\theta \, \phi_{\mu\nu,\rho} = \partial_\rho \, \theta_{\mu\nu} - \partial_{[\mu} \, \theta_{\nu]\rho}$$

$$\epsilon_{\mu\nu} = \epsilon_{\nu\mu} \, , \qquad \theta_{\mu\nu} = -\theta_{\nu\mu}$$

Gauge for gauge symmetry

$$\epsilon_{\mu\nu}(\xi) = \partial_{(\mu} \, \xi_{\nu)} \, , \qquad \theta_{\mu\nu}(\xi) = \partial_{[\mu} \, \xi_{\nu]}$$

Hook field in (A)dS

Surprise – only one of the gauge symmetries survives.

$$\big(\mathcal{G}^{\Lambda}_{s}\,\phi\big)_{\mu\nu,}{}^{\rho} = -\frac{1}{2}\,\delta^{\rho\lambda\gamma\delta}_{\mu\nu\alpha\beta}\,\nabla^{\alpha}\,\nabla_{\lambda}\,\phi_{\gamma\delta,}{}^{\beta}\,, \qquad \big(\mathcal{G}^{\Lambda}_{a}\,\phi\big)_{\mu\nu,}{}^{\rho} = \big(\mathcal{G}^{\Lambda}_{s}\,\phi\big)_{\mu\nu,}{}^{\rho} - m^{2}_{\Lambda}\,\mathcal{I}$$

$$m^{2}_{\Lambda} = -\frac{4(d-3)}{(d-1)(d-2)}\,\Lambda\,, \qquad (\mathcal{I}\,\phi)_{\mu\nu,}{}^{\rho} = \frac{1}{2}\,\delta^{\rho\beta\gamma}_{\mu\nu\alpha}\,\phi_{\beta\gamma,}{}^{\alpha} = \phi_{\mu\nu,}{}^{\rho} - 2\,\delta^{\rho}_{[\nu}\,\phi_{\mu]\alpha,}{}^{\alpha}$$

Gauge symmetry

$$\delta^{\Lambda}_{s}\,\phi_{\mu\nu,\rho} = 2\,\nabla_{[\mu}\,\epsilon_{\nu]\rho}\,, \qquad \delta^{\Lambda}_{a}\,\phi_{\mu\nu,\rho} = \nabla_{\rho}\,\theta_{\mu\nu} - \nabla_{[\mu}\,\theta_{\nu]\rho}$$

Spectrum in terms of flat massless little group irreps

$$\ydiagram{2,1} \;\oplus\; \Lambda\,\ydiagram{1,1} \qquad\qquad \ydiagram{2,1} \;\oplus\; (-\Lambda)\,\ydiagram{2}$$

1. Is there any theory of hook field that preserves both of these gauge symmetries in (A)dS?

2. Is there any theory of hook field that has a smooth flat space limit with the same number of propagating D.o.F.?

3. Is there any theory of hook field that has the same spectrum around dS and AdS?

Realization of both symmetries in (A)dS

Take following gauge parameters:

$$\epsilon_{\mu\nu}(\alpha) = \nabla_{(\mu}\,\alpha_{\nu)}\,, \qquad \theta_{\mu\nu}(\alpha) = -\frac{1}{3}\,\nabla_{[\mu}\,\alpha_{\nu]}$$

And form a linear combination

$$\left(\delta_{\epsilon(\alpha)} + \delta_{\theta(\alpha)}\right)\phi_{\mu\nu,\rho} = \frac{\Lambda}{(d-1)(d-2)}\,g^{\Lambda}_{\rho[\mu}\,\alpha_{\nu]}$$

Any theory of hook field, that has both gauge symmetries, has a Weyl symmetry!

What used to be flat space gauge-for-gauge symmetry became local Weyl symmetry in (A)dS.

Weyl action for hook

Spectrum: combines the two short irreps in (A)dS

$$\mathcal{S}^{\Lambda}_{W}[\phi] = -\frac{1}{2}\int d^dx\sqrt{|g|}\,\phi_{\mu\nu,\rho}\,(\mathcal{G}^{\Lambda}_{s}\mathcal{I}^{-1}\mathcal{G}^{\Lambda}_{a}\,\phi)^{\mu\nu,\rho}$$

Symmetry: both gauge symmetries (+ Weyl symmetry)

$$\delta^{\Lambda}_{s}\,\phi_{\mu\nu,\rho} = 2\,\nabla_{[\mu}\,\epsilon_{\nu]\rho}\,, \qquad \delta^{\Lambda}_{a}\,\phi_{\mu\nu,\rho} = \nabla_{\rho}\,\theta_{\mu\nu} - \nabla_{[\mu}\,\theta_{\nu]\rho}$$

Flat space limit: smooth, same number of D.o.F.

$$\mathcal{S}_{W}[\phi] = -\frac{1}{2}\int d^dx\sqrt{|g|}\,\phi_{\mu\nu,\rho}\,(\mathcal{G}\mathcal{I}^{-1}\mathcal{G}\,\phi)^{\mu\nu,\rho}$$

4d Weyl gravity action, written in any dimension

$$W^{\mu\nu,\rho\lambda}\,W_{\mu\nu,\rho\lambda} = \mathcal{L}_{GB} + \frac{4(d-3)}{d-2}\left(R^{\mu\nu}\,R_{\mu\nu} - \frac{d}{4(d-1)}\,R^2\right),$$

$$\mathcal{L}_{GB} = \frac{1}{4}\,\delta^{\mu_1\,\mu_2\,\mu_3\,\mu_4}_{\nu_1\,\nu_2\,\nu_3\,\nu_4}\,R_{\mu_1\mu_2,}{}^{\nu_1\nu_2}\,R^{\nu_3\nu_4,}{}_{\mu_3\mu_4}\,.$$

Similarly for the Weyl action of the hook

$$<\mathcal{W}\,\phi\,|\,\mathcal{W}\,\phi>=\frac{1}{12}\int d^dx\sqrt{|g^\Lambda|}\,\mathcal{L}^\Lambda_{\mathrm{GB}}(\phi) + \frac{d-4}{d-3}\,<\phi\,|\,\mathcal{G}^\Lambda\,\mathcal{I}^{-1}\,\mathcal{G}^\Lambda\,\phi>,$$

$$\mathcal{L}^\Lambda_{GB}(\phi) = \frac{1}{2!\,3!}\,\delta^{\mu_1\,\mu_2\,\mu_3\,\mu_4\,\mu_5}_{\nu_1\,\nu_2\,\nu_3\,\nu_4\,\nu_5}\,(\mathcal{R}^\Lambda_s\,\phi)_{\mu_1\mu_2\mu_3,}{}^{\nu_1\nu_2}\,(\mathcal{R}^\Lambda_a\,\phi)^{\nu_3\nu_4\nu_5,}{}_{\mu_4\mu_5}\,.$$

Spectrum: Two short irreps of (A)dS

$$\mathcal{S}^{\Lambda}_{W}[\phi] \simeq m^2_{\Lambda}\left(\mathcal{S}^{\Lambda}_{E\,(s)}[h] - \mathcal{S}^{\Lambda}_{E\,(a)}[f]\right)$$

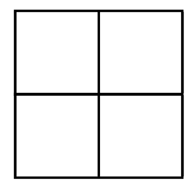
$$\left(\,\yng(2,1) \;\oplus\; (\Lambda)\,\yng(1,1)\,\right) \;\ominus\; \left(\,\yng(2,1) \;\oplus\; (-\Lambda)\,\yng(2)\,\right)$$

Arbitrary d>4: in both dS and AdS there is one ghost hook. The spectrum is the same in any constant curvature background.

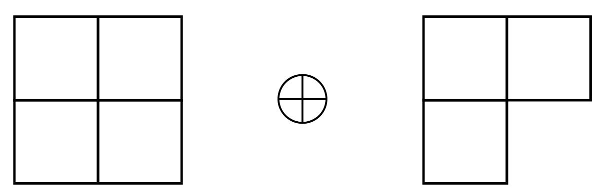
In 4d: unitary theory of spin two and scalar.

What about Window?

For window diagrams there is smooth deformation to (A)dS for massless action with same D.o.F.



In addition, there is partially massless (A)dS window with the following D.o.F.



The Weyl action combines these two D.o.F.'s, similarly to Conformal Gravity.

Weyl action for symmetric fields: all the short irreps of (A)dS combined.

E.Joung and K.M. '12, Metsaev '14
(See also Tseytlin '13, Nutma and Taronna '14)

Does this statement generalize to MS fields? We proved it in the case of two columns. Generic MS fields – work in progress.

Introducing mass term: 4d example

1. Gauge symmetry with symmetric parameter only

$$\mathcal{S}_s[\phi, m] = \mathcal{S}_W^{\Lambda}[\phi] + m^2\, \mathcal{S}_{E\,s}^{\Lambda}[\phi]\,,$$

Spectrum: Massive spin two and a massless scalar.
Unitary in AdS, non-unitary in dS.

2. Gauge symmetry with antisymmetric parameter only

$$\mathcal{S}_a[\phi, m] = \mathcal{S}_W^{\Lambda}[\phi] + m^2\, \mathcal{S}_{E\,a}^{\Lambda}[\phi]\,.$$

Spectrum: Massive spin two and a massless spin two.
Unitary in dS, non-unitary in AdS.

Thank you