Twistors and Conformal Higher-Spin Theory

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Given the deep connections between twistors, the conformal group and conformal geometry it natural to ask if there is a twistor description of conformal higher-spin theories.

Main idea of twistor theory is to relate differential field equations in space-time to holomorphic structures on a complex manifold

Twistor Theory

Starting point is the twistor equation

$$\boxed{\nabla_{A'}^{(A}\omega^{B)} = 0}$$

- A, $A' = 0,1$ are two-component spinor indices.

- $\nabla_{AA'} = \nabla_a$ is the covariant derivative where we make use of the bi-spinor notation for vectors. The metric is: $g_{ab} = \epsilon_{AB}\epsilon_{A'B'}$

- Equation is conformally invariant. Under $g_{ab} \mapsto \hat{g}_{ab} = \Omega^2 g_{ab}$ we have $\hat{\omega}^A = \omega^A$ so that

$$\hat{\nabla}_{A'}^{(A}\hat{\omega}^{B)} = \Omega^{-1}\nabla_{A'}^{(A}\omega^{B)}$$

- Only consistent for: $\Psi_{ABCD}\omega^D = 0$ where

$$C_{abcd} = \Psi_{ABCD}\epsilon_{A'B'}\epsilon_{C'D'} + \tilde{\Psi}_{A'B'C'D'}\epsilon_{AB}\epsilon_{CD}$$

- The space of solutions form a four dimensional vector space over the complex numbers.

Flat Twistor Space

In Minkowski space the twistor equation has non-trivial solutions:

$$\omega^A = \overset{\circ}{\omega}{}^A + i x^{AA'} \overset{\circ}{\pi}_{A'}$$
$$\pi_{A'} = \overset{\circ}{\pi}_{A'}$$

- $\overset{\circ}{\omega}{}^A$, $\overset{\circ}{\pi}_{A'}$ are constant spinor fields defining a four complex-dimensional vector space of solutions called twistor space, $\mathbb{T}_\alpha \simeq \mathbb{C}^4$.
- We denote elements of this solution space W_α and we can represent them in a non-conformally invariant fashion by the pair of fields

$$\boxed{W_\alpha = (\pi_{A'}, \omega^A)}$$

- We can similarly define dual twistors as solutions of $\nabla_A^{(A'} \mu^{B')} = 0$

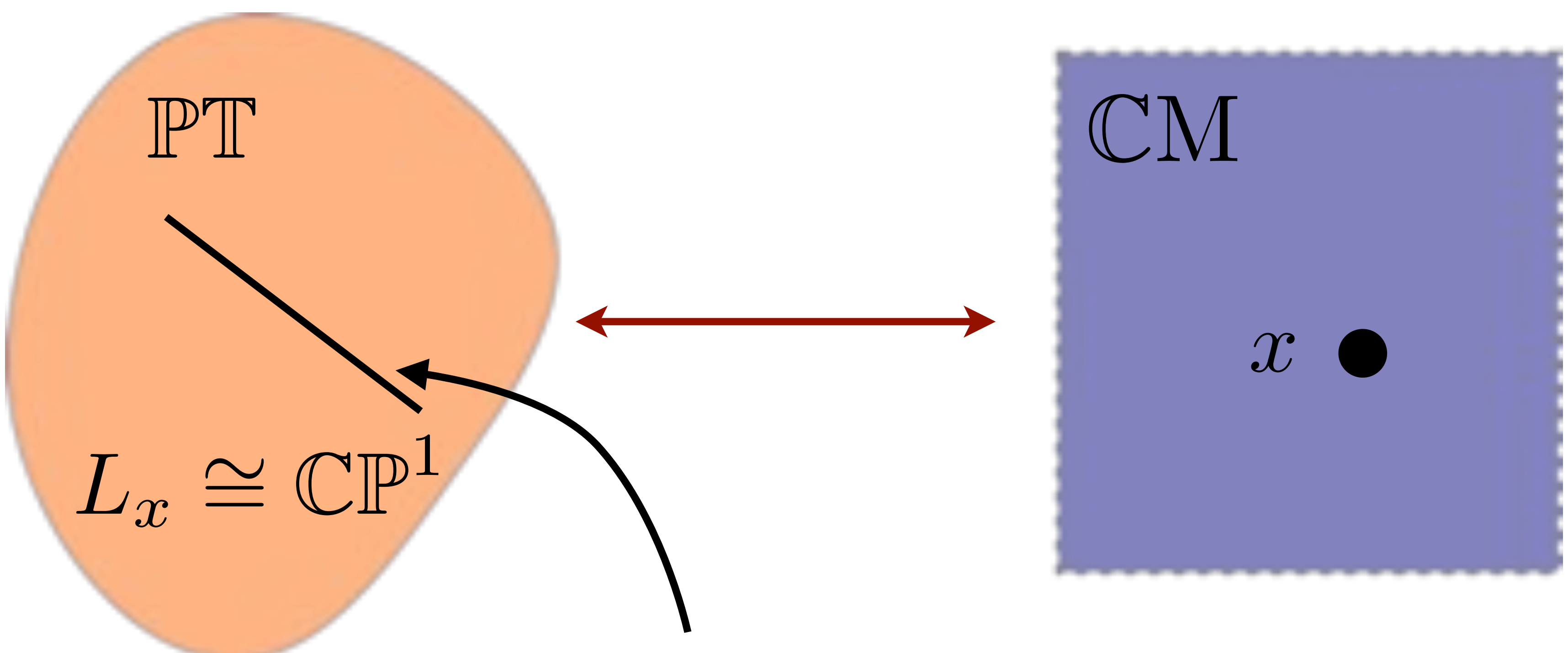
$$\boxed{Z^\alpha = (\mu^{A'}, \lambda_A)}$$

Twistor Theory

Projective Twistor space $\mathbb{PT} \cong \mathbb{CP}^3$ corresponds to the identification

$$Z^\alpha \sim tZ^\alpha \ , \quad t \in \mathbb{C}^*$$

We identify points in $\mathbb{C}M$ with complex projective lines in $\mathbb{PT}$ via the incidence relation



$$\mu^{A'} = i x^{A'A} \lambda_A$$

Twistor Theory

One application of twistor theory gives an identification between complex data on Twistor space and solutions of linear massless equations *[Hitchen '80, Eastwood et al '81]*

$$\bar{\partial}\alpha = 0 \qquad \longleftrightarrow \qquad \nabla^{A_1' A_1}\phi_{A_1 \ldots A_{2h}} = 0$$

α is a (0,1)-form with values in $\mathcal{O}(-2h-2)$.

$\phi_{A_1 \ldots A_{2h}}$ is a space-time field of helicity 2h.

via the Penrose transform

$$\phi_{A_1 \ldots A_{2h}} = \frac{1}{2\pi i}\int_{L_x} \lambda_{A_1} \ldots \lambda_{A_{2h}} \alpha \wedge D\lambda$$

where $D\lambda = \epsilon^{AB}\lambda_A d\lambda_B \equiv \langle \lambda d\lambda \rangle$ is the volume form on $\mathbb{CP}^1$.
Can prove *[Hitchen '80, Atiyah '79]*

$$\boxed{\mathrm{H}^{0,1}(\mathbb{PT};\mathcal{O}(-2h-2)) \cong \begin{array}{c}\text{space of solutions on } \mathbb{CM} \text{ of} \\ \nabla^{A_1' A_1}\phi_{A_1 \ldots A_{2h}} = 0\end{array}}$$

Self-dual Actions

Generalized Weyl $C^{s}_{\mu(s)\,\nu(s)}$ curvature tensors can be decomposed as:

$$C^{(s)}_{A(s)A'(s)\,B(s)B'(s)} = \epsilon_{A_I B_I}\,\widetilde{\Psi}_{A'_I B'_I} + \epsilon_{A'_I B'_I}\,\Psi_{A_I B_I}$$

For YM and Weyl Gravity (and linearized HS)

$$S^s[\phi] = \frac{1}{2\varepsilon^2}\int d^4x \left(\Psi_{A_I B_I}\Psi^{A_I B_I} + \widetilde{\Psi}_{A'_I B'_I}\widetilde{\Psi}^{A'_I B'_I}\right)$$

Schematically we can write this as

$$S^s[\phi] = \frac{1}{\varepsilon^2}\int d^4x\,\Psi\Psi + \text{Boundary term}$$

or introducing Lagrange multiplier

$$\boxed{S^s[\phi, G] = \int d^4x\, G_{A_I B_I}\,\Psi^{A_I B_I} - \frac{\varepsilon^2}{2}\int d^4x\, G_{A_I B_I}\, G^{A_I B_I}\,.}$$

which has E.o.M.: $\Psi_{A_I B_I} = \varepsilon^2\, G_{A_I B_I}\,, \qquad \nabla^{A_I A'_I} G_{A_I B_I} = 0\,.$

Provides an expansion about self-dual sector: $\varepsilon = 0$

Complex curved projective twistor spaces $\mathbb{P}\mathcal{T}$ $\xleftrightarrow{\text{Penrose/Ward}}$ Self-dual space-times: $\Psi_{ABCD} = 0$

Form curved twistor spaces $\mathcal{T}$ by deforming the complex structure

$$\bar\partial = d\bar{Z}^{\alpha}\frac{\partial}{\partial \bar{Z}^{\alpha}} \longrightarrow \bar\partial_f = \bar\partial + f$$

where the deformation is a $(1{,}0)$-vector valued $(0{,}1)$-form

$$f = f^{\alpha}_{\bar\alpha}\; d\bar{Z}^{\bar\alpha} \otimes \partial_{\alpha}$$

To be well-defined on projective space, $\mathbb{P}\mathcal{T}$, f^{α} must have homogeneity 1 under coordinate rescalings. Also has the invariance

$$f^{\alpha} \cong f^{\alpha} + Z^{\alpha}\Lambda$$

The condition for the deformation to give rise to an integrable complex structures is

$$\boxed{\bar\partial_f^2 = (\bar\partial f^{\alpha} + f^{\beta}\wedge\partial_{\beta} f^{\alpha})\partial_{\alpha} = 0}$$

Write an action, [Berkovits/Witten '04], by introducing a Lagrange multiplier field

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Write an action, [Berkovits/Witten '04], by introducing a Lagrange multiplier field

$$S_{S.D.} = \int_{\mathbb{PT}} \Omega \wedge g_\alpha \wedge (\bar{\partial} f^\alpha + f^\beta \wedge \partial_\beta f^\alpha)$$

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Holomorphic (3,0) volume
form of homogeneity 4. In terms of homogeneous coordinates

$$D^3 Z = \epsilon_{\alpha_1 \alpha_2 \alpha_3 \alpha_4} Z^{\alpha_1} Z^{\alpha_2} Z^{\alpha_3} dZ^{\alpha_4}$$

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(0,1)-form with homogeneity -5
satisfying constraint $g_\alpha Z^\alpha = 0$

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$$S_{S.D.} = \int_{\mathbb{P}\mathcal{T}} \Omega \wedge g_\alpha \wedge (\bar{\partial} f^\alpha + f^\beta \wedge \partial_\beta f^\alpha)$$

Equation of motion for tensor field

$$\boxed{\bar{\partial}_f g_\alpha = 0} \quad \xleftrightarrow{\text{P.T.}} \quad \boxed{\nabla^C_{A'} \nabla^D_{B'} G_{ABCD} = 0}$$

Describes a helicity-(-2) particle moving in a self-dual background.
Corresponds to self-dual sector of Weyl gravity

$$S_{S.D.} = \int d^4x \sqrt{-g}\, G^{ABCD} \Psi_{ABCD}$$

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To include the anti-self-dual interactions we add term

$$\boxed{S_{A.S.D.} = -\epsilon \int d^4x \sqrt{-g}\, G^{ABCD} G_{ABCD}}$$

This is equivalent to usual Weyl action up to topological terms.

Write an action, [Berkovits/Witten '04], by introducing a Lagrange multiplier field

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To include the anti-self-dual interactions we add twistor term constructed by using the Penrose transform involving the deformation which gives rise to a MHV vertex like expansion.

Can be used to very efficiently calculate all Einstein gravity MHV scattering amplitude.

Following Maldacena's argument for truncating conformal gravity to Einstein gravity with a non-zero cosmological constant, Adamo and Mason gave a twistor prescription for extracting EG amplitudes by restricting the fields to a "Unitary" sector :

$$\left. \boxed{\begin{aligned} f^\alpha &= I^{\beta\alpha}\partial_\beta h \\ g_\alpha &= I_{\alpha\beta} Z^\beta \tilde{h} \end{aligned}} \right\} \quad h,\ \tilde{h} \text{ describe } \pm 2 \text{ helicity gravitons}$$

Use infinity twistor for AdS space,

$$I^{\alpha\beta} = \begin{pmatrix} \Lambda\epsilon_{AB} & 0 \\ 0 & \epsilon^{A'B'} \end{pmatrix}$$

plane-wave wave-functions for h and $\tilde{h}$ and produces AdS analogue of scattering amplitudes which in the flat limit reproduce Hodges formula for MHV amplitudes after accounting for overall powers of Λ.

Can we generalise this to Higher Spin Fields?

Higher Spin Deformations

Consider deformations of Dolbeault operator for a curved twistor space corresponding to an arbitrary self-dual space time

$$\bar{\partial} = d\bar{Z}^{\alpha}\frac{\partial}{\partial \bar{Z}^{\alpha}} \longrightarrow \bar{\partial}_f = \bar{\partial} + f$$

Higher Spin Deformations

Consider deformations of Dolbeault operator for a curved twistor space corresponding to an arbitrary self-dual space time

$$\bar{\partial}_f = \bar{\partial} + f^{\alpha_I} \otimes \partial_{\alpha_I}$$

Spin-(|I|+1)

using a multi-index notation.

f^{α_I} is a rank-n symmetric tensor valued (0,1)-form of homogeneity n on $\mathcal{T}$. In order to be defined on $\mathbb{P}\mathcal{T}$ it must have the invariance

$$f^{\alpha_1 \dots \alpha_n} \to f^{\alpha_1 \dots \alpha_n} + Z^{(\alpha_1} \Lambda^{\alpha_2 \dots \alpha_n)}$$

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$\Lambda^{(\alpha_1 \dots \alpha_{n-1})}$ symmetric tensor of homogeneity n-1

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Spin-(|I|+1)

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We impose the equation of motion

$$(\bar{\partial} f^{\alpha_I} + f^{\beta_I} \wedge \partial_{\beta_I} f^{\alpha_I}) = 0 \neq \bar{\partial}^2 = 0$$

and can write an action function by introducing a Lagrange multiplier field

$$S_{S.D.} = \int_{\mathbb{P}\mathcal{T}} \Omega \wedge g_{\alpha_I} \wedge (\bar{\partial} f^{\alpha_I} + f^{\beta_I} \wedge \partial_{\beta_I} f^{\alpha_I})$$

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$g_{\alpha_I} = g_{(\alpha_1 \dots \alpha_n)}$ is a (0,1)-form of homogeneity -n-4 s.t. $g_{\alpha_1 \dots \alpha_n} Z^{\alpha_1} = 0$

Linearized Deformations

At a linearized level the deformation and the Lagrange multiplier define elements of cohomology groups

$$\mathrm{H}^{0,1}(\mathbb{P}\mathcal{T};\mathcal{O}(-n-4)) \quad \& \quad \mathrm{H}^{0,1}(\mathbb{P}\mathcal{T};\mathcal{O}(n))$$

Via the Penrose transform for homogeneous tensors [Eastwood, Mason] we have

$$\boxed{\bar{\partial} g_{\alpha_1\ldots\alpha_n} = 0} \quad \xleftrightarrow{\text{P.T.}} \quad \boxed{\nabla^{A_1' A_1} \ldots \nabla^{A_n' A_n} G_{A_1\ldots A_n B_1\ldots B_n} = 0}$$

$$\boxed{\bar{\partial} f^{\alpha_1\ldots\alpha_n} = 0} \quad \longleftrightarrow \quad \boxed{\nabla^{A_1'}_{(A_1} \ldots \nabla^{A_n'}_{A_n} \phi_{B_1\ldots B_n)A_1'\ldots A_n'} = 0}$$

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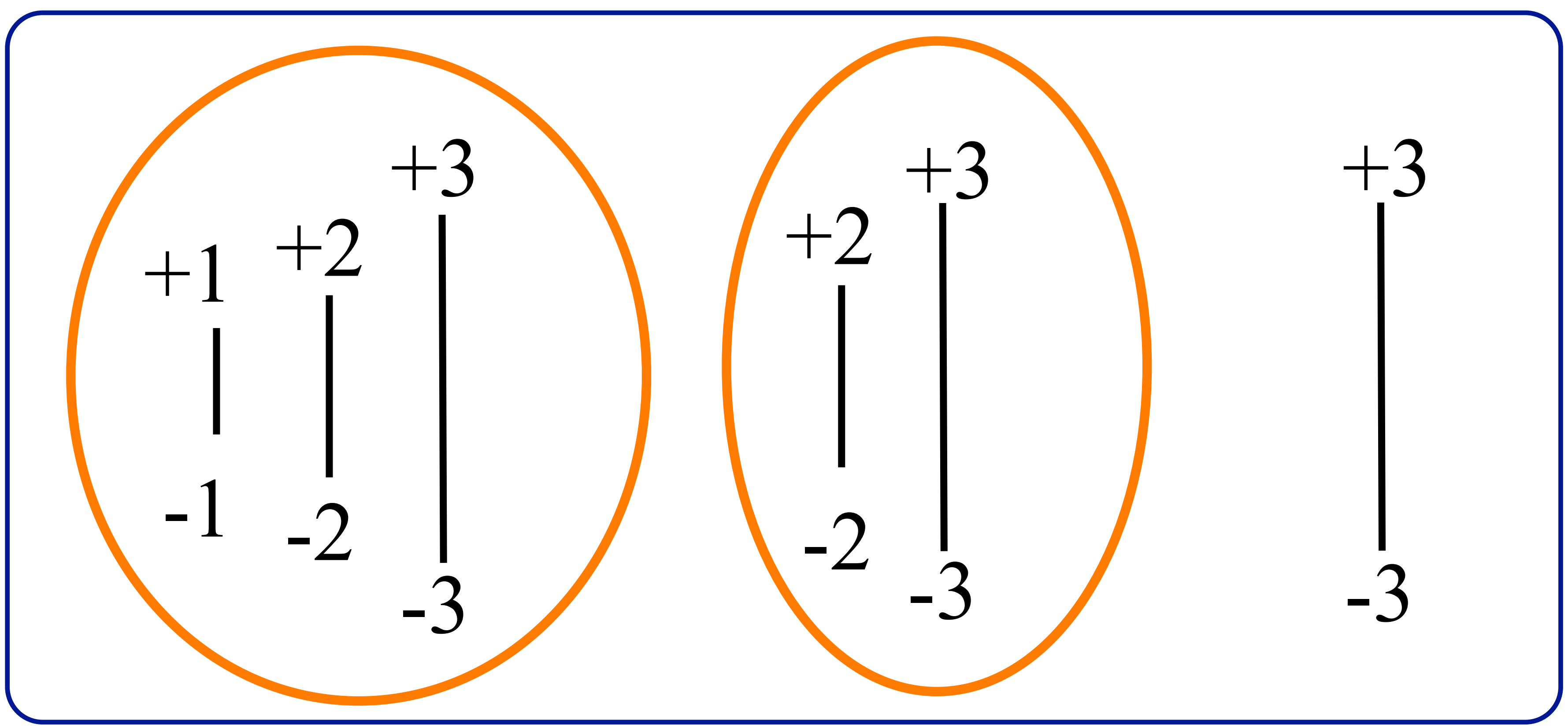
Straightforward to calculate flat-space on-shell spectrum *á la Witten & Berkovits* : e.g. spin-3



= 12 on-shell states

Linearized Conserved Charges

There is a nice trick for calculating conserved charges in a linearised spin-$n/2$ massless theory. Given a field satisfying

$$\nabla^{A_1 A_1'} \phi_{A_1 \ldots A_n} = 0$$

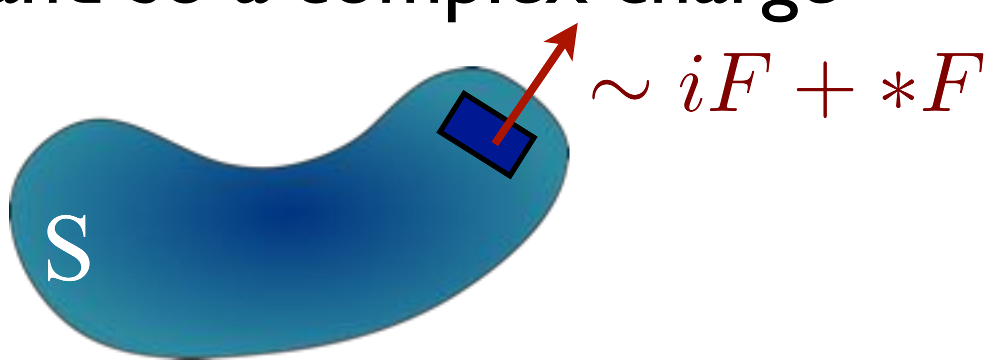
and a solution of the twistor equation

$$\nabla^{B_1'(B_1} \lambda^{B_1 \ldots B_{n-2})} = 0$$

we can form a solution to Maxwell equation

$$\chi_{A_1 A_2} = \phi_{A_1 \ldots A_n} \lambda^{A_3 \ldots A_n}$$

and so a complex charge



$$\mu + iq \quad = \quad \frac{1}{4\pi} \oint_S \chi_{AB} \epsilon_{A'B'} dx^{AA'} \wedge dx^{BB'}$$

Hence for every solution of the twistor equation we find two conserved charges (half are actually zero).

Linearized Conserved Charges

For conformal higher-spin theory we can do a similar construction.

Given a conformal higher-spin field $G^{\alpha_1\ldots\alpha_{s-1}}{}_{B_1\ldots B_{s+1}}$
we can use a symmetric trace-free $\left[\begin{smallmatrix} s \\ s \end{smallmatrix}\right]$-twistor

$$\hat{W}_{\alpha_1\ldots\alpha_{s-1}}{}^{\beta_1\ldots\beta_{s-1}}$$

to form a solution of the Maxwell equation

$$\chi_{B_1 B_2} = G^{\alpha_1\ldots\alpha_{s-1}}{}_{B1B_2\ldots B_{s+1}} W_{\alpha_1\ldots\alpha_{s-1}}{}^{B_3\ldots B_{s+1}}$$

Each charge corresponds to a solution of the twistor equation. Easy to count in flat-space that there are

$$\tfrac{1}{12}s^2(s+1)^2(2s+1)^2$$

This is essentially the number of conformal Killing tensors and matches with the calculation of Eastwood. Here it is easy to generalise to arbitrary self-dual backgrounds.

At linearized level twistor action appears to describe conformal higher-spin theory.

What about non-linear terms?

Infinite-Spin Deformations

We define a deformed Dolbeault operator involving all spins

$$\bar{\partial}_f = \bar{\partial} + \sum_{|J|=0}^{\infty} f^{\beta_J} \partial_{\beta_J}$$

then demand that each term vanishes

$$\boxed{\bar{\partial}_f^2 = 0 \Leftrightarrow \; \bar{\partial} f^{\alpha_I} + \sum_{|J|=0}^{|I|} \sum_{|K|=0}^{\infty} C_{|K||J|} f^{(\alpha_J \beta_K} \wedge \partial_{\beta_K} f^{\alpha_{I-J})} = 0}$$

Lower-spin fields source higher-spin fields and can't truncate to just e.g. spin-3 fields. Self-dual action is simply the sum of constraints.

Defines a holomorphic structure on the infinite jet bundle of symmetric product of dual tangent bundles of twistor space.

Can straightforwardly write an action by introducing Lagrange multiplier fields.

Unitary Self-Interactions

On-shell flat space spectrum forms a non-diagonalizable representation of Poincaré algebra. We can identify a "unitary" sub-sector analogous to EG in CG

$$f^{\alpha_1\ldots\alpha_n} = I^{\beta_1\alpha_1}\ldots I^{\beta_n\alpha_n}\partial_{\beta_1}\ldots\partial_{\beta_n}h_s$$

$$g_{\alpha_1\ldots\alpha_n} = I_{\alpha_1\beta_1}\ldots I^{\alpha_n\beta_n}Z^{\beta_1}\ldots Z^{\beta_n}\tilde{h}_s$$

h_s & $\tilde{h}_s$ describe helicity $\pm s$ states.

Choose AdS background, evaluate action on plane-wave solutions to find the AdS analogue of 3-pt MHV-bar "amplitudes"

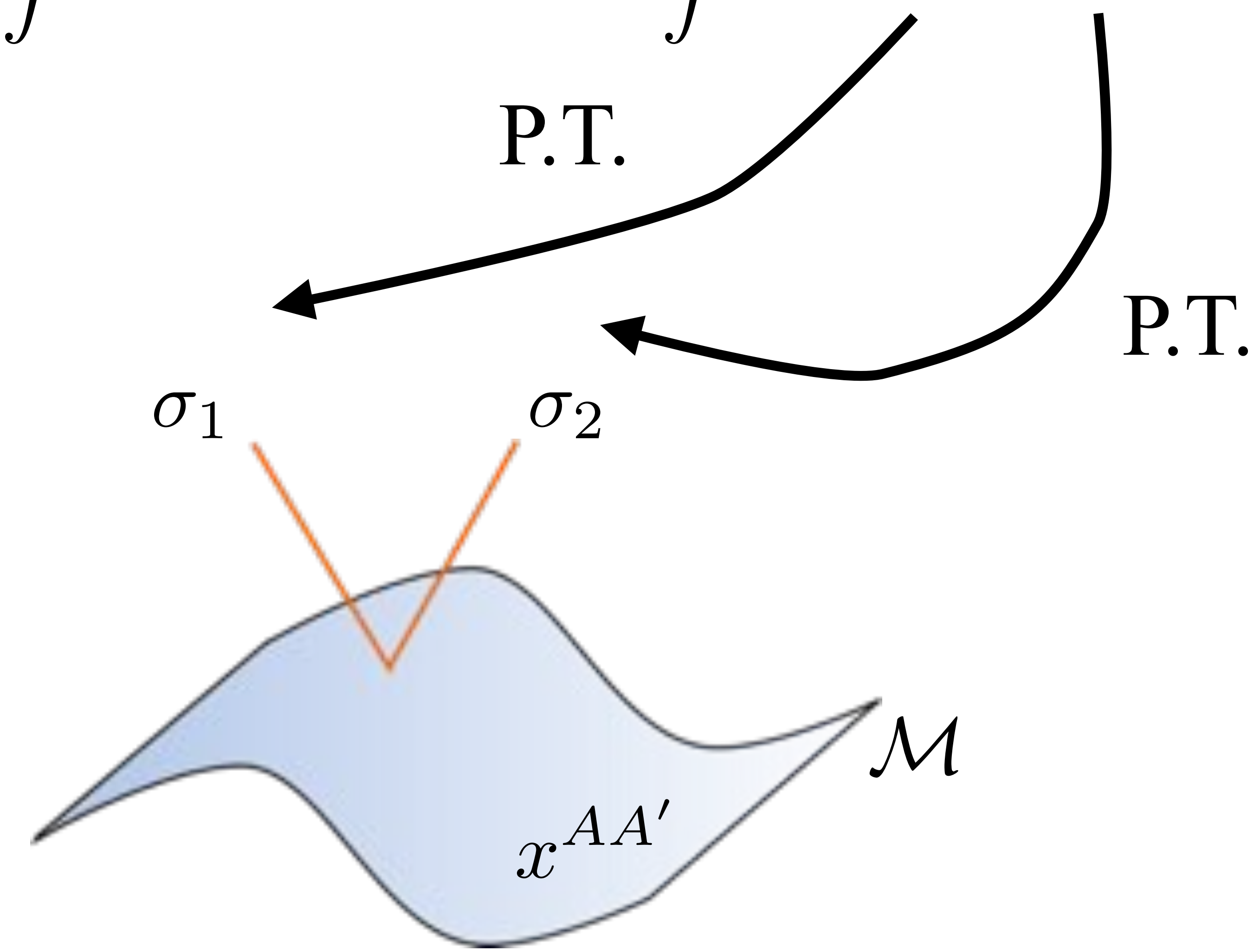
$$\widetilde{\mathcal{M}}_{3,-1}(-s_1,+s_2,+s_3) = \Lambda^{s_1-1}\,\frac{\widetilde{\mathcal{N}}^{(s_1,s_2,s_3)}\,[2\,3]^{s_1+s_2+s_3}}{[1\,2]^{s_1+s_3-s_2}\,[3\,1]^{s_1+s_2-s_3}}\,(1+\Lambda\,\Box_P)^{s_{23|1}}\;\delta^4(P)$$

Thus accounting for powers of Λ we reproduce the unique flat space answer consistent with Poincaré symmetry and helicity constraints.

ASD interactions

So far we have only considered the self-dual sector to motivate the interactions consider the quadratic case

$$S_{\mathrm{spin-n}} = \int d^4x\, G^{A_I}\Gamma_{A_I} - \lambda \int d^4x\, G^{A_I} G_{A_I}$$



It is convenient to picture real, Euclidean space-times. In this case $\mathbb{P}\mathcal{T}$ is fibered over the space-time $\mathcal{M}$ thus we see the action corresponds to a fibre wise product of twistor spaces $\mathbb{PT} \times_M \mathbb{PT}$

ASD interactions

We can use the twistor formulation to propose a full ASD action but it's rather involved. The interaction term for arbitrary negative helicity CHS fields on a conformal gravity background is relatively pleasing

$$S^{(2)}_{\rm int}[f^{(2)}, g] = \int_{\mathbb{PT}\times_{\mathcal{M}}\mathbb{PT}} \Omega\wedge\Omega' \sum_{I=0}^{\infty} Z^{\alpha_I}\, Z'^{\beta_I}\, g_{\beta_I}(Z)\, g_{\alpha_I}(Z')$$

and can be expanded to compute arbitrary (-s, -s, 2, 2, 2, ...) MHV amplitudes in a flat-space limit:

$$\lim_{\Lambda\to 0}\frac{\widetilde{\mathcal{M}}_{n,0}}{\Lambda} \propto \delta^4(P)\,\frac{\langle 12\rangle^{2s+2}}{\langle 1i\rangle^2\,\langle 2i\rangle^2}\,\left|\Phi^{12i}_{12i}\right|$$, where

$$\Phi_{ij} = \frac{[i\,j]}{\langle i\,j\rangle}\,, \quad \text{for } i\neq j$$

$$\Phi_{ii} = -\sum_{j\neq i}\frac{[i\,j]}{\langle i\,j\rangle}\frac{\langle \xi\, j\rangle^2}{\langle \xi\, i\rangle^2}\,.$$

However here it is defined for arbitrary self-dual backgrounds and so provides a general quadratic coupling of CHS fields to conformal gravity.

Conclusions/Outlook

- Described a twistor action to describe higher-spin fields on arbitrary self-dual backgrounds.
- Flat space-time action/spectrum is that of conformal higher-spin theory and produces linearised charges.
- Interactions involve one copy of all spins s>0. Have interpretation of holomorphic structure for infinite jet bundle of homogeneous tensors.
- Identified a "unitary" sub-sector, analogues of EG inside CG, which can be used to reproduce MHV and MHV-bar 3-pt amplitudes up powers of cosmological constant. Can be used to calculate n-point (-s, -s, 2, 2, 2, ...) MHV amplitude.

- What is the full non-linear interacting theory. CHS theory of Segal? What is the "unitary" subsector?
- Can we calculate something interesting? All MHV "amplitudes"? Correlation functions? Partition functions?
- Classical solutions? Instantons? Black Holes?