Local Current Interactions from Nonlinear Higher-Spin Equations in the One-form Sector II

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Higher Spin Theory and Holography-5

Plan

I It will be recalled how interactions of massless fields of all spins with conserved currents result from a linear problem that describes a gluing between rank-one massless system and rank-two current system in the unfolded dynamics approach.

OG, Vasiliev, arXiv:1012.3143

II Outline the current progress in the reconstruction of current interactions in the gauge sector from nonlinear HS equations

OG, Vasiliev, work in progress

AdS_4 background connections

Flat $sp(4)$ connection $w = (\omega^L{}_{\alpha\beta}, \overline{\omega}^L{}_{\dot\alpha\dot\beta}, h_{\alpha\dot\beta})$:

Lorentz connection $\omega^L{}_{\alpha\beta}$, $\overline{\omega}^L{}_{\dot\alpha\dot\beta}$ + vierbein $h_{\alpha\dot\beta}$

Zero curvature conditions

$$R_{\alpha\beta} = \mathrm{d}\omega^L{}_{\alpha\beta} + \omega^L{}_{\alpha\gamma}\omega^L{}_{\beta}{}^{\gamma} - \lambda^2 H_{\alpha\beta} = 0,$$

$$R_{\alpha\dot\beta} = \mathrm{d}h_{\alpha\dot\beta} + \omega^L{}_{\alpha\gamma}h^{\gamma}{}_{\dot\beta} + \overline{\omega}^L{}_{\dot\beta\dot\delta}h_{\alpha}{}^{\dot\delta} = 0.$$

$\lambda^{-1} = \rho$ **radius of** AdS_4

$H^{\alpha\beta} = H^{\beta\alpha} := h^{\alpha\dot\alpha}h^{\beta}{}_{\dot\alpha}\,,\qquad \overline{H}^{\dot\alpha\dot\beta} = \overline{H}^{\dot\beta\dot\alpha} := h^{\alpha\dot\alpha}h_{\alpha}{}^{\dot\beta}$ **the basis 2-forms**

Central on-shell theorem

Massless fields are described by $1-$**forms** $\omega(y,\bar{y}|x)$ **and** $0-$**forms** $C(y,\bar{y}|x)$

Free unfolded equations =Central on-shell theorem Vasiliev (1989)

$$\begin{cases} D^{ad}\omega(y,\bar{y}|x) = i\Big(\eta\overline{H}^{\dot\alpha\dot\beta}\frac{\partial^2}{\partial\bar{y}^{\dot\alpha}\partial\bar{y}^{\dot\beta}}\ \bar{C}(0,\bar{y}|x) + \bar{\eta}H^{\alpha\beta}\frac{\partial^2}{\partial y^\alpha\partial y^\beta}\ C(y,0|x)\Big) \\ D^{tw}C(y,\bar{y}|x) = 0 \end{cases}$$

$$D^{ad}\omega(y,\bar{y}|x) := D^L\omega(y,\bar{y}|x) + \lambda h^{\alpha\dot\beta}\Big(y_\alpha\frac{\partial}{\partial\bar{y}^{\dot\beta}} + \frac{\partial}{\partial y^\alpha}\bar{y}_{\dot\beta}\Big)\omega(y,\bar{y}|x)\,,$$

$$D^{tw}C(y,\bar{y}|x) := D^LC(y,\bar{y}|x) - i\lambda h^{\alpha\dot\beta}\Big(y_\alpha\bar{y}_{\dot\beta} - \frac{\partial^2}{\partial y^\alpha\partial\bar{y}^{\dot\beta}}\Big)C(y,\bar{y}|x)\,,$$

$$D^Lf(y,\bar{y}|x) := \mathsf{d}f(y,\bar{y}|x) + \Big(\omega^{L\ \alpha\beta}y_\alpha\frac{\partial}{\partial y^\beta} + \overline{\omega}^{L\ \dot\alpha\dot\beta}\bar{y}_{\dot\alpha}\frac{\partial}{\partial\bar{y}^{\dot\beta}}\Big)f(y,\bar{y}|x)\,.$$

η **and** $\bar{\eta}$ **complex conjugated free parameters**

$\mathsf{d} = dx^n\frac{\partial}{\partial x^n}$

Current equations and Current deformations

Rank-two unfolded equations in AdS_4 **= current equations**

$$D_{cur}{}^{tw}\,\mathcal{J}(y,\bar{y}|x)\;=0$$

OG, Vasiliev (2003)

$$D^{tw}_{cur}=D^{L}+\lambda e^{\alpha\dot{\beta}}\Big(y^{1}{}_{\alpha}\,\bar{y}^{1}{}_{\dot{\beta}}-y^{2}{}_{\alpha}\,\bar{y}^{2}{}_{\dot{\beta}}-\frac{\partial^{2}}{\partial y^{1\alpha}\partial\bar{y}^{1\dot{\beta}}}+\frac{\partial^{2}}{\partial y^{2\alpha}\partial\bar{y}^{2\dot{\beta}}}\Big)\,.$$

In the unfolded dynamics approach current interactions result from a nontrivial mixing between fields of ranks one and two

Schematically for the flat connection $D=\mathrm{d}+w$

$$\begin{cases} D\omega+L(C,\overline{C},w)=0\\[2ex] DC=0\\[2ex] D_{2}\mathcal{J}=0\end{cases}\quad\Rightarrow\quad\begin{cases} D\omega+L(C,\overline{C},w)+\Gamma_{cur}(w,\mathcal{J})=0\\[2ex] DC+\mathcal{H}_{cur}(w,\mathcal{J})=0\\[2ex] D_{2}\mathcal{J}=0\end{cases}$$

$\Gamma_{cur}(w,\mathcal{J})$ **and** $\mathcal{H}_{cur}(w,\mathcal{J})$ **glue rank-one and rank-two modules**

Consistency conditions $\Rightarrow$ Deformed equations in AdS_4 OG,Vasiliev [1012.3143]

Example : for integer spin s

$$\begin{cases} D^{ad}\omega_{\mathrm{s}}(y,\bar{y}|x) = i\overline{H}^{\dot{\alpha}\dot{\beta}}\bar{\partial}_{\dot{\alpha}}\bar{\partial}_{\dot{\beta}}\overline{C}(0,\bar{y}\mid x) \\ \quad +a_{\mathrm{s}}\overline{H}^{\dot{\alpha}\dot{\beta}}\bar{\partial}_{-\dot{\alpha}}\bar{\partial}_{-\dot{\beta}}\sum_{k=0}^{s-2}\frac{\left(\mathcal{N}_-\right)^{s+k}\left(\overline{\mathcal{N}}_-\right)^{s-k-2}}{(s+k)!}\left(f_+\right)^k\mathcal{J}\Big|_{y^\pm=\bar{y}^\pm=0} \qquad + \ cc \\ D^{tw}C(y,\bar{y}\mid x) + \lambda a_{\mathrm{s}} h^{\mu\dot{\beta}}\mathfrak{F}^{\mathrm{s}}(\mathcal{N}_\pm,\overline{\mathcal{N}}_\pm)\, y^-{}_\alpha\bar{\partial}_{-\dot{\beta}}\left(f_+\right)^{\mathrm{s}-1}\mathcal{J}\Big|_{y^\pm=\bar{y}^\pm=0} = 0 \\ D_2^{tw}\mathcal{J} = 0 \end{cases}$$

$$\mathfrak{F}^{\mathrm{s}} = \left(\mathcal{N}_-\right)^{2\mathrm{s}}\sum_{m\geq 0}\frac{\left(\overline{\mathcal{N}}_+\mathcal{N}_- + \overline{\mathcal{N}}_-\mathcal{N}_+\right)^m}{m!(m+2\mathrm{s}+1)!} \qquad \mathcal{N}_\pm = y^\alpha\partial_{\pm\alpha}\,, \qquad \overline{\mathcal{N}}_\pm = \bar{y}^{\dot{\alpha}}\bar{\partial}_{\pm\dot{\alpha}}$$

$$f_+ = y^{+\nu}y^-{}_\nu - \frac{\partial^2}{\partial\bar{y}^{+\dot{\nu}}\partial\bar{y}^-{}_{\dot{\nu}}}$$

$$y^\pm \sim y_1 \pm y_2, \qquad \bar{y}^\pm \sim \bar{y}_1 \pm \bar{y}_2$$

a_s : arbitrary coefficients, $\eta = \bar{\eta} = 1$

The deformation is consistent in the flat limit

Quadratic corrections from nonlinear equations

In the 0-form sector

$$D^{tw}C + [\omega\,, C]_* + \mathcal{H}_\eta(w,\mathcal{J}) + \mathcal{H}_{\bar\eta}(w,\mathcal{J}) \quad = 0$$

Vasiliev (2015)

contains arbitrary degrees of $\partial_{1\alpha}\partial_2{}^\alpha\bar\partial_{1\dot\alpha}\bar\partial_2{}^{\dot\alpha} \quad \sim$ **non-local**

Modulo field redefinition $C =: C + \Phi_\eta(\mathcal{J}) + \bar\Phi_{\bar\eta}(\mathcal{J})$

$$\begin{aligned}\widetilde{\mathcal{H}}_\eta(w,\mathcal{J}) &= \mathcal{H}_\eta(w,\mathcal{J}) + D^{tw}\Phi_\eta(\mathcal{J})\\ \Phi_\eta(\mathcal{J}) &= \frac{1}{2}\eta\int\frac{dSdT}{}\exp iS_AT^A\int d\tau_i\prod_{i=1}^{3}\theta(\tau_i)\delta'\Big(1-\sum_{i=1}^{3}\tau_i\Big)\\ &\quad \mathcal{J}(\tau_3 s+\tau_1 y, t-\tau_2 y; \bar y+\bar s, \bar y+\bar t; K)*k\end{aligned}$$

$\widetilde{\mathcal{H}}_\eta(w,\mathcal{J}) + \widetilde{\mathcal{H}}_{\bar\eta}(w,\mathcal{J})$ **reproduce the local result** OG,Vasiliev [1012.3143]

In the 1-form sector

$$\Gamma = \Gamma_{\eta\eta}(w,\mathcal{J}) + \Gamma_{\bar\eta\bar\eta}(w,\mathcal{J})\ , \qquad \Gamma_{\eta\bar\eta}(w,\mathcal{J}) = 0$$

$$\begin{aligned}
\Gamma_{\eta\eta} = & \frac{i\eta^2}{2^3}\int_0^1 d\tau \int dS\, dT \int \exp(is_\alpha t^\alpha + i\bar s_{\dot\alpha}\bar t^{\dot\alpha})\Big[\exp(i(\tau s_\alpha - t_\alpha)y^\alpha) \\
& \Big\{(\omega_{L\nu}^{\alpha}\omega_L{}^{\beta\nu})(\tau s_\alpha - t_\alpha)(\tau s_\beta - t_\beta) + 2h_\nu{}^{\dot\alpha}\,\omega_L{}^{\nu\beta}(\tau s_\beta - t_\beta)(\bar t - \bar s)_{\dot\alpha} \\
- & \overline{H}^{\dot\alpha\dot\beta}\,(\bar t - \bar s)_{\dot\alpha}(\bar t - \bar s)_{\dot\alpha}\Big] + \overline{H}^{\dot\alpha\dot\beta}(\bar t - \bar s)_{\dot\alpha}(\bar t - \bar s)_{\dot\beta}\Big\}\mathcal{J}(-\tau s,\, t,\, \bar y + \bar s,\, \bar y + \bar t) \\
+ & \frac{\eta^2}{2^3}\int dS\, dT \int_0^1 d\tau_1 \int_0^1 d\tau_2\, \exp(is_\alpha t^\alpha + i\bar s_{\dot\alpha}\bar t^{\dot\alpha})\Big(\exp i(\tau_1 s_\gamma - \tau_2 t_\gamma)y^\gamma\Big) \\
& \Big\{\Big(\omega_L{}^{\alpha}{}_{\gamma}\omega_L{}^{\gamma\beta}\Big)s_\mu t^\mu \tau_1\tau_2\Big(-\tau_1 s_\alpha s_\beta - \tau_2 t_\alpha t_\beta + 2t_\alpha s_\beta\Big) \\
- & 2(\omega_{L\nu}{}^{\beta}h^{\nu\dot\beta})\tau_1\tau_2 s_\mu t^\mu(t_\beta \bar s_{\dot\beta} + s_\beta \bar t_{\dot\beta}) \\
+ & 2(\omega_L^{\alpha\gamma}h^{\mu\dot\nu})(\tau_1\tau_2 - 1)(-\tau_1 t_\alpha s_\gamma s_\mu \bar s_{\dot\nu} + \tau_2 s_\alpha t_\gamma t_\mu \bar t_{\dot\nu}) \\
- & \bar H^{\dot\alpha\,\dot\beta}s_\nu t^\nu\Big((\tau_1\tau_2 - 1)\bar s_{\dot\alpha}\bar t_{\dot\beta} + 2\bar s_{\dot\alpha}\bar t_{\dot\beta} - \tau_1\bar s_{\dot\alpha}\bar s_{\dot\beta} - \tau_2\bar t_{\dot\beta}\bar t_{\dot\alpha}\Big) \\
- & H^{\alpha\,\beta}s_\alpha t_\beta(\bar s_{\dot\nu}\bar t^{\dot\nu} - 2i)(\tau_1\tau_2 - 1)\Big\}\,\mathcal{J}(-\tau_1 s, +\tau_2 t, \bar y + \bar s, \bar y + \bar t)
\end{aligned}$$

In different form the deformation Γ were obtained by Boulanger, Kessel, Skvortsov and Taronna (2015)

Let

$$\Omega = \quad i\,\frac{\eta^2}{4}\omega_L{}^{\mu\,\nu}\int_0^1 d\tau_1\int_0^1 d\tau_2\int dSdT\,\mathrm{exp}(is_\alpha t^\alpha + i\bar{s}_{\dot\alpha}\bar{t}^{\dot\alpha})$$
$$s_\nu\, t_\mu\ \tau_2\tau_1\ \mathrm{exp}(i(\tau_1 s_\gamma - \tau_2 t_\gamma)y^\gamma)\mathcal{J}(-\tau_1 s, +\tau_2 t, \bar{y}+\bar{s}, \bar{y}+\bar{t})$$

$$\Psi = \quad -i\frac{\eta^2}{4}h^{\alpha\,\dot\beta}\int_0^1 d\tau_1\int_0^1 d\tau_2\int dSdT\mathrm{exp}(is_\beta t^\beta + i\bar{s}_{\dot\alpha}\bar{t}^{\dot\alpha})$$
$$s_\alpha\,\bar{s}_{\dot\beta}\,\tau_1\,\mathrm{exp}(i(\tau_1 s_\gamma - \tau_2 t_\gamma)y^\gamma)\mathcal{J}(-\tau_1 s, \tau_2 t, \bar{y}+\bar{s}, \bar{y}+\bar{t})$$

Field redefinition

$$\widetilde{\Gamma}_{\eta\eta}(\mathcal{J}) \ = \ \Gamma_{\eta\eta} - \mathcal{D}_{ad}\Big\{\Omega + \Psi\Big\} = \ -\frac{i\eta^2}{8}\bar{H}^{\dot\alpha\,\dot\beta}\int_0^1 d\tau\int dS\,dT\int\ \mathrm{exp}(is_\alpha t^\alpha + i\bar{s}_{\dot\gamma}\bar{t}^{\dot\gamma})$$
$$(\bar{t}-\bar{s})_{\dot\alpha}(\bar{t}-\bar{s})_{\dot\beta}\mathcal{J}(-\tau s,\ t,\ \bar{y}+\bar{s},\ \bar{y}+\bar{t})$$

cancels

$$i\eta\overline{H}^{\dot\alpha\dot\beta}\frac{\partial^2}{\partial\overline{y}^{\dot\alpha}\partial\overline{y}^{\dot\beta}}\ \Phi_\eta(\mathcal{J})(0,\overline{y}|x)$$

resulting from the field redefinition in the 0-form sector via First on-shell Theorem. CC is analogous

η^2,$\bar{\eta}^2$-independence

The obtained η^2,$\bar{\eta}^2$-independence is in accordance with the result obtained for lower-spin currents from analysis in the 0-form sector

$\Gamma\eta\bar{\eta} = 0 \quad \Rightarrow$ **Quadratic correction in the 1-form sector** $\sim \eta\bar{\eta}$:

$$\widetilde{\Gamma}_{\eta\bar{\eta}}(\mathcal{J}) =: i\bar{\eta} H^{\alpha\beta}\frac{\partial^2}{\partial y^\alpha \partial y^\beta}\Phi_\eta(\mathcal{J})(y,0|x) + i\eta \overline{H}^{\dot{\alpha}\dot{\beta}}\frac{\partial^2}{\partial \bar{y}^{\dot{\alpha}}\partial \bar{y}^{\dot{\beta}}}\bar{\Phi}_{\bar{\eta}}(\mathcal{J})(0,\bar{y}|x)\,,$$

resulting from the field redefinition in the 0-form sector

via First on-shell Theorem.

As a result

$$\widetilde{\Gamma}_{\eta\bar{\eta}}(\mathcal{J}) = -\frac{i}{8}\eta\bar{\eta}H^{\alpha\beta}\frac{\partial^2}{\partial y^\alpha \partial y^\beta}\int dS dT \exp i[s_\beta t^\beta + \bar{s}_{\dot{\beta}}\bar{t}^{\dot{\beta}}]\int d\tau_i \prod_{i=1}^{3}\theta(\tau_i)$$
$$\delta'\Big(\Big(1-\sum_{i=1}^{3}\tau_i\Big)\Big)\mathcal{J}(\tau_3 s + \tau_1 y, t - \tau_2 y; \bar{\tau}_3\bar{s} + \bar{\tau}_1\bar{y}, \bar{t} - \bar{\tau}_2\bar{y}; K)\Big|_{y=0} + cc$$

Nonlocal deformation should be shifted to a local one modulo exact forms. Ways to do this are different.

Ansatz to reproduce results of 2010

$$\Lambda(\mathcal{J}) \;=\; \frac{-i}{4}\,h^{\alpha\dot\beta}\int dSdT\,\exp iS_AT^A\int\!\!\int d^3\bar\tau d^3\tau\sum_{i,j}g_{i\,j}\partial_{i\alpha}\bar\partial_{j\dot\beta}$$

$$\exp i\Big(\tau_3\partial_{1\alpha}\partial_2{}^{\alpha}+\bar\tau_3\bar\partial_{1\dot\alpha}\bar\partial_2{}^{\dot\alpha}\Big)\qquad \mathcal{J}(\tau_1 y,-\tau_2 y;\bar\tau_1\bar y,-\bar\tau_2\bar y;K)\,,$$

$$g_{11}=\Big\{\bar\tau_1(\bar\tau_2+\tau_1\bar\tau_3)\delta(x)\delta'(\bar x)+\tau_1(\tau_2+\tau_3\bar\tau_1)\delta'(x)\delta(\bar x)-\tau_1\bar\tau_1\delta(x)\delta(\bar x)\Big\}\delta(Z)\Upsilon$$

$$g_{21}=-\Big\{\bar\tau_1(\bar\tau_1+\tau_2\bar\tau_3)\delta(x)\delta'(\bar x)+\tau_2(\tau_2+\tau_3\bar\tau_1)\delta'(x)\delta(\bar x)-\tau_2\bar\tau_1\delta(x)\delta(\bar x)\Big\}\delta(Z)\Upsilon$$

$$g_{22}=\Big\{\bar\tau_2(\bar\tau_1+\tau_2\bar\tau_3)\delta(x)\delta'(\bar x)+\tau_2(\tau_1+\tau_3\bar\tau_2)\delta'(x)\delta(\bar x)-\tau_2\bar\tau_2\delta(x)\delta(\bar x)\Big\}\delta(Z)\Upsilon$$

$$g_{12}=-\Big\{\bar\tau_2(\bar\tau_2+\tau_1\bar\tau_3)\delta(x)\delta'(\bar x)+\tau_1(\tau_1+\tau_3\bar\tau_2)\delta'(x)\delta(\bar x)-\tau_1\bar\tau_2\delta(x)\delta(\bar x)\Big\}\delta(Z)\Upsilon$$

$$Z=\tau_1\bar\tau_1-\tau_2\bar\tau_2\,,\qquad \Upsilon=\prod_{i=1,2,3}\theta(\tau_i)\bar\theta(\tau_i)$$

$$\tilde{\tilde{\Gamma}}_{\eta\bar\eta}(\mathcal{J}) \;=\; D_{ad}\Big(\Lambda(\mathcal{J})\Big)+\tilde\Gamma_{\eta\bar\eta}(\mathcal{J})\;,\qquad \tilde{\tilde{\Gamma}}_{\eta\bar\eta}(\mathcal{J})\;-\;\textbf{Local}$$

$$\tilde{\tilde{\Gamma}}_{\eta\bar\eta}(\mathcal{J}) \;=\; \frac{i}{4}\bar H^{\dot\alpha\dot\beta}\Big[(\bar\partial_2\tau_1-\bar\partial_1\tau_2)_{\dot\alpha}(\bar\partial_2\tau_1-\bar\partial_1\tau_2)_{\dot\beta}\delta'(x)\delta(\bar x)$$

$$+(\bar\partial_2\tau_1-\bar\partial_1\tau_2)_{\dot\alpha}(\bar\partial_2\bar\tau_2-\bar\partial_1\bar\tau_1)_{\dot\beta}\delta(x)\delta'(\bar x)\Big]$$

$$\delta(\tau_3)\delta(Z)\theta_1\theta_2\prod_j\bar\theta_j\exp i\Big(\bar\tau_3\bar\partial_{1\dot\alpha}\bar\partial_2{}^{\dot\alpha}\Big)\mathcal{J}(\tau_1 y,-\tau_2 y;\bar\tau_1\bar y,-\bar\tau_2\bar y;K)\;+cc.$$

Howe dual algebra

To classify currents $\sim$ current deformations it is convenient to use two mutally commutative algebras ${}^{v}\mathfrak{sl}_2$ (vertical)

$$f_+ = y^{+\nu} y^-{}_\nu - \frac{\partial^2}{\partial \bar{y}^{+\dot\nu} \partial \bar{y}^-{}_{\dot\nu}}\,, \qquad f_- = -\frac{\partial^2}{\partial y^{+\gamma} \partial y^-{}_\gamma} + \bar{y}^{+\dot\gamma} \bar{y}^-{}_{\dot\gamma}\,,$$

$$f_0 = y^{+\alpha} \frac{\partial}{\partial y^{+\alpha}} + y^{-\alpha} \frac{\partial}{\partial y^{-\alpha}} - \bar{y}^{+\dot\alpha} \frac{\partial}{\partial \bar{y}^{+\dot\alpha}} - \bar{y}^{-\dot\alpha} \frac{\partial}{\partial \bar{y}^{-\dot\alpha}}\,,$$

and ${}^{h}\mathfrak{sl}_2$ (horizontal)

$$g_+ = y^{+\alpha} \frac{\partial}{\partial y^{-\alpha}} - \bar{y}^{+\dot\alpha} \frac{\partial}{\partial \bar{y}^{-\dot\alpha}}\,, \qquad g_- = y^{-\alpha} \frac{\partial}{\partial y^{+\alpha}} - \bar{y}^{-\dot\alpha} \frac{\partial}{\partial \bar{y}^{+\dot\alpha}}\,,$$

$$g_0 = y^{+\alpha} \frac{\partial}{\partial y^{+\alpha}} + \bar{y}^{+\dot\alpha} \frac{\partial}{\partial \bar{y}^{+\dot\alpha}} - y^{-\alpha} \frac{\partial}{\partial y^{-\alpha}} - \bar{y}^{-\dot\alpha} \frac{\partial}{\partial \bar{y}^{-\dot\alpha}}$$

are dual to the rank-two covariant derivative, mapping current to current

$\Rightarrow$ Deformed equations form ${}^{v}\mathfrak{sl}_2 \otimes {}^{h}\mathfrak{sl}_2$ representation.

The Cartan operator $f_0 \in {}^{v}\mathfrak{sl}_2$ is the rank-two helicity operator

The Cartan operator $g_0 \in {}^{h}\mathfrak{sl}_2$ is the current helicity operator

Elimination of torsion

Torsion-like terms of deformation for spin s gauge one form are proportional to $y^{\mathbf{s-1}}\bar{y}^{\mathbf{s-1}}$

Currents $\mathcal{J}_0$ with zero rank-two helicity do not contribute to torsion-like terms because of pre-factors $H^{\dot\alpha\dot\beta}\bar\partial_{i\dot\alpha}\bar\partial_{j\dot\beta}$ and $H^{\alpha\beta}\partial_{i\alpha}\partial_{j\beta}$

Using manifest formulae for trivial deformations OG,Vasiliev [1012.3143] it can be shown that a current deformation dependent on $\mathcal{J}_h$ with positive integer rank-two helicity h are equivalent to a current deformation dependent on $\widetilde{\mathcal{J}}_0 \sim (f_-)^h\mathcal{J}_h$ modulo D^{ad} -exact forms.

Analogously, current deformation dependent on $\mathcal{J}_h$ with negative integer rank-two helicity h are equivalent to a current deformation dependent on $\widetilde{\mathcal{J}}_0 \sim (f_+)^{-h}\mathcal{J}_h$.

Trivial deformations were constructed for ${}^h\mathfrak{sl}_2$ lower-weight deformations and can be generalized by action of upper ${}^h\mathfrak{sl}_2$ generators

Conclusion

Current interactions result from a linear problem via bilinear substitution

Modulo field redefinitions quadratic corrections in nonlinear equations in the 1-form-sector do not depend on η^2 and $\bar{\eta}^2$

Modulo field redefinitions quadratic corrections do not contribute to torsion-like terms