Holographic instantaneous heating up in 2d CFT.

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Thermalization

- Thermalization in arbitrary interacting quantum field theories is a challenging problem.
- Thermalization is the process, when quantum field initially given by some zero-temperature vacuum state under approach thermal equilibrium

Temperature change

Thermalization means that initial state of quantum system under consideration is taken at vacuum state at zero temperature

In my talk we consider more difficult problem — initial state is arbitrary state taken at arbitrary thermal equilibrium.

This is a typical situation in condensed matter theory and especially in open quantum system (photosynthesis)

AdS/CFT and temperature

AdS/CFT is a natural language to describe quantum field theory at finite temperature — Black hole in AdS describes quantum field theory on the boundary in the same way as AdS describes it at zero temperature

$$ds^2 = -\left(r^2 + R^2 - \frac{M}{r^{d-2}}\right) d\tau^2 + \frac{R^2 dr^2}{R^2 + r^2 - M/r^{d-2}} + r^2 d\Omega_{d-1}^2.$$

Rethermalization in AdS/CFT correspondense

So, the thermalization process in AdS/CFT coressponds to black hole formation process.

From this point of view the problem of our interest — rethermalization (temperature regimes change) corresponds to the space where initial and final states are black holes with different horizons (temperatures).

BTZ-Vaidya metric

In this talk we will focus on the two-dimensional conformal field theories due their relative simplicity with respect to our problem

$$v = t - z_h \operatorname{arctanh}\frac{z}{z_h}$$

$$v < 0$$

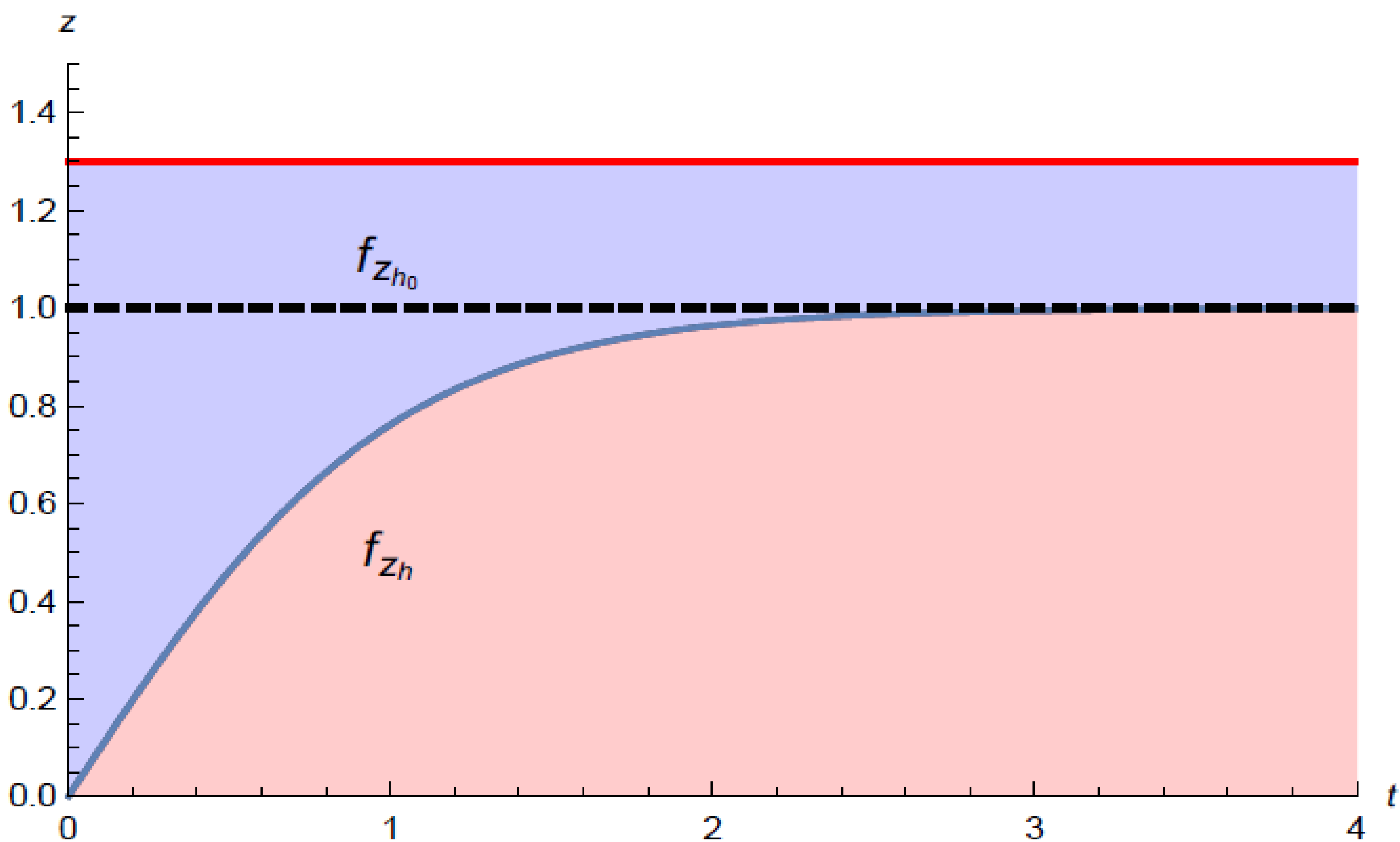
$$ds^2 = \frac{1}{z^2}\left(-f_h(z)dv^2 - 2dvdz + dx^2\right)$$

$$v > 0$$

$$ds^2 = \frac{1}{z^2}\left(-f_{h_0}(z)dv^2 - 2dvdz + dx^2\right)$$

BTZ-Vaidya metric

Black hole-black hole collapse in 2+1 dimensions



Entanglement entropy

Entanglement entropy $S_A(t)$ of a generic state $|\psi\rangle$ for a subsystem by the replica method.

$$S_A(t) = -\frac{\partial}{\partial n}\bigg|_{n=1} \mathrm{tr}\rho_A^n(t).$$

The reduce density matrix ρ_A for a subsystem A is defined as

$$\rho_A = \mathrm{tr}_{A^c}[\rho] \qquad \rho = e^{iHt}|\psi\rangle\langle\psi|e^{-iHt},$$

where A^c denotes the complement of a subsystem A.

Entanglement entropy in 2d CFT

In two dimensional CFTs on a Riemann surface Σ, computations of traces of reduced density matrices boil down to computation of correlation functions of a twisted operator

For example, when the subsystem A is an interval $[l_1, l_2]$,the relation is

$$\mathrm{tr}\rho_A^n(\tau) = \langle \mathcal{O}(l_1, \tau)\mathcal{O}(l_2, \tau)\rangle_\Sigma.$$

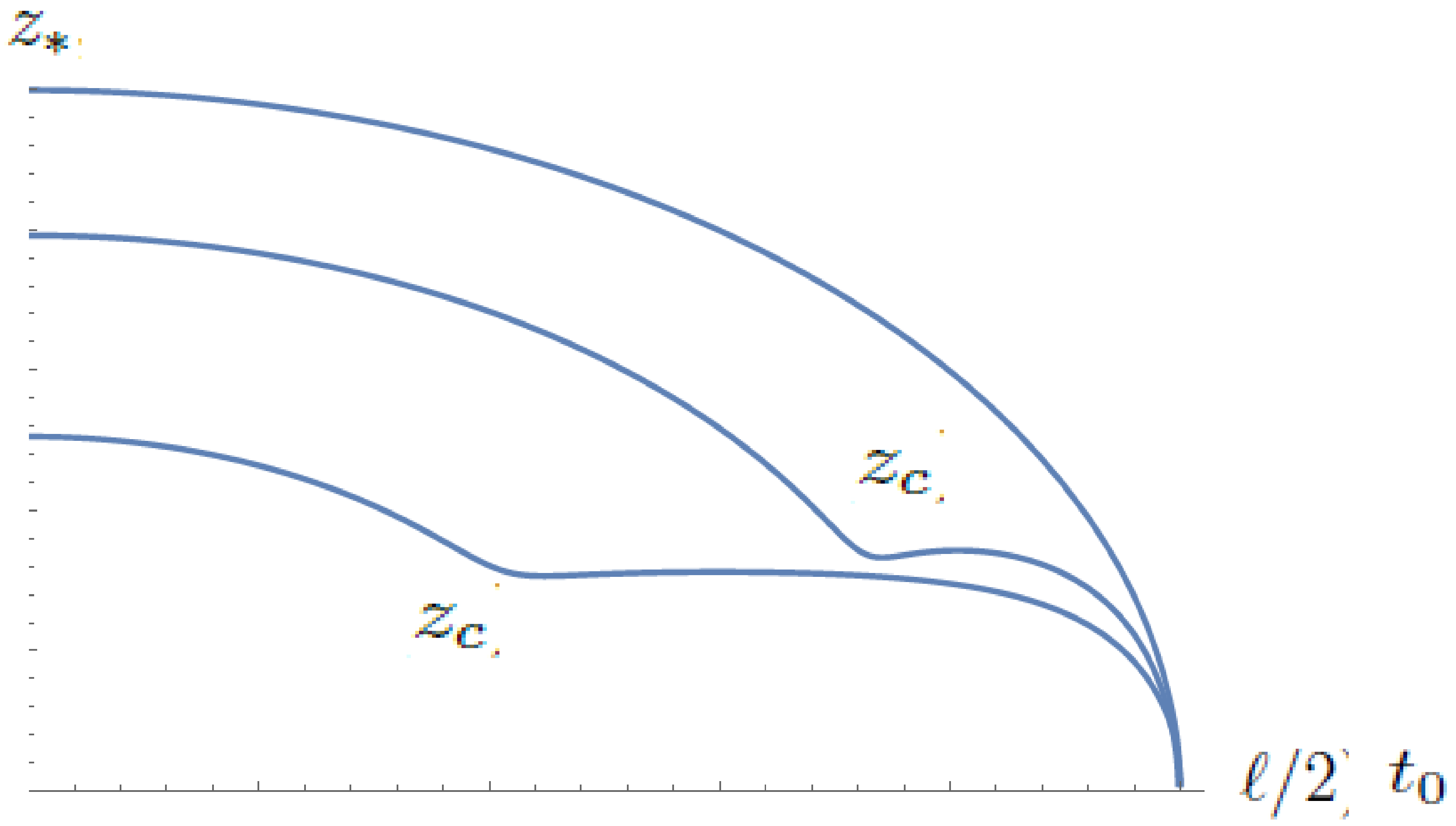
The conformal weights of the operator are $\Delta_n = \bar{\Delta}_n = \frac{c}{24}\left(n - \frac{1}{n}\right)$

Entanglement entropy in AdS/CFT

In arbitrary interacting quantum field theories the calculation of EE is difficult, but one of the most intriguing breakthroughs of AdS/CFT is Ryu-Takayanagi formula. It relates minimal hypersurface (i.e. g $\mathcal{A}_\Sigma(\mathrm{t})$ sic in our case) area and entanglement Σ tropy of the region on the boundary of AdS.

$$S_\Sigma(\mathrm{t}) = \frac{\mathcal{A}_\Sigma(\mathrm{t})}{4G_N}$$

Geodesics in BTZ-Vaidya



Geodesics in BTZ-Vaidya

The action
$$S(L) = \int_{-L/2}^{L/2} dx \frac{\sqrt{Q}}{z},$$
$$Q = 1 - 2v'z' - f(z,v)v'^2$$

$$z(\pm\ell/2) = 0, \qquad z(0) = z_*,$$
$$v(\pm\ell/2) = t_0. \qquad v(0) = v_*.$$

The integrals of motion
$$\mathcal{E} = z' + fv',$$
$$\mathcal{J} = z\sqrt{Q}.$$

Geodesics in BTZ-Vaidya

To find the action of the relevant geodesic, let us denote the corresponding integrals of motion for the part of geodesic under the shell ($v < 0$) as $\mathcal{E} = E$, $\mathcal{J} = J$ and over the shell ($v > 0$) as $\mathcal{E} = E_0$ and $\mathcal{J} = J_0$

The integral of motion $\mathcal{J}$ is conserved due to x-independence of the system. The integral of motion $\mathcal{E}$ is not conserved, but we can match it in the crossing point

$$z_* = z\sqrt{1 - 2v'_- z'_- - f_-(z) v'^2_-}$$

$$z_* = z\sqrt{1 - 2v'_+ z'_+ - f_+(z) v'^2_+}$$

$$0 = z'_- + f_-(z) v'_-$$

$$z'_{+c} + f_+(z_c) v'_{+c} = z'_+ + f_+(z) v'_+$$

Thermalization limit (zero temperature initial state):

when the initial state is zero-temperature vacuum the formulae simplifies to the know simple answer (hep-th/1103.2683, V. Balasubramanian et.al.)

$$\delta\mathcal{L}(t_0,\ell) = 2\ln\left[\frac{\sinh(r_H t_0)}{r_H s(\ell, t_0)}\right]$$

$$\ell = \frac{1}{r_H}\left[\frac{2c}{s\rho} + \ln\left(\frac{2(1+c)\rho^2 + 2s\rho - c}{2(1+c)\rho^2 - 2s\rho - c}\right)\right]$$

Explicit formulae for time

After integrals matching it is sthraightforward to find out the boundary time explicitly

$$t_0 = \int_0^{z_c} \frac{dz}{f(z)} \left(\frac{E_+ z}{\sqrt{(z_*^2 - z^2) f(z) + E_+^2 z^2}} - 1 \right)$$

$$E = \frac{c\gamma^2}{2\Delta\rho s}$$

$$z_* = \frac{z_c}{s}, \quad z_c = \frac{z_h}{\rho} \quad \Delta = \sqrt{\rho^2 - \kappa^2}$$

$$z_{h_0} = \frac{z_h}{\kappa},$$

$$\coth\frac{t_0}{z_h} = \frac{(-\kappa^2 + 2\rho^2 + 1)\, c + 2\rho\Delta}{2\,(\Delta + \rho c)}$$

$$\gamma^2 = \kappa^2 - 1.$$

Explicit formulae for action

Then we need the length(action) of the geodesic.

$$S(L,t) = 2(\mathcal{L}_0 + \mathcal{L})$$

$$E = \frac{c\gamma^2}{2\Delta\rho s}$$

$$\mathcal{L}_0 = -\frac{1}{2}\log\left(\frac{\kappa^2 - s^2\rho^2}{\kappa^2 - \rho\left(c^2\rho + 2c\Delta + \rho\right)}\right)$$

$$z_* = \frac{z_c}{s}, \quad z_c = \frac{z_h}{\rho}$$

$$\mathcal{L} = \frac{1}{2}\log\left(\frac{16 z_h^2 \Delta^2}{\left(4\Delta\rho\sqrt{4c^2\Delta^2\left(\rho^2-1\right)+c^2\gamma^4} + c^2\gamma^4 - 4\Delta^2\left(\rho^2\left(s^2-2\right)+1\right)\right)}\right)$$

$$\Delta = \sqrt{\rho^2 - \kappa^2}$$

$$z_{h_0} = \frac{z_h}{\kappa},$$

$$\gamma^2 = \kappa^2 - 1.$$

Then we need the formula for spatial separation

$$\ell_0 = \frac{z_h}{2\kappa} \log\left(\frac{\rho^2 s^2 - \kappa^2}{(c\kappa + \Delta s)^2}\right)$$

$$E = \frac{c\gamma^2}{2\Delta\rho s}$$

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$$\ell = \frac{z_h}{2} \log\left(\frac{c^2\gamma^4 - 4\Delta\left(-s\sqrt{4c^2\Delta^2(\rho^2-1) + c^2\gamma^4} + \Delta(\rho^2-2)s^2 + \Delta\right)}{c^2\gamma^4 - 4\Delta^2(\rho s - 1)^2}\right)$$

$$z_* = \frac{z_c}{s}, \quad z_c = \frac{z_h}{\rho}$$

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$$\Delta = \sqrt{\rho^2 - \kappa^2}$$

$$z_{h_0} = \frac{z_h}{\kappa}, \ 1.$$

$$\gamma^2 = \kappa^2 - 1.$$

The thermalization time

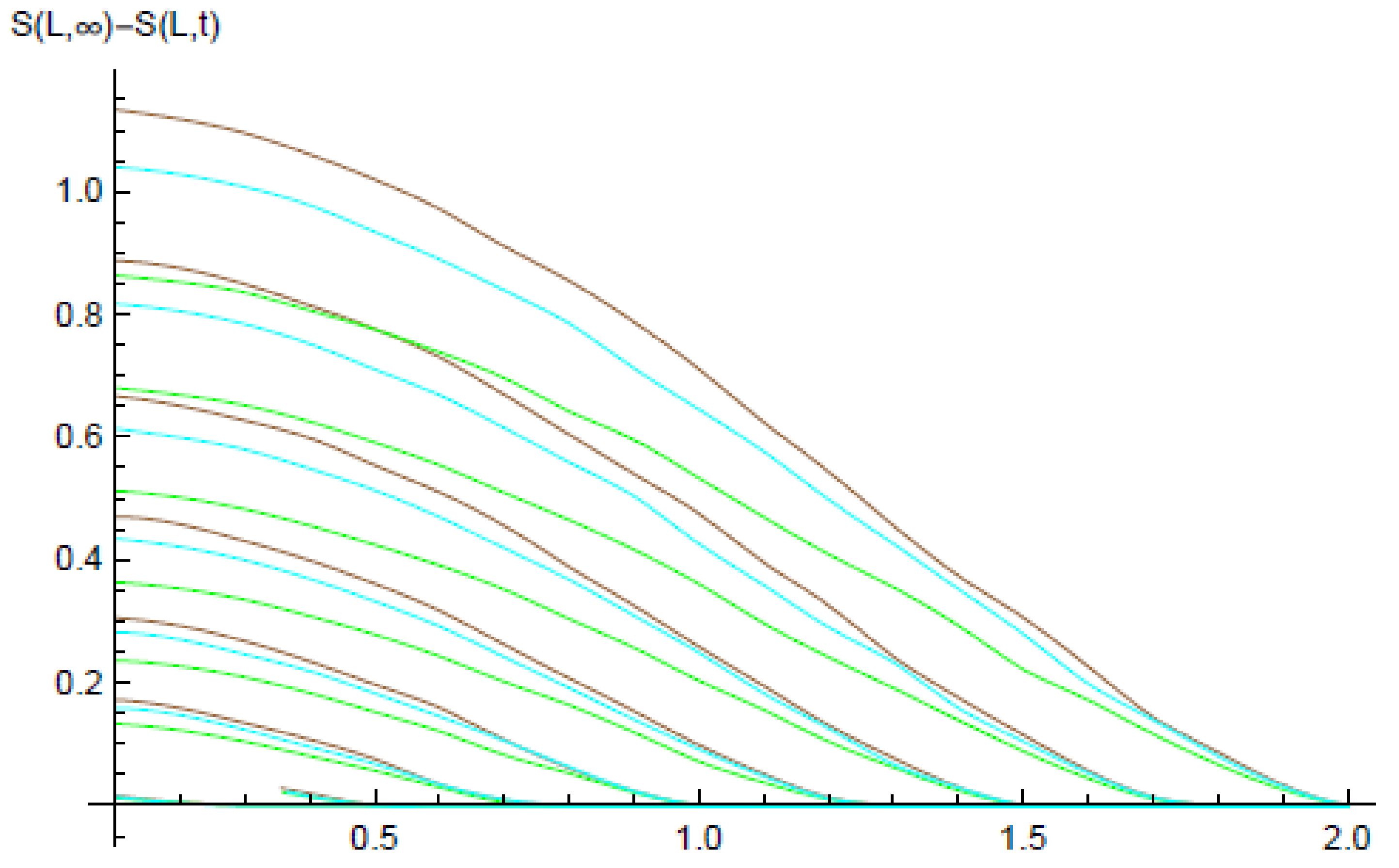
In AdS-Vaidya model of thermalization the thermalization time depends on scale linearly with coefficient 1/2.

Quite unusual feature of 2d CFT holographic model thermalization that this result does not depend of temperature.

From our formulae it follows, that rethermalization <u>has the same dependence, without any dependence on temperature difference.</u>

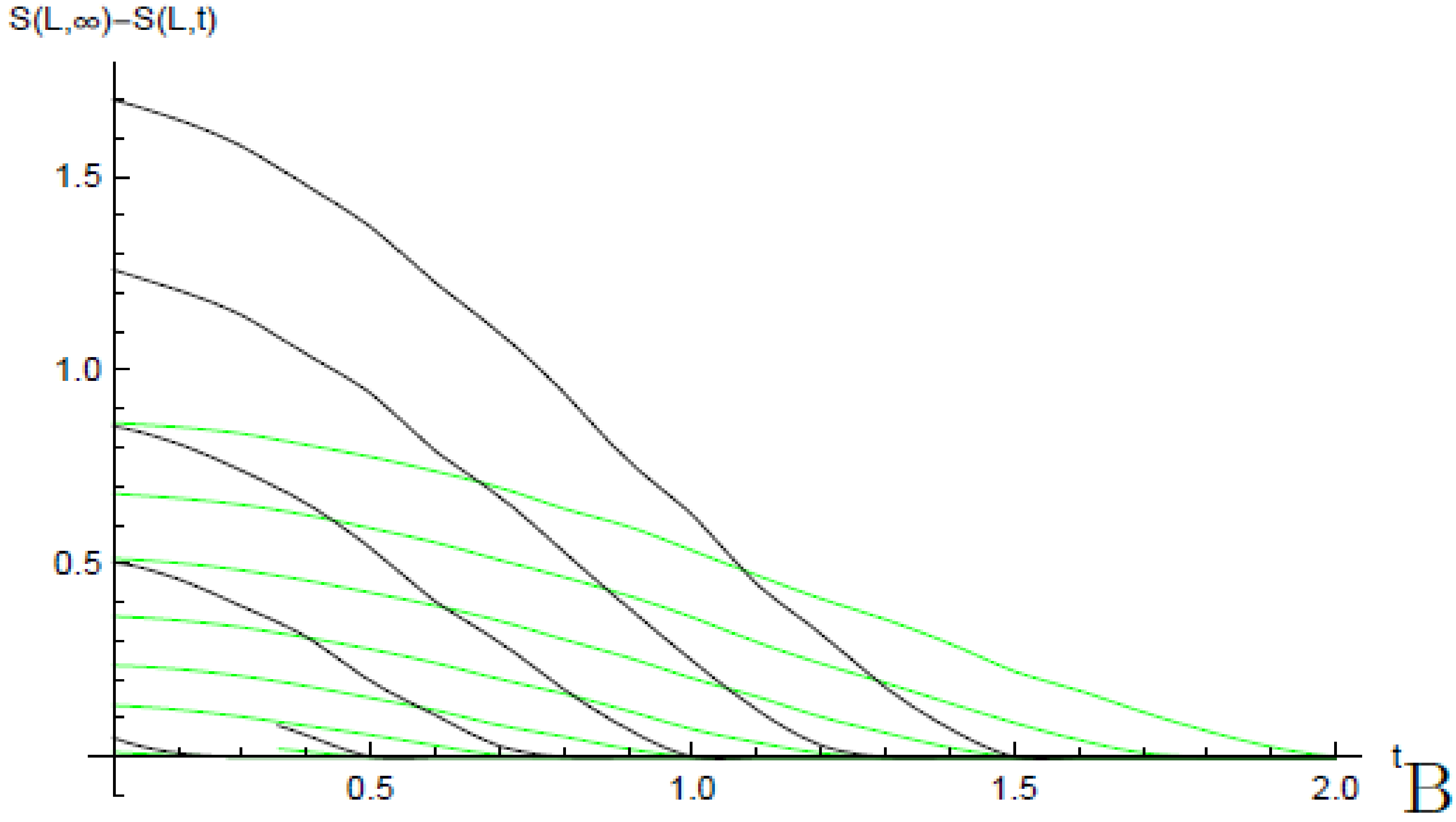
The EE dynamics

The dynamics with the same final initial state and different temperature changes



The EE dynamics

With the same temperature difference. It follows, that in 2d CFT the final state is much more important



Mutual information in holography and CFT

$$I(A;B) = \begin{cases} \mathcal{I}(A;B), & \text{if } \mathcal{I}(A;B) \geq 0 \\ 0 & \text{if } \mathcal{I}(A;B) \leq 0 \end{cases}$$

$$\mathcal{I}(A;B) \equiv S(A) + S(B) - S(A + x + B) - S(x).$$

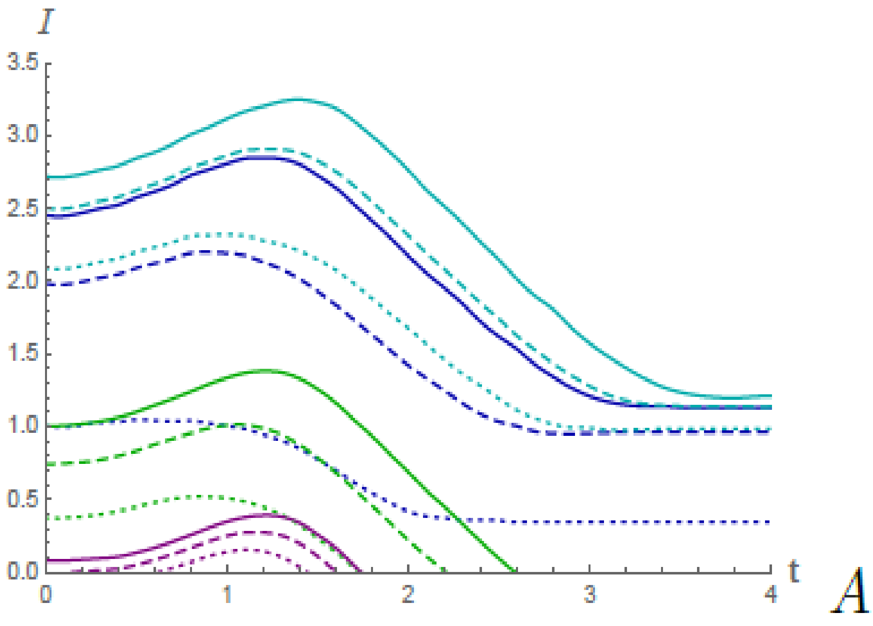
$$\langle \sigma_n(z_1)\tilde{\sigma}_n(z_2)\sigma_n(z_3)\tilde{\sigma}_n(z_4)\rangle$$

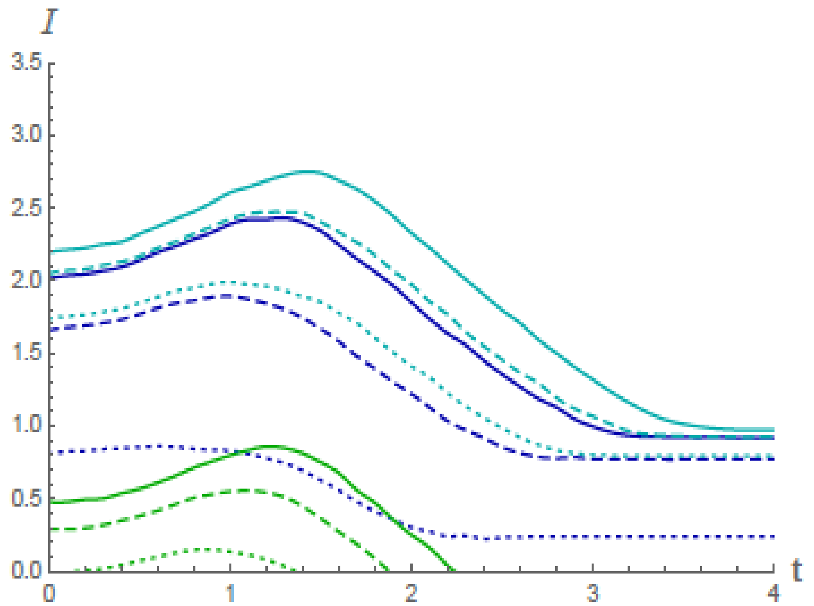
Mutual information in holography

Left — thermalization

Right — heating up (rethermalization)

Some configuration are insensitive, some very sensitive to temperature difference





Conclusion and future work:

-higher dimensional generalizations?

-application to open quantum systems (imprints of conformal theories in biology and phothosynthesis?)

Thank your for attention