Based on 1603.05387

Collaboration with Jin-beom BAE and Shailesh LAL

One Loop Test of Free Adjoint Model Holography

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(Free) Vector Model Duality

Sezgin, Sundell, Klebanov, Polyakov

O(N)/U(N)
Vector Models

Single trace operators

$$T^{\mu\nu}$$

$$J^{\mu}$$

$$J^{\mu_1 \cdots \mu_s}$$

$$\cancel{\mathcal{O}_{\Delta}^{\mu_1 \cdots \mu_s}}$$

Vasiliev Theories

Fields (single particles)

$$G_{MN}$$

$$A_M$$

$$\varphi_{M_1 \cdots M_s}$$

$$\cancel{\phi_{M_1 \cdots M_s}}$$

Free Adjoint Scalar Model Duality

Free Adjoint Scalar

$$S_{\rm CFT}[\phi] = \int d^d x \, {\rm Tr}\left[\phi^\dagger \, \Box \, \phi\right]$$

<u>Single trace operators</u>

$T^{\mu\nu}$

J^{μ}

$J^{\mu_1 \cdots \mu_s}$

$\mathcal{O}_{\Delta}^{\mu_1 \cdots \mu_s}$

$${\rm Tr}\Big[(\partial_{\mu_1}\partial_{\mu_2}\cdots\partial_{\mu_l}\phi)(\partial_{\nu_1}\partial_{\nu_2}\cdots\partial_{\nu_m}\phi)\cdots(\partial_{\rho_1}\partial_{\rho_2}\cdots\partial_{\rho_n}\phi)\Big]$$

HS Gauge Theories + HS Matters

<u>Fields (single particles)</u>

G_{MN}

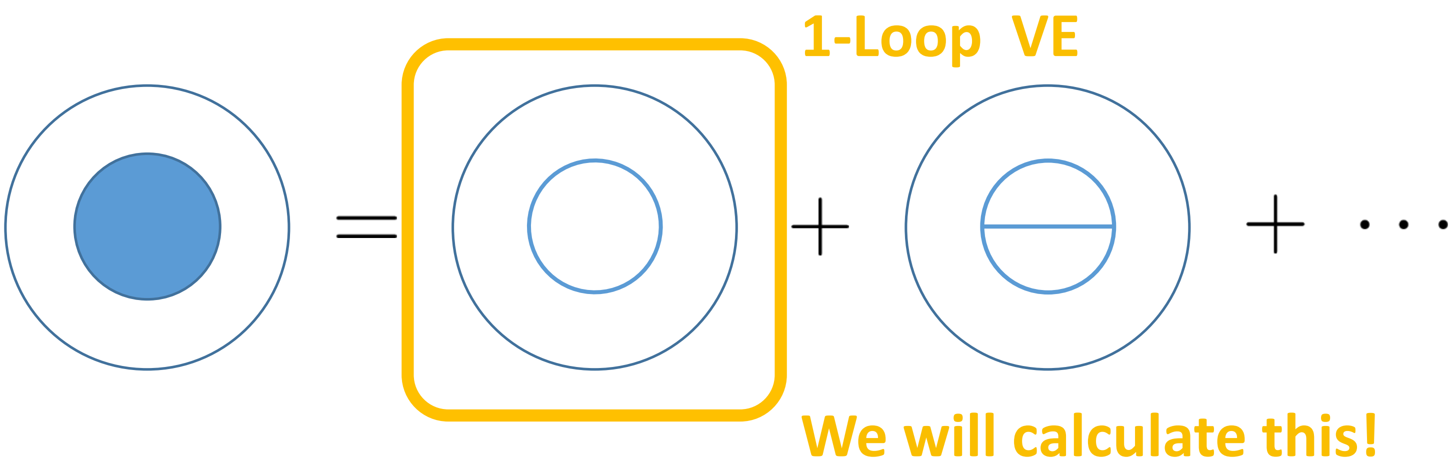
A_M

$\varphi_{M_1 \cdots M_s}$

$\phi_{M_1 \cdots M_s}$

What Can We Do About This Theory?

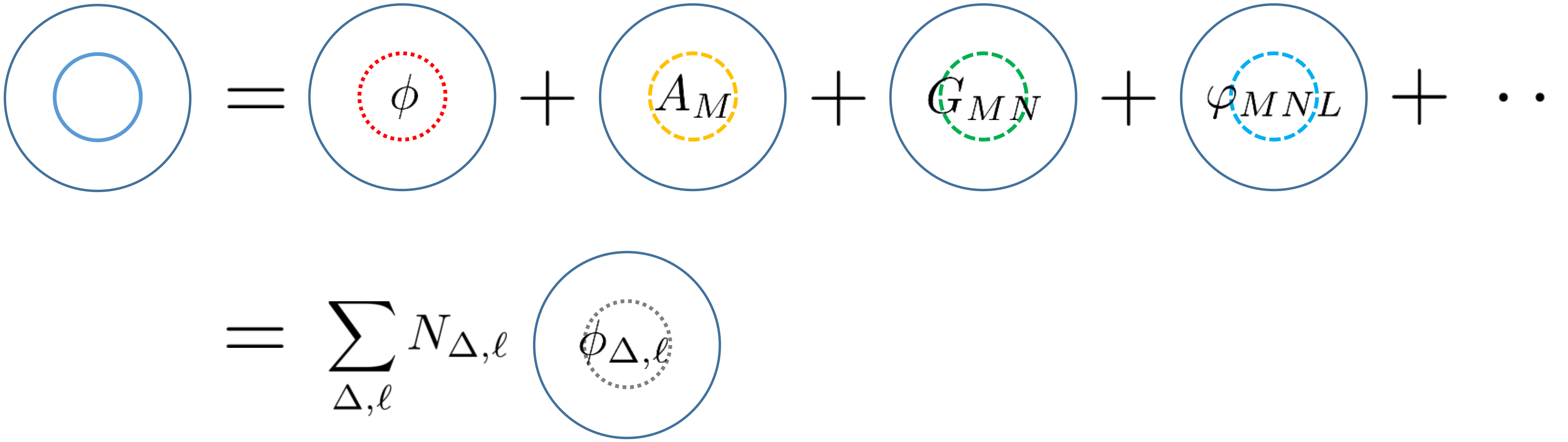
No External Leg: AdS Vacuum (or Free) Energy



- No interaction involved
- We only need the spectrum of AdS theory
- Expect to vanish

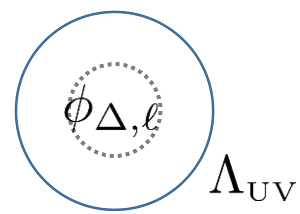
Vector Model Case

- Calculated by Giombi, Klebanov, Safidi; Tseytlin [2013-…]
- VE vanishes for non-min HS, but not for min HS (shift N → N-1)



We need to know

- Multiplicity $N_{\Delta,s}$
- Vacuum (or Free) Energy of AdS Field: $\Gamma^{(1)}_{\Delta,\ell}(z) =$



$\boxed{\frac{1}{z} = \log(\Lambda_{\mathrm{UV}}\, R)}$

❖ Zeta Function

$$\Gamma^{(1)}_{\Delta,\boldsymbol{\ell}}(\Lambda) = -\frac{1}{2}\,\Gamma(z)\,\zeta_{\Delta,\boldsymbol{\ell}}(z)\,, \qquad \zeta_{\Delta,\boldsymbol{\ell}}\,(z) = \int_0^\infty \frac{dt}{t}\,\frac{t^z}{\Gamma(z)}\,K_{\Delta,\boldsymbol{\ell}}(t)$$

Zeta Function (AdS with Sphere bd)

- UV divergence $$\Gamma^{(1)\,\mathrm{div}}_{\Delta,\ell} = -\frac{\boxed{\zeta_{\Delta,\ell}(0)}}{2}\log\frac{\Lambda_{\mathrm{UV}}}{\mu}$$
- Renormalized VE $$\Gamma^{(1)\,\mathrm{ren}}_{\Delta,\ell} = -\frac{\zeta_{\Delta,\ell}(0)}{2}\log(\mu R) - \frac{\boxed{\zeta'_{\Delta,\ell}(0)}}{2}$$
- Integral Representation for $\zeta_{\Delta,\ell}(z)$ Camporesi, Higuchi; Beccaria, Tseytlin

Vector Model Case

- Simple Multiplicity (Flato-Fronsdal)

A

- UV divergent VE $$\sum_s \zeta_{s+d-2,s}(0) = 0 + \cancel{\infty}$$

Additional Regularization

- Renormalized VE $$\sum_s \zeta'_{s+d-2,s}(0) = 0 + \cancel{\infty}$$

B

- Total Zeta Funtion $$\zeta_{\mathrm{AdS}}(z) = \sum_s \zeta_{s+d-2,s}(z) = \mathcal{O}(z^2)$$

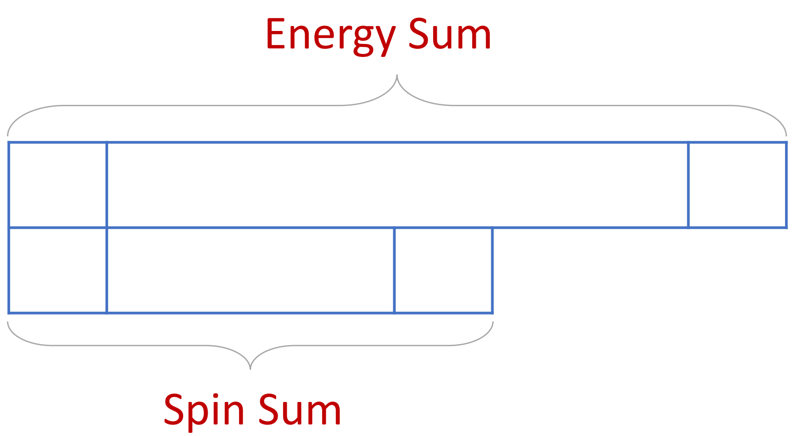
Summatioin Order of Energy vs Spin

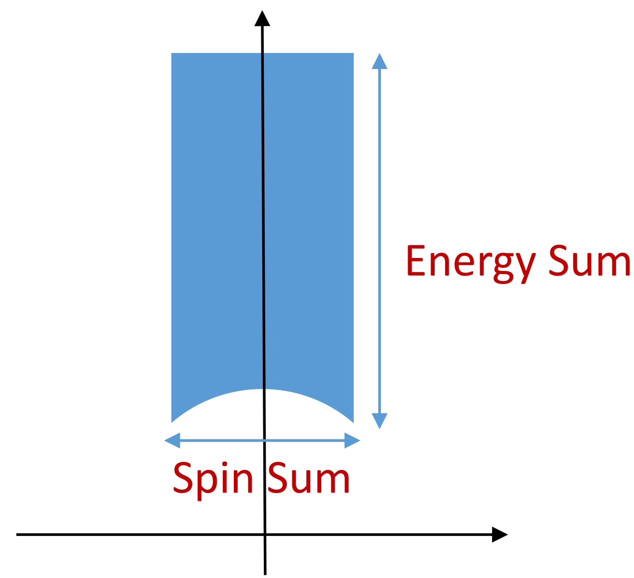
A

1) Energy Sum : **Divergent** → **Regularization**
2) Spin Sum : **Divergent** → **Regularization**

B

1) Spin Sum : **Convergent**
2) Energy Sum : **Convergent**





Regularization controlled by HS Symmetry?

VE vanishes for each HS multiplet?

I. Single Particle Multiplicities

of Each Massive HS Multiplets

II. Vacuum Energy

of Each Massive HS Multiplets

I. Single Particle Multiplicities

of Each Massive HS Multiplets

$(\text{Singleton})^1$ 'Fund. Rep.' of HS Sym

$$\partial_{\mu_1}\partial_{\mu_2}\cdots\partial_{\mu_l}\phi \Leftrightarrow \mathrm{Rac} = \mathcal{D}(\tfrac{1}{2},0)$$

$(\text{Singleton})^2$ Massless HS Multiplet

$$\left(\partial_{\mu_1}\partial_{\mu_2}\cdots\partial_{\mu_l}\phi\right)\left(\partial_{\nu_1}\partial_{\nu_2}\cdots\partial_{\nu_m}\phi\right) \Leftrightarrow \mathrm{Rac}\otimes\mathrm{Rac}$$

$(\text{Singleton})^n$ Massive HS Multiplet

$$\left(\partial_{\mu_1}\partial_{\mu_2}\cdots\partial_{\mu_l}\phi\right)\left(\partial_{\nu_1}\partial_{\nu_2}\cdots\partial_{\nu_m}\phi\right)\cdots\left(\partial_{\rho_1}\partial_{\rho_2}\cdots\partial_{\rho_n}\phi\right) \Leftrightarrow \mathrm{Rac}^{\otimes n}$$

Flato-Fronsdal Theorem (F^2 Thm.)

$$\mathrm{Rac}^{\otimes 2} = \bigoplus_{s=0}^{\infty} \mathcal{D}(s+1,s)\,, \qquad \mathrm{Di}^{\otimes 2} = \mathcal{D}(2,0) \oplus \bigoplus_{s=1}^{\infty} \mathcal{D}(s+1,s)$$

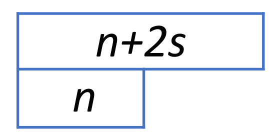
massless spin *s*
in AdS4

Higher Power Extension (F^n Thm.)

- Oscillator Method Konstein, Vasiliev

$$(\mathrm{Di} \oplus \mathrm{Rac})^{\otimes p} = \bigoplus_{n=0}^{\infty} \bigoplus_{2s=0}^{\infty} \dim\left(\pi^{O(p)}_{(n+2s,n)}\right) \mathcal{D}\left(s+n+\frac{p}{2}, s\right)$$

$$\mathrm{Di}^{\otimes q} \otimes \mathrm{Rac}^{\otimes p-q} = \bigoplus_{n=0}^{\infty} \bigoplus_{2s=0}^{\infty} N^{[q,p-q]}_{(n+2s,n)}\, \mathcal{D}\left(s+n+\frac{p}{2}, s\right)$$



- Character Method Barabanschikov, Grant, Huang, Raju; Newton, Spradlin

- Concrete Cases

Order Three (F^3) or '2nd Regge Trajectory'

$$\mathrm{Rac}^{\otimes 3} = \bigoplus_{s=0}^{\infty} (s+1) \left[\mathcal{D}(s+\tfrac{3}{2}, s) \oplus \mathcal{D}(s+\tfrac{7}{2}, s+1) \right]$$

Order Four (F^4) or '3rd Regge Trajectory'

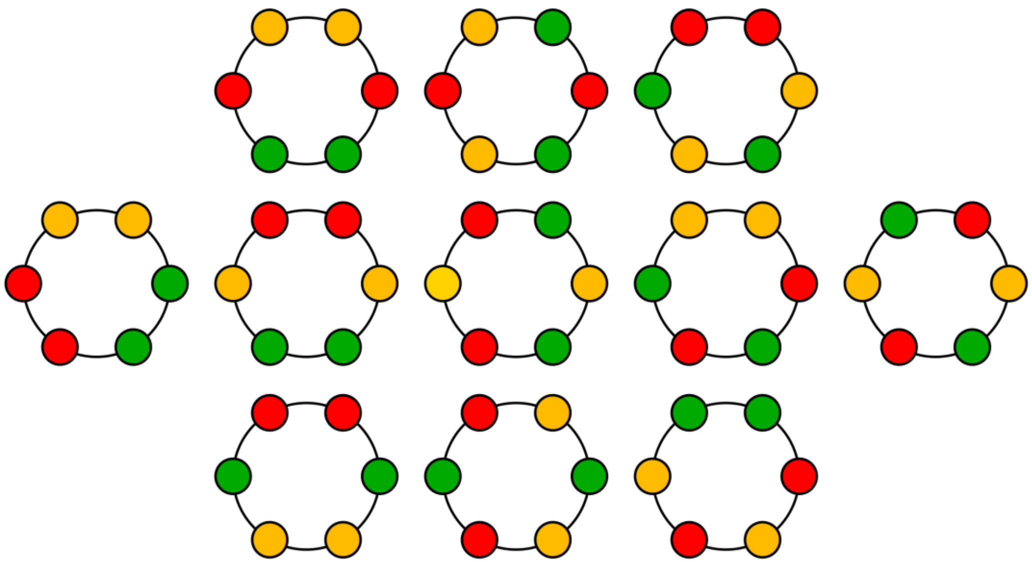
$$\mathrm{Rac}^{\otimes 4} = \bigoplus_{s=0}^{\infty} \frac{(1+s)(2+s)}{2} \mathcal{D}(s+2, s)$$

$$\oplus \bigoplus_{s=0}^{\infty} \bigoplus_{n=1}^{\infty} \frac{(2n+2s+1)(2s+1)+3(-1)^n}{4} \mathcal{D}(s+n+2, s)$$

Single Trace Operators

$$\mathrm{Tr}\left[(\partial_{\mu_1}\partial_{\mu_2}\cdots\partial_{\mu_l}\phi)(\partial_{\nu_1}\partial_{\nu_2}\cdots\partial_{\nu_m}\phi)\cdots(\partial_{\rho_1}\partial_{\rho_2}\cdots\partial_{\rho_n}\phi)\right]$$

Counting Number of Necklaces



Polya Enumeration Thm. Polyakov; Bianchi, Morales, Samtleben, Beisert; Spradlin, Volovich

SU(N) Trace : Cyclic Tensor Product

$$\boxed{\chi_{\mathrm{cyc}^n}(g) = \frac{1}{n}\sum_{k|n}\varphi(k)\left(\chi_V(g^k)\right)^{\frac{n}{k}}}$$

Euler totient fn:
# of relative prime of k in {1,…,k}

$$T^{(2)}_{\mathrm{cyc}}(\mathrm{Rac}) = \bigoplus_{n=0}^{\infty}\mathcal{D}(2n+1, 2n)$$

$$T^{(3)}_{\mathrm{cyc}}(\mathrm{Rac}) = \bigoplus_{s=0}^{\infty}\left(s+1+2\left[-\tfrac{s}{3}\right]\right)\left[\mathcal{D}(s+\tfrac{3}{2}, s)\oplus\mathcal{D}(s+\tfrac{7}{2}, s+1)\right]$$

Order four formula does not fit in a single page

II. Vacuum Energy

of Each Massive HS Multiplets

A
- UV divergent VE $\sum_{\Delta,s} N_{\Delta,s}\,\zeta_{\Delta,s}(0)$
- Renormalized VE $\sum_{\Delta,s} N_{\Delta,s}\,\zeta'_{\Delta,s}(0)$

B
- Total Zeta Funtion $\zeta_{\mathrm{AdS}}(z) = \sum_{\Delta,s} N_{\Delta,s}\,\zeta_{\Delta,s}(z)$

Ⓐ Toy Model : Rac^3

$$\mathrm{Rac}^{\otimes 3} = \bigoplus_{s=0}^{\infty} (s+1)\left[\mathcal{D}(s+\tfrac{3}{2}, s) \oplus \mathcal{D}(s+\tfrac{7}{2}, s+1)\right]$$

$$\zeta_{\Delta,s}(0) = \frac{2s+1}{24}\left[\left(\Delta-\frac{3}{2}\right)^4 - \left(\frac{2\,s+1}{2}\right)^2\left(2\left(\Delta-\frac{3}{2}\right)^2+\frac{1}{6}\right)-\frac{7}{240}\right]$$

$$\zeta'_{\Delta,s}(0) = \frac{2\,s+1}{3}\left[\frac{1}{8}\left(\Delta-\frac{3}{2}\right)^4+\frac{1}{48}\left(\Delta-\frac{3}{2}\right)^2+c_3+\left(\frac{2\,s+1}{2}\right)^2 c_1\right.$$
$$\left.+\int_0^{\Delta-\frac{3}{2}} dx\left(\left(\frac{2\,s+1}{2}\right)^2-x^2\right)x\,\psi\left(x+\frac{1}{2}\right)\right].$$

$$c_n = \int_0^{\infty} du\,\frac{2\,u^n\,\ln u}{e^{2\pi u}+1}, \qquad \psi(x) = \int_0^{\infty} dt\left(\frac{e^{-t}}{t}-\frac{e^{-xt}}{1-e^{-t}}\right)$$

CONTENT CONTROL
CENSORED
CONTENT CONTROL

TECHNICALITY
CENSORSHIP
PROTECTING YOU FROM REALITY

A Toy Model : Rac^3

UV divergent part of Vacuum Energy

$$\zeta_{\mathrm{Rac}^{\otimes 3}}(0) = \lim_{\alpha\to 0} \sum_{s=0}^{\infty} \left(\Delta - \frac{3}{2}\right)^{-\alpha} \zeta_{\Delta,s}(0)$$

$$= \lim_{\alpha\to 0} \sum_{s=0}^{\infty} s^{-\alpha} \zeta_{s+\frac{3}{2},s}(0) + \lim_{\alpha\to 0} \sum_{s=0}^{\infty} (s+1)^{-\alpha} \zeta_{s+\frac{5}{2},s}(0)$$

$$\vdots$$

$$= -\frac{7}{13824} + \frac{7}{13824} = 0$$

Ⓐ Toy Model : Rac^3

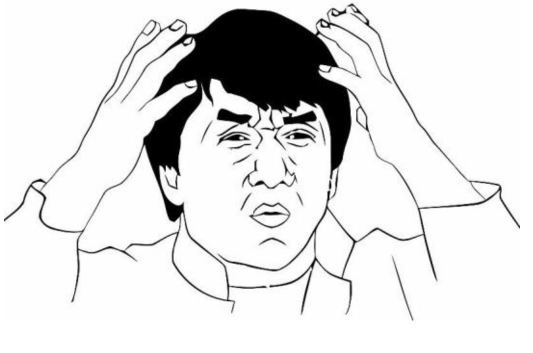
Renormalized Vacuum Energy

$$\zeta'_{\mathrm{Rac}^{\otimes 3}}(0) = \lim_{\alpha\to 0}\sum_{s=0}^{\infty}\int_0^{\infty} dt \left[s^{-\alpha}(s+1)\int_0^{s} dx + (s+1)^{-\alpha}\, s\int_0^{s+1} dx \right] \times$$

$$\times \frac{2}{3}\left(\frac{2s+1}{2}\right)\left[\left(\frac{2s+1}{2}\right)^2 x - x^3\right]\left(\frac{e^{-t}}{t} - \frac{e^{-(x+\frac{1}{2})t}}{1-e^{-t}}\right)$$

$$\vdots$$

$$= -\frac{\ln 2}{128} - \frac{19\,\zeta(3)}{3840\,\pi^2} + \frac{25\,\zeta(5)}{256\,\pi^4} - \frac{63\,\zeta(7)}{512\,\pi^6}$$



Rac^4

UV divergent part of Vacuum Energy

$$\zeta_{\mathrm{Rac}^{\otimes 4}}(0) = \lim_{\alpha \to 0} \sum_{s=0}^{\infty} \sum_{n=1}^{\infty} \left(s + n + \frac{1}{2} \right)^{-\alpha} \frac{2\,n\,(2\,s+1) + 3\,(-1)^n + (2\,s+1)^2}{4}\, \zeta_{s+n+2,s}(0) + \lim_{\alpha \to 0} \sum_{s=0}^{\infty} \left(s + \frac{1}{2} \right)^{-\alpha} \frac{(s+1)\,(s+2)}{2}\, \zeta_{s+2,s}(0)$$

$$= \quad \cdots \quad = -\frac{7963}{232243200} + \frac{7963}{232243200} = 0$$

Renormalized Vacuum Energy

$$\zeta'_{\mathrm{Rac}^{\otimes 4}}(0) = \quad \bullet \quad \bullet \quad \bullet$$

$$= -\frac{29\,\zeta(3)}{2520\,\pi^2} - \frac{\zeta(5)}{60\,\pi^4} + \frac{\zeta(7)}{8\,\pi^6} - \frac{\zeta(9)}{8\,\pi^8}$$

Cyclic Tensor Products ?

Hard to control the regularization scheme

No Way to Proceed ?

Rac^n
Character

Decomposition

Multiplicities

Resummation

Zeta Function

?

this works in Thermal AdS
Giombi, Klebanov, Tseyltlin

New Method to Calculate

Zeta Function

We introduce
Character Integral Representation for Zeta Function B

- For any spectra $\mathcal{H}$
- Identify the corresponding character $\chi_{\mathcal{H}}(\beta,\alpha)$
- Total zeta function of $\mathcal{H}$ given by

$$\tilde{\zeta}_{\mathcal{H}}(z) = \int_0^\infty d\beta \, \frac{\beta^{2z-1}}{\Gamma(2z)} \, \frac{\cosh\frac{\beta}{2}}{\sinh\frac{\beta}{2}} \left(1 + \sinh^2\frac{\beta}{2}\,\partial_\alpha^2\right) \chi_{\mathcal{H}}(\beta,\alpha)\Big|_{\alpha=0}$$

Great simplification !

since we usually identify the spectra using character

Revisit (test of formula)

Vector Model

$$\tilde{\zeta}_{\text{non-min}}(z) = 0 \qquad\qquad \tilde{\zeta}_{\text{min}}(z) = 4^{-z}\,\tilde{\zeta}_{\text{Rac}}(z) = \tilde{\zeta}_{\text{Rac}}(z) + \mathcal{O}(z^2)$$

integrand itself vanishes

$$\tilde{\zeta}_{\text{Rac}}(z) = \zeta(2z-2,\tfrac{1}{2}) + \frac{1}{4}\,\zeta(2z,\tfrac{1}{2}) = -\left[\frac{\ln 2}{4} - \frac{3\,\zeta(3)}{8\,\pi^2}\right] z + \mathcal{O}(z^2)$$

Coincide with bd 1-loop calculation

more discussion: Becarria, Tseytlin

Rac^3 and Rac^4

$$\begin{aligned}\tilde{\zeta}_{\text{Rac}^{\otimes 3}}(z) &= \frac{\zeta(2\,z,\frac{3}{2})}{128} - \frac{19\,\zeta(2\,z-2,\frac{3}{2})}{1440} - \frac{5\,\zeta(2\,z-4,\frac{3}{2})}{72} - \frac{\zeta(2\,z-6,\frac{3}{2})}{90}\\ &= z\left(-\frac{\ln 2}{128} - \frac{19\,\zeta(3)}{3840\,\pi^2} + \frac{25\,\zeta(5)}{256\,\pi^4} - \frac{63\,\zeta(7)}{512\,\pi^6}\right) + \mathcal{O}(z^2)\,,\end{aligned}$$

$$\begin{aligned}\tilde{\zeta}_{\text{Rac}^{\otimes 4}}(z) &= \frac{29\,\zeta(2\,z-2)}{1260} - \frac{\zeta(2\,z-4)}{90} - \frac{\zeta(2\,z-6)}{90} - \frac{\zeta(2\,z-8)}{1260}\\ &= z\left(-\frac{29\,\zeta(3)}{2520\,\pi^2} - \frac{\zeta(5)}{60\,\pi^4} + \frac{\zeta(7)}{8\,\pi^6} - \frac{\zeta(9)}{8\,\pi^8}\right) + \mathcal{O}(z^2)\,.\end{aligned}$$

Coincide with the previous calculations A

Free SU(N) Adjoint Scalar : '2nd Regge Trajectory'

$$
\begin{aligned}
\tilde{\zeta}_{\mathrm{cyc}^3}(z) = \; & \frac{\zeta\left(2z,\frac{3}{2}\right)}{1152} - \frac{19\,\zeta\left(2z-2,\frac{3}{2}\right)}{17280} - \frac{5\,\zeta\left(2z-4,\frac{3}{2}\right)}{216} - \frac{\zeta\left(2z-6,\frac{3}{2}\right)}{270} \\
& + 2^{2z}\,3^{-2z-1}\left[\zeta(2z)+\zeta(2z-2)\right] - 3^{-2z-1}\left[\zeta(2z)+4\,\zeta(2z-2)\right] \\
& + 3^{-2z-3}\,\frac{57}{32}\left[\zeta\left(2z,\tfrac{1}{6}\right)+\zeta\left(2z,\tfrac{5}{6}\right)-8\,\zeta\left(2z-1,\tfrac{1}{6}\right)+8\,\zeta\left(2z-1,\tfrac{5}{6}\right)\right] \\
& + 3^{-2z-1}\left[\zeta\left(2z-2,\tfrac{1}{6}\right)+\zeta\left(2z-2,\tfrac{5}{6}\right)-3\,\zeta\left(2z-3,\tfrac{1}{6}\right)+3\,\zeta\left(2z-3,\tfrac{5}{6}\right)\right]
\end{aligned}
$$

UV divergent part of Vacuum Energy

$$\tilde{\zeta}_{\mathrm{cyc}^3}(0) = 0$$ ☺

Renormalized Vacuum Energy

$$
\begin{aligned}
\tilde{\zeta}_{\mathrm{cyc}^3}{}'(0) &= -\frac{43\,\ln 2}{128} + \frac{1487\,\zeta(3)}{3840\,\pi^2} + \frac{25\,\zeta(5)}{768\,\pi^4} - \frac{21\,\zeta(7)}{512\,\pi^6} + \frac{4\,\pi}{27\sqrt{3}} - \frac{19\,\psi^{(1)}\left(\frac{1}{3}\right)}{72\sqrt{3}\,\pi} + \frac{\psi^{(3)}\left(\frac{1}{3}\right)}{96\sqrt{3}\,\pi^3} \\
&\simeq -0.311588\,,
\end{aligned}
$$

☹

‘3rd Regge Trajectory’

UV divergent part of Vacuum Energy

$\tilde{\zeta}_{\mathrm{cyc}^4}(0) = 0$ ☺

Renormalized Vacuum Energy

$$\begin{aligned}\tilde{\zeta}_{\mathrm{cyc}^4}{}'(0) &= -\frac{25\,\ln 2}{64} - \frac{4573\,\zeta(3)}{20160\,\pi^2} + \frac{457\,\zeta(5)}{1920\,\pi^4} + \frac{\zeta(7)}{32\,\pi^6} - \frac{\zeta(9)}{32\,\pi^8} + \frac{\pi}{32} - \frac{5\,\psi^{(1)}\left(\frac{1}{4}\right)}{96\,\pi} + \frac{\psi^{(3)}\left(\frac{1}{4}\right)}{384\,\pi^3} \\ &\simeq -0.353518\,.\end{aligned}$$

☹

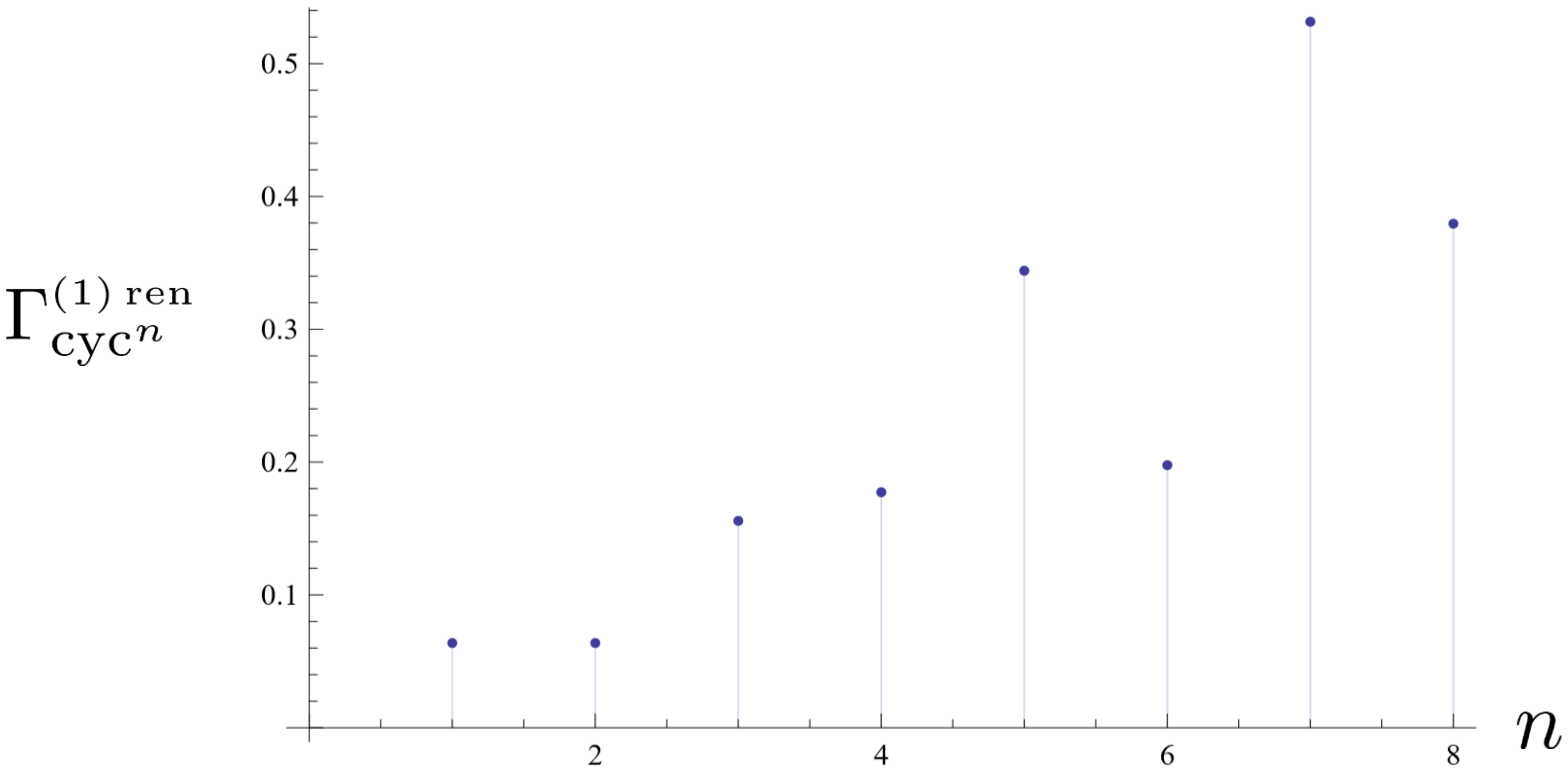
Conjecture of Free **Adjoint** Model Holography:

Maybe, we need to sum over ***all trajectories***

Q

- Why each massive **HS multiplet** do not show any special property?
- No particular role of **HS sym**?
- What about **Multiptcl HS** sym?

VE of First Few Trajectories



pattern not so clear

AdS_5/CFT_4

Character Integral Representation for Zeta Function

$$\zeta_{\mathcal{H}}(z) := \zeta_{\mathcal{H}|2}(z) + \zeta_{\mathcal{H}|1}(z) + \zeta_{\mathcal{H}|0}(z)$$

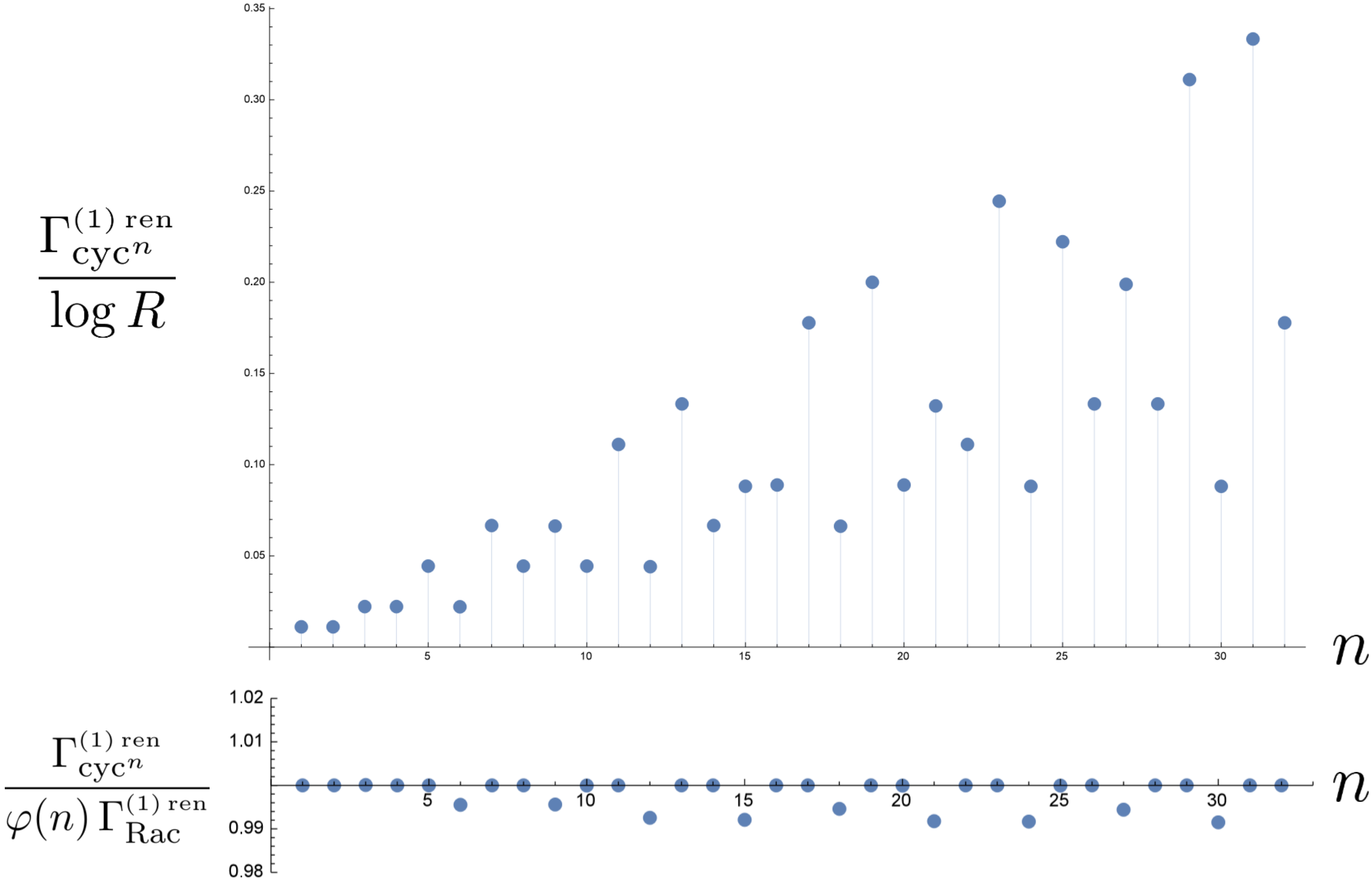
$$\frac{\Gamma(z)\,\zeta_{\mathcal{H}|n}(z)}{\log R} = \int_0^\infty d\beta\,\frac{\left(\frac{\beta}{2}\right)^{2(z-1-n)}}{\Gamma(z-n)}\,f_{\mathcal{H}|n}(\beta)$$

$$f_{\mathcal{H}|2}(\beta) = \frac{\sinh^4\frac{\beta}{2}}{2}\,\chi_{\mathcal{H}}(\beta,0,0)$$

$$f_{\mathcal{H}|1}(\beta) = \sinh^2\frac{\beta}{2}\left[\frac{\sinh^2\frac{\beta}{2}}{3} - 1 - \sinh^2\frac{\beta}{2}\left(\partial_{\alpha_1}^2 + \partial_{\alpha_2}^2\right)\right]\chi_{\mathcal{H}}(\beta,\alpha_1,\alpha_2)\Big|_{\alpha_i=0}$$

$$f_{\mathcal{H}|0}(\beta) = \Bigg[1 + \frac{\sinh^2\frac{\beta}{2}\left(3-\sinh^2\frac{\beta}{2}\right)}{3}\left(\partial_{\alpha_1}^2 + \partial_{\alpha_2}^2\right) - \frac{\sinh^4\frac{\beta}{2}}{3}\left(\partial_{\alpha_1}^4 - 12\,\partial_{\alpha_1}^2\partial_{\alpha_2}^2 + \partial_{\alpha_2}^4\right)\Bigg]\chi_{\mathcal{H}}(\beta,\alpha_1,\alpha_2)\Big|_{\alpha_i=0}$$

VE of First Few Trajectories



Certain pattern. But not yet clear how to deal with it

Slicing Vacuum Energy Differently

Total VE in Different Slicings

Sum over Trajectories

$$\chi_{\mathrm{adj}} = \sum_{n=2}^{\infty} \chi_{\mathrm{cyc}^n}$$

Sum over ‘log slice’s

$$\chi_{\mathrm{adj}} = -\chi_{\mathrm{Rac}} + \sum_{k=1}^{\infty} \frac{\varphi(k)}{k} \chi_{\log,k}$$

$$\chi_{\log,k}(\beta, \alpha_1, \alpha_2) = -\log[1 - \chi_{\mathrm{Rac}}(k\,\beta, k\,\alpha_1, k\,\alpha_2)]$$

A Subtlety

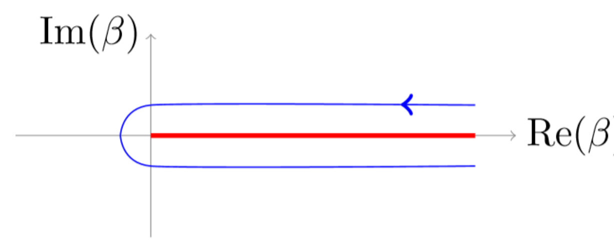
$$\boxed{\frac{\Gamma(z)\,\zeta_{\mathcal{H}|n}(z)}{\log R} = \int_0^\infty d\beta\, \frac{\left(\frac{\beta}{2}\right)^{2(z-1-n)}}{\Gamma(z-n)}\, f_{\mathcal{H}|n}(\beta)}$$

1. Extract β^{2n+1} coefficients from $f_{\mathcal{H}|n}(\beta)$

2. Integrate with analytic continuation on z

3. Contour integral

$$\frac{i}{2\,\sin(2\pi z)} \oint_C d\beta\, \frac{\left(-\frac{\beta}{2}\right)^{2(z-1-n)}}{\Gamma(z-n)}\, f_{\mathcal{H}|n}(\beta)$$



They are **equivalent** for **single ptcl**
and for each **trajectory**, but **not** for **each 'log slice'**

$$\chi_{\log,k}(\beta,\alpha_1,\alpha_2) = -\log[1-\chi_{\mathrm{Rac}}(k\,\beta,k\,\alpha_1,k\,\alpha_2)]$$

No β^5

1 $$f_{\log,k|2}(\beta) = \frac{-3\,\log\beta+\log(-\frac{2}{k^3})}{32}\,\beta^4 + \frac{-3\,\log\beta+\log(-\frac{2}{k^3})}{192}\,\beta^6 - \frac{k^3}{64}\,\beta^7 + \mathcal{O}(\beta^8)$$

No β^3

$$f_{\log,k|1}(\beta) = \frac{3\,\log\beta-\log(-\frac{2}{k^3})+1}{8}\,\beta^2 - \frac{k^2-2}{96}\,\beta^4 + \frac{k^3}{8}\,\beta^5 + \mathcal{O}(\beta^6)$$

No β

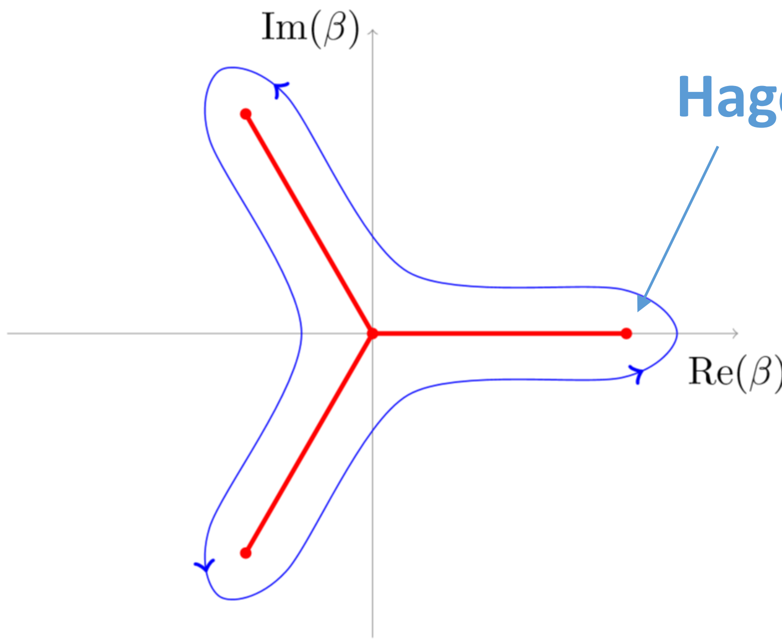
$$f_{\log,k|0}(\beta) = -3\log\beta+\log(-\tfrac{2}{k^3}) - \frac{3}{2} + \frac{k^2-1}{12}\,\beta^2 - \frac{3\,k^2}{2}\,\beta^3 + \mathcal{O}(\beta^4)$$

2 can be done using an additional regularization

Vacuum Energy vanishes for each k 'log slice' !

$$\chi_{\log,k}(\beta,\alpha_1,\alpha_2) = -\log[1-\chi_{\text{Rac}}(k\,\beta,k\,\alpha_1,k\,\alpha_2)]$$

3



Hagedorn phase transition T

But, β is integration parameter here !

Any interpretation ?

OUTLOOK

- SO(N) & Sp(N) Adjoint, Bivector Models
- Free Yang-Mills, $\mathcal{N}$=4 SYM
- Understand the branch cut contributions
- Finite N?
- …

Thank you for the attention!