Unfolded Form of Current Interactions of $4d$ Higher-Spin Fields

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Plan

I It will be recalled how interactions of massless fields of all spins with conserved currents result from a solution of the linear problem that describes a gluing between rank-one massless system and rank-two current system in the unfolded dynamics approach.

OG, Vasiliev, arXiv:1012.3143

II Outline the current progress in the reconstruction of current interactions in the gauge sector from nonlinear HS equations

OG, Vasiliev, work in progress

AdS_4 background connections

Flat $sp(4)$ **connection** $w = (\omega^L{}_{\alpha\beta}, \overline{\omega}^L{}_{\dot\alpha\dot\beta}, h_{\alpha\dot\beta})$ **:**

Lorentz connection $\omega^L{}_{\alpha\beta}$, $\overline{\omega}^L{}_{\dot\alpha\dot\beta}$ + **vierbein** $h_{\alpha\dot\beta}$

Zero curvature conditions

$$R_{\alpha\beta} = \mathrm{d}\omega^L{}_{\alpha\beta} + \omega^L{}_{\alpha\gamma}\omega^L{}_{\beta}{}^{\gamma} - \lambda^2 H_{\alpha\beta} = 0,$$

$$\overline{R}_{\dot\alpha\dot\beta} = \mathrm{d}\overline{\omega}^L{}_{\dot\alpha\dot\beta} + \overline{\omega}^L{}_{\dot\alpha\dot\gamma}\overline{\omega}^L{}_{\dot\beta}{}^{\dot\gamma} - \lambda^2 \overline{H}_{\dot\alpha\dot\beta} = 0\,,$$

$$R_{\alpha\dot\beta} = \mathrm{d}h_{\alpha\dot\beta} + \omega^L{}_{\alpha\gamma}h^{\gamma}{}_{\dot\beta} + \overline{\omega}^L{}_{\dot\beta\dot\delta}h_{\alpha}{}^{\dot\delta} = 0.$$

$\lambda^{-1} = \rho$ **radius of** AdS_4

$\varepsilon_{\alpha\beta} = -\varepsilon_{\beta\alpha}$, $\varepsilon_{12} = 1$: $A^\alpha = \varepsilon^{\alpha\beta}A_\beta$, $A_\alpha = A^\beta\varepsilon_{\beta\alpha}$

$H^{\alpha\beta} = H^{\beta\alpha} := h^{\alpha\dot\alpha}h^{\beta}{}_{\dot\alpha}\,,\qquad \overline{H}^{\dot\alpha\dot\beta} = \overline{H}^{\dot\beta\dot\alpha} := h^{\alpha\dot\alpha}h_{\alpha}{}^{\dot\beta}$ **the basis 2-forms**

Central on-shell theorem

Massless fields are described by 1$-$**forms** $\omega(y,\bar{y}|x)$ **and** 0$-$**forms** $C(y,\bar{y}|x)$

Rank-one unfolded equations =Central on-shell theorem Vasiliev (1989)

$$\begin{cases} D^{ad}\omega(y,\bar{y}|x) = i\Big(\eta\overline{H}^{\dot\alpha\dot\beta}\frac{\partial^2}{\partial\bar{y}^{\dot\alpha}\partial\bar{y}^{\dot\beta}}\ \bar{C}(0,\bar{y}|x) + \bar\eta H^{\alpha\beta}\frac{\partial^2}{\partial y^{\alpha}\partial y^{\beta}}\ C(y,0|x)\Big) \\ D^{tw}C(y,\bar{y}|x) = 0 \end{cases}$$

$$D^{ad}\omega(y,\bar{y}|x) := D^{L}\omega(y,\bar{y}|x) + \lambda h^{\alpha\dot\beta}\Big(y_\alpha\frac{\partial}{\partial\bar{y}^{\dot\beta}} + \frac{\partial}{\partial y^{\alpha}}\bar{y}_{\dot\beta}\Big)\omega(y,\bar{y}|x)\,,$$

$$D^{tw}C(y,\bar{y}|x) := D^{L}C(y,\bar{y}|x) - i\lambda h^{\alpha\dot\beta}\Big(y_\alpha\bar{y}_{\dot\beta} - \frac{\partial^2}{\partial y^{\alpha}\partial\bar{y}^{\dot\beta}}\Big)C(y,\bar{y}|x)\,,$$

$$D^{L}f(y,\bar{y}|x) := \mathsf{d}f(y,\bar{y}|x) + \Big(\omega^{L\,\alpha\beta}y_\alpha\frac{\partial}{\partial y^{\beta}} + \overline{\omega}^{L\,\dot\alpha\dot\beta}\bar{y}_{\dot\alpha}\frac{\partial}{\partial\bar{y}^{\dot\beta}}\Big)f(y,\bar{y}|x)\,.$$

η **and** $\bar\eta$ **complex conjugated free parameters**

$\mathsf{d} = dx^n\frac{\partial}{\partial x^n}$

Rank-two equations and conserved currents

Rank-two unfolded equations in AdS_4 **= current equations**

$$D_{cur}{}^{tw}\,\mathcal{J}(y,\bar{y}|x) = 0$$

OG, Vasiliev (2003)

$$D^{tw}_{cur} = D^L + \lambda e^{\alpha\dot\beta}\Big(y^1{}_\alpha\,\bar{y}^1{}_{\dot\beta} - y^2{}_\alpha\,\bar{y}^2{}_{\dot\beta} - \frac{\partial^2}{\partial y^{1\alpha}\partial\bar{y}^{1\dot\beta}} + \frac{\partial^2}{\partial y^{2\alpha}\partial\bar{y}^{2\dot\beta}}\Big)\,.$$

Three-form $\Omega(\mathcal{J}) = \mathcal{H}^{\alpha\dot\beta}\frac{\partial^2}{\partial y^{-\alpha}\partial\bar{y}^{-\dot\beta}}\,\mathcal{J}(y,\bar{y}|x)\Big|_{y=\bar{y}=0}$

$$y^{\pm}\sim y_1\pm y_2\,,\qquad \bar{y}^{\pm}\sim\bar{y}_1\pm\bar{y}_2\,,\qquad \mathcal{H}^{\alpha\dot\beta} = h^{\alpha}{}_{\dot\alpha}\wedge h^{\beta\dot\alpha}\wedge h_{\beta}{}^{\dot\beta}$$

is closed by virtue of current equations $\Longrightarrow$

$$J(y_1,y_2,\bar{y}_1,\bar{y}_2|x) = C_1(y_1,\bar{y}_1|x)C_2(y_2,\bar{y}_2|x)$$

AdS_4 **bilinear conserved charges**

$$Q = \int_{\Sigma^3}\Omega\,(J)\,,$$

Current deformation

In the unfolded dynamics approach current interactions result from a nontrivial mixing between fields of ranks one and two

Schematically for the flat connection $D = \mathrm{d} + w$

$$\begin{cases} D\omega + L(C, \overline{C}, w) = 0 \\ DC = 0 \\ D_2\mathcal{J} = 0 \end{cases} \quad \Rightarrow \quad \begin{cases} D\omega + L(C, \overline{C}, w) + \Gamma_{cur}(w, \mathcal{J}) = 0 \\ DC + \mathcal{H}_{cur}(w, \mathcal{J}) = 0 \\ D_2\mathcal{J} = 0 \end{cases}$$

$\Gamma_{cur}(w, \mathcal{J})$ and $\mathcal{H}_{cur}(w, \mathcal{J})$ glue rank-one and rank-two modules

$$L = -i\Big(\eta \overline{H}^{\dot\alpha\dot\beta} \frac{\partial^2}{\partial \overline{y}^{\dot\alpha} \partial \overline{y}^{\dot\beta}} \; \bar{C}(0, \overline{y}|x) + c.c.\Big)$$

Deformed equations in AdS_4 for integer spin s result from consistency conditions

OG, Vasiliev (2010)

$$\begin{cases} D^{ad}\omega(y,\bar{y}|x) = i\overline{H}^{\dot\alpha\dot\beta}\bar\partial_{\dot\alpha}\bar\partial_{\dot\beta}\overline{C}(0,\bar{y}\mid x) + iH^{\alpha\beta}\partial_\alpha\partial_\beta C(y,0\mid x) + \\ \quad a_{\mathrm{s}}\overline{H}^{\dot\alpha\dot\beta}\bar\partial_{-\dot\alpha}\bar\partial_{-\dot\beta}\sum_{k=0}^{s-2}\frac{\left(\mathcal{N}_-\right)^{s+k}\left(\overline{\mathcal{N}}_-\right)^{s-k-2}}{(s+k)!}\left(f_+\right)^k\mathcal{J}_{\mathrm{s}}\Big|_{y^\pm=\bar{y}^\pm=0}, \qquad + c.c. \\ D^{tw}C(y,\bar{y}\mid x) + \lambda a_{\mathrm{s}} h^{\mu\dot\beta}\mathfrak{F}_1{}^{\mathrm{s}}(\mathcal{N}_\pm,\overline{\mathcal{N}}_\pm)\, y^-{}_\alpha\bar\partial_{-\dot\beta}\left(f_+\right)^{\mathrm{s}-1}\mathcal{J}_{\mathrm{s}}\Big|_{y^\pm=\bar{y}^\pm=0} = 0 \\ D_2^{tw}\mathcal{J}_{\mathrm{s}} = 0 \end{cases}$$

$$\mathfrak{F}_K{}^{\mathrm{s}} = \left(\mathcal{N}_-\right)^{2\mathrm{s}}\sum_{m\geq 0}\frac{\left(\overline{\mathcal{N}}_+\mathcal{N}_- + \overline{\mathcal{N}}_-\mathcal{N}_+\right)^m}{m!(m+2\mathrm{s}+K)!}$$

spin-s current field $$\left(y^{+\alpha}\frac{\partial}{\partial y^{+\alpha}} + \bar{y}^{+\dot\alpha}\frac{\partial}{\partial\bar{y}^{+\dot\alpha}} - y^{-\alpha}\frac{\partial}{\partial y^{-\alpha}} - \bar{y}^{-\dot\alpha}\frac{\partial}{\partial\bar{y}^{-\dot\alpha}}\right)\mathcal{J}_{\mathrm{s}} = -2\mathrm{s}\mathcal{J}_{\mathrm{s}}$$

$$\mathcal{N}_\pm = y^\alpha\partial_{\pm\alpha}\,, \qquad \overline{\mathcal{N}}_\pm = \bar{y}^{\dot\alpha}\bar\partial_{\pm\dot\alpha}$$

a_s **: arbitrary coefficients,** $\eta = \bar\eta = 1$

The deformation is consistent in the flat limit

Trivial deformations

The operators

$$f_- = -\frac{\partial^2}{\partial y^{+\gamma}\partial y^{-}{}_{\gamma}} + \bar{y}^{+\dot{\gamma}}\bar{y}^{-}{}_{\dot{\gamma}}\,, \qquad f_+ = \overline{f_-}\,, \qquad\qquad f_0 = [f_+, f_-]$$

commute with $D_2^{tw} \Rightarrow$ form Howe dual "vertical" $\mathfrak{sl}_2$

$\mathcal{J}$-conserved current $\Rightarrow P(f_-, f_+)\mathcal{J}$-conserved current

The following deformed equation is trivial

$$D^{tw}C(y,\bar{y}|x) + h^{\mu\dot{\beta}}\mathfrak{F}_1{}^s\Big(y^+{}_\mu\bar{\partial}_{+\dot{\beta}} - y^-{}_\mu\bar{\partial}_{-\dot{\beta}}\Big)\, f_-\,\mathcal{J}\,\Big|_{y^\pm=\bar{y}^\pm=0} = 0$$

It follows from a local field redefinition of

OG, Vasiliev (2014)

$$D^{tw}\Big\{C(y,\bar{y}|x) - \lambda^{-1}\,\mathfrak{F}_0{}^s\,J(y^\pm,\bar{y}^\pm|x)\Big\}\Big|_{y^\pm=\bar{y}^\pm=0} = 0$$

The same is true in the flat limit

Complex conjugate formulas are analogous

ϕ^3 vertices are zero according to the results of Sezgin and Sundell (2003) and our resent results (2014)

Yukawa interaction,

Maxwell equation with nonzero current, and

linearised Einstein equation with stress tensor

follow from the deformed equations

Homotopy integrals

Using

$$\int_0^1 d\tau\, \tau^m (1-\tau)^n = \frac{m!n!}{(m+n+1)!}$$

deformations in 0-form sector can be rewritten as

$$\mathcal{H}_{cur}(w,\mathcal{J}) = \int_0^1 d\tau \sum_{h_1,h_2,h_{\mathcal{J}}} a(h_1,h_2,h_{\mathcal{J}}) \int \frac{d\bar{s}d\bar{t}}{(2\pi)^2} \exp i[\bar{s}_{\dot\beta}\bar{t}^{\dot\beta}]$$

$$h^{\alpha\dot\beta} y_\alpha (\tau \bar{s}_{\dot\beta} + (1-\tau)\bar{t}_{\dot\beta}) \mathcal{J}_{h_1,h_2,h_{\mathcal{J}}}(\tau y, -(1-\tau)y, \bar{y}+\bar{s}, \bar{y}+\bar{t}) + c.c.\,,$$

$\mathcal{J}_{h_1,h_2,h_{\mathcal{J}}}$ **is the projection of** $\mathcal{J}$ **to the helicities** $h_1, h_2, h_{\mathcal{J}}$.

Coefficients $a(h_1,h_2,h_{\mathcal{J}})$ **remain undetermined representing arbitrary coefficient in front of different vertices representing ambiguous coefficients in front of different vertices**

Nonlinear higher-spin equations in AdS_4

$\mathcal{W}(Z;Y;k,\bar{k}|x) = \mathrm{d} + W + S\,, \qquad W = dx^n W_n\,, \qquad S = \theta^\alpha S_\alpha + \bar{\theta}^{\dot{\alpha}} \bar{S}_{\dot{\alpha}}$ Vasiliev 1992

$$\mathcal{W} \star \mathcal{W} = i(\theta^A \theta_A + \eta \theta^\alpha \theta_\alpha B \star k \star \kappa + \bar{\eta} \bar{\theta}^{\dot{\alpha}} \bar{\theta}_{\dot{\alpha}} B \star \bar{k} \star \bar{\kappa})$$

$$\mathcal{W} \star B = B \star \mathcal{W}\,, \qquad B = B(Z;Y;k,\bar{k}|x)\,, \qquad \theta = dz$$

Inner Klein operators $\kappa = \exp(i z_\alpha y^\alpha)\,, \qquad \bar{\kappa} = \exp(i \bar{z}_{\dot{\alpha}} \bar{y}^{\dot{\alpha}})$

star product $\quad (f \star g)(Z,Y) = \int \exp(i S_A T^A) f(Z+S, Y+S)\, g(Z-T, Y+T)\,.$

Exterior Klein operators k and $\bar{k}$ $\quad kk = 1, \quad kf(z^\alpha; y^\alpha; \theta^\alpha) = f(-z^\alpha; -y^\alpha; -\theta^\alpha)\, k$

Integration measure is implicitly normalized in such a way that $f \star 1 = f$, $S = (s, \bar{s})$, $T = (t, \bar{t})$.

Quadratic corrections in the 0-form sector from nonlinear equations

In the 0-form sector the deformation is

$$D^{tw}C + [\omega\,,C]_* + \mathcal{H}_\eta(w,\mathcal{J}) + \mathcal{H}_{\bar\eta}(w,\mathcal{J}) = 0$$

obtained from nonlinear equations — Vasiliev (2015)

using the homotopy technique — Didenko, Misuna and Vasiliev (2015)

Modulo field redefinition $C =: C + \Phi_\eta(\mathcal{J}) + \bar\Phi_{\bar\eta}(\mathcal{J})$ — see Vasiliev's talk

$$\begin{aligned}
\widetilde{\mathcal{H}}_\eta(w,\mathcal{J}) &= \mathcal{H}_\eta(w,\mathcal{J}) + D^{tw}\Phi_\eta(\mathcal{J}) \\
\Phi_\eta(\mathcal{J}) &= \eta\int \frac{dSdT}{(2\pi)^4}\exp iS_AT^A\int d\tau_i \prod_{i=1}^{3}\theta(\tau_i)\delta\Big(1-\sum_{i=1}^{3}\tau_i\Big) \\
&\quad \frac{\partial}{\partial\tau_3}\mathcal{J}(\tau_3 s+\tau_1 y, t-\tau_2 y; \bar y+\bar s, \bar y+\bar t; K) * k \\
&- \quad \eta\int \frac{d\bar s d\bar t}{(2\pi)^2}\exp i[\bar s_{\dot\beta}\bar t^{\dot\beta}]\int_0^1 d\tau\,\mathcal{J}(\tau y, -(1-\tau)y; \bar y+\bar s, \bar y+\bar t; K) * k
\end{aligned}$$

$\widetilde{\mathcal{H}}_\eta(w,\mathcal{J}) + \widetilde{\mathcal{H}}_{\bar\eta}(w,\mathcal{J})$ reproduced the above local result with the definite coefficients $a(h_1,h_2,h_{\mathcal{J}}) = \eta$

Quadratic corrections in the 1-form sector from nonlinear equations

In the 1-form sector the deformation is

$$D^{ad}\omega + [\omega\,,\omega]_* + \Gamma(w,\mathcal{J}) \;=\; i\eta\overline{H}^{\dot\alpha\dot\beta}\frac{\partial^2}{\partial\overline{y}^{\dot\alpha}\partial\overline{y}^{\dot\beta}}\,(\bar C + \bar\Phi_{\bar\eta}(\mathcal{J}) + \Phi_\eta(\mathcal{J}))(0,\overline{y}|x)$$
$$+\; i\bar\eta H^{\alpha\beta}\frac{\partial^2}{\partial y^{\alpha}\partial y^{\beta}}\,(C + \Phi_{\bar\eta}(\mathcal{J}) + \bar\Phi_\eta(\mathcal{J}))(y,0|x)$$

$$\mathcal{J}(y_1,y_2;\bar y_1,\bar y_2;K|x) = C(y_1,\bar y_1;K|x)C(y_2,\bar y_2;K|x)$$

A straightforward computation using the homotopy technique gives

$$\Gamma = \Gamma_{\eta\eta}(w,\mathcal{J}) + \Gamma_{\bar\eta\bar\eta}(w,\mathcal{J})$$
$$\Gamma_{\eta\bar\eta}(w,\mathcal{J}) = 0$$

In different form the deformation Γ were obtained by Boulanger, Kessel, Skvortsov and Taronna (2015)

Preliminary analysis: independence on η^2 and $\bar{\eta}^2$

Modulo field redefinition $\omega =: \omega + \Psi_{\eta\eta}(\mathcal{J})$

$$\widetilde{\Gamma}_{\eta\eta}(\mathcal{J}) = \Gamma_{\eta\eta}(\mathcal{J}) + \mathcal{D}^{ad}\Psi_{\eta\eta}(\mathcal{J})$$

$\widetilde{\Gamma}_{\eta\eta}(\mathcal{J})$ **just compensates $\eta\eta$ -term resulting from the field redefinitions in the 0-form sector** $i\eta\overline{H}^{\dot{\alpha}\dot{\beta}}\frac{\partial^2}{\partial\overline{y}^{\dot{\alpha}}\partial\overline{y}^{\dot{\beta}}}\Phi_\eta(\mathcal{J})(0,\overline{y}|x)$

This is in accordance with the result obtained for lower-spin currents from analysis in the 0-form sector

$\Rightarrow$ **It remains to consider $\eta\bar{\eta}$ terms**

$$i\bar{\eta}H^{\alpha\beta}\frac{\partial^2}{\partial y^\alpha \partial y^\beta}\Phi_\eta(\mathcal{J})(y,0|x) + i\eta\overline{H}^{\dot{\alpha}\dot{\beta}}\frac{\partial^2}{\partial\overline{y}^{\dot{\alpha}}\partial\overline{y}^{\dot{\beta}}}\bar{\Phi}_{\bar{\eta}}(\mathcal{J})(0,\overline{y}|x)$$

Conclusion

Current interactions result from a linear problem via bilinear substitution

Modulo field redefinitions quadratic corrections in nonlinear equations in the 1-form-sector do not depend on η^2 and $\bar{\eta}^2$