Partially massless spin 3 in the Fradkin-Vasiliev formalism

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Outlook

1. Kinematics
   - Massless case
   - Partially massles case
2. Gravitational interaction
   - Massless case
   - Minimal case
   - Non-minimal case

Frame-like formalism for massless spin 3

- Frame-like Lagrangain for massless spin-3 in AdS_d:

$$\mathcal{L}_0 = -\frac{1}{6}\left\{ {}^{\mu\nu}_{ab} \right\}[\Omega_\mu{}^{cd,a}\Omega_\nu{}^{cd,b} + 2\Omega_\mu{}^{ac,d}\Omega_\nu{}^{bc,d}] - \frac{2}{3}\left\{ {}^{\mu\nu\alpha}_{abc} \right\}\Omega_\mu{}^{ad,b}D_\nu\Phi_\alpha{}^{cd} + (d-1)\kappa\left\{ {}^{\mu\nu}_{ab} \right\}\Phi_\mu{}^{ac}\Phi_\nu{}^{bc}$$

- It is invariant under the following gauge transformations:

$$\delta\Omega_\mu{}^{ab,c} = D_\mu\eta^{ab,c} + \zeta^{ab,c}{}_\mu + \frac{(d-1)\kappa}{(d-2)}[2e_\mu{}^c\xi^{ab} - e_\mu{}^{(a}\xi^{b)c} - Tr]$$

$$\delta\Phi_\mu{}^{ab} = D_\mu\xi^{ab} + \eta^{ab}{}_\mu$$

- We will need also an extra field (that do not enter the free Lagrangian):

$$\delta\Sigma_\mu{}^{ab,cd} = D_\mu\zeta^{ab,cd} + \kappa[\eta^{ab,(c}e_\mu{}^{d)} + \eta^{cd,(a}e_\mu{}^{b)} - Tr]$$

Gauge invariant objects and Lagrangian

- There are three gauge invariant two-forms:

$$
\begin{aligned}
\mathcal{R}_{\mu\nu}{}^{ab,cd} &= D_{[\mu}\Sigma_{\nu]}{}^{ab,cd} - \kappa[\Omega_{[\mu}{}^{ab,(c}e_{\nu]}{}^{d)} + \Omega_{[\mu}{}^{cd,(a}e_{\nu]}{}^{b)} - Tr] \\
R_{\mu\nu}{}^{ab,c} &= D_{[\mu}\Omega_{\nu]}{}^{ab,c} - \Sigma_{[\mu}{}^{ab,c}{}_{\nu]} + \frac{(d-1)\kappa}{(d-2)}[2e_{[\mu}{}^{c}\Phi_{\nu]}{}^{ab} + \dots \\
\mathcal{T}_{\mu\nu}{}^{ab} &= D_{[\mu}\Phi_{\nu]}{}^{ab} - \Omega_{[\mu}{}^{ab}{}_{\nu]}
\end{aligned}
$$

- On-shell we have:

$$
\mathcal{R}_{\mu\nu}{}^{ab,c} \approx 0, \quad \mathcal{T}_{\mu\nu}{}^{ab} \approx 0 \quad \Rightarrow \quad D_{[\mu}\mathcal{R}_{\nu\alpha]}{}^{ab,cd} \approx 0, \quad \mathcal{R}_{[\mu\nu}{}^{ab,c}{}_{\alpha]} \approx 0
$$

- Free Lagrangian can be rewritten as follows:

$$
\mathcal{L}_0 = \left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} [a_1 \mathcal{R}_{\mu\nu}{}^{ae,bf}\mathcal{R}_{\alpha\beta}{}^{ce,df} + a_2 R_{\mu\nu}{}^{ae,b} R_{\alpha\beta}{}^{ce,d}]
$$

$$
a_1 \sim \frac{1}{\kappa^2}, \qquad a_2 \sim \frac{1}{\kappa}
$$

From massive to partially massless spin-3

- Massive spin-3 = massless spin-3,2,1,0

$$\begin{pmatrix} \Sigma_\mu{}^{ab,cd} \\ \Omega_\mu{}^{ab,c} \\ \Phi_\mu{}^{ab} \end{pmatrix} \oplus \begin{pmatrix} W^{ab,cd} \\ \Omega_\mu{}^{a,b} \\ f_\mu{}^a \end{pmatrix} \oplus \begin{pmatrix} F^{ab,c} \\ F^{a,b} \\ A_\mu \end{pmatrix} \oplus \begin{pmatrix} \pi^{ab} \\ \pi^a \\ \varphi \end{pmatrix}$$

- Partially massless spin-3 of depth one = massless spin-3,2

$$\begin{pmatrix} \Sigma_\mu{}^{ab,cd} \\ \Omega_\mu{}^{ab,c} \\ \Phi_\mu{}^{ab} \end{pmatrix} \oplus \begin{pmatrix} W^{ab,cd} \\ \Omega_\mu{}^{ab} \\ f_\mu{}^a \end{pmatrix}$$

Lagrangian and gauge transformations

$$\begin{aligned}
\mathcal{L}_0 &= -\frac{1}{6}\left\{ {}^{\mu\nu}_{ab} \right\}[\Omega_\mu{}^{cd,a}\Omega_\nu{}^{cd,b} + 2\Omega_\mu{}^{ac,d}\Omega_\nu{}^{bc,d}] - \frac{2}{3}\left\{ {}^{\mu\nu\alpha}_{abc} \right\}\Omega_\mu{}^{ad,b}D_\nu\Phi_\alpha{}^{c} \\
&+\frac{1}{2}\left\{ {}^{\mu\nu}_{ab} \right\}\Omega_\mu{}^{ac}\Omega_\nu{}^{bc} - \frac{1}{2}\left\{ {}^{\mu\nu\alpha}_{abc} \right\}\Omega_\mu{}^{ab}D_\nu f_\alpha{}^{c} \\
&+m\left\{ {}^{\mu\nu}_{ab} \right\}[\Omega_\mu{}^{ac,b}f_\nu{}^{c} - \Omega_\mu{}^{ac}\Phi_\nu{}^{bc}] \\
&+m^2\left\{ {}^{\mu\nu}_{ab} \right\}[\frac{3(d-2)}{2d}\Phi_\mu{}^{ac}\Phi_\nu{}^{bc} - \frac{9(d-2)}{4(d-1)}f_\mu{}^{a}f_\nu{}^{b}], \qquad m^2 = \frac{d\kappa}{3}
\end{aligned}$$

$$\begin{aligned}
\delta\Omega_\mu{}^{ab,c} &= D_\mu\eta^{ab,c} + \frac{3m^2}{2d}[2e_\mu{}^{c}\xi^{ab} - e_\mu{}^{(a}\xi^{b)c} - Tr] - \frac{3m}{2d}[e_\mu{}^{(a}\eta^{b)c} - Tr] \\
\delta\Phi_\mu{}^{ab} &= D_\mu\xi^{ab} + \eta^{ab}{}_\mu + \frac{3m}{2(d-1)}[e_\mu{}^{(a}\xi^{b)} - \frac{2}{d}g^{ab}\xi_\mu] \\
\delta\Omega_\mu{}^{ab} &= D_\mu\eta^{ab} - m\eta_\mu{}^{[a,b]} + \frac{9m^2}{2(d-1)}e_\mu{}^{[a}\xi^{b]} \\
\delta f_\mu{}^{a} &= D_\mu\xi^{a} + \eta_\mu{}^{a} + m\xi_\mu{}^{a}
\end{aligned}$$

Gauge invariant objects

- Correspondingly, we have six gauge invariant objects:

$$
\begin{aligned}
\mathcal{R}_{\mu\nu}{}^{ab,cd} &= D_{[\mu}\Sigma_{\nu]}{}^{ab,cd} - \frac{3m}{2d}[W^{ab,(c}{}_{[\mu}e_{\nu]}{}^{d)} + W^{cd,(a}{}_{[\mu}e_{\nu]}{}^{b)}] \\
\mathcal{R}_{\mu\nu}{}^{ab,c} &= D_{[\mu}\Omega_{\nu]}{}^{ab,c} - \Sigma_{[\mu}{}^{ab,c}{}_{\nu]} - \frac{3m}{2d}[e_{[\mu}{}^{(a}\Omega_{\nu]}{}^{b)c} - Tr] + \dots \\
\mathcal{T}_{\mu\nu}{}^{ab} &= D_{[\mu}\Phi_{\nu]}{}^{ab} - \Omega_{[\mu}{}^{ab}{}_{\nu]} + \frac{3m}{2(d-1)}[e_{[\mu}{}^{(a}f_{\nu]}{}^{b)} + \frac{2}{d}g^{ab}f_{[\mu,\nu]}] \\
\mathcal{W}_{\mu}{}^{ab,cd} &= D_\mu W^{ab,cd} - 2m\Sigma_\mu{}^{ab,cd} \\
\mathcal{R}_{\mu\nu}{}^{ab} &= D_{[\mu}\Omega_{\nu]}{}^{ab} + m\Omega_{[\mu,\nu]}{}^{[a,b]} + \frac{9m^2}{2(d-1)}e_{[\mu}{}^{[a}f_{\nu]}{}^{b]} + W_{[\mu}{}^{a,b}{}_{\nu]} \\
\mathcal{T}_{\mu\nu}{}^{a} &= D_{[\mu}f_{\nu]}{}^{a} - \Omega_{[\mu,\nu]}{}^{a} - m\Phi_{[\mu,\nu]}{}^{a}
\end{aligned}
$$

- On-shell:

$$
\mathcal{R}_{\mu\nu}{}^{ab,c} \approx 0, \quad \mathcal{T}_{\mu\nu}{}^{ab} \approx 0, \quad \mathcal{R}_{\mu\nu}{}^{a,b} \approx 0, \quad \mathcal{T}_{\mu\nu}{}^{a} \approx 0
$$

Skvortsov-Vasiliev version

- We have four one-forms only:

$$(\Omega_\mu{}^{ab,c}, \Phi_\mu{}^{ab}) \oplus (\Omega_\mu{}^{ab}, f_\mu{}^a)$$

- and four gauge invariant two-forms:

$$\begin{aligned}
\mathcal{R}_{\mu\nu}{}^{ab,c} &= D_{[\mu}\Omega_{\nu]}{}^{ab,c} - \frac{3m}{2d}[e_{[\mu}{}^{(a}\Omega_{\nu]}{}^{b)c} - Tr] + \frac{3m^2}{2d}[2e_{[\mu}{}^c\Phi_{\nu]}{}^{ab} + \dots \\
\mathcal{T}_{\mu\nu}{}^{ab} &= D_{[\mu}\Phi_{\nu]}{}^{ab} - \Omega_{[\mu}{}^{ab}{}_{\nu]} + \frac{3m}{2(d-1)}[e_{[\mu}{}^{(a}f_{\nu]}{}^{b)} + \frac{2}{d}g^{ab}f_{[\mu,\nu]}] \\
R_{\mu\nu}{}^{ab} &= D_{[\mu}\Omega_{\nu]}{}^{ab} + m\Omega_{[\mu,\nu]}{}^{[a,b]} + \frac{9m^2}{2(d-1)}e_{[\mu}{}^{[a}f_{\nu]}{}^{b]} \\
\mathcal{T}_{\mu\nu}{}^a &= D_{[\mu}f_{\nu]}{}^a - \Omega_{[\mu,\nu]}{}^a - m\Phi_{[\mu,\nu]}{}^a
\end{aligned}$$

- Free Lagrangian can be written as:

$$\mathcal{L}_0 = \left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} [a_2\mathcal{R}_{\mu\nu}{}^{ae,b}\mathcal{R}_{\alpha\beta}{}^{ce,d} + a_3\mathcal{R}_{\mu\nu}{}^{ab}\mathcal{R}_{\alpha\beta}{}^{cd}]$$

What we can expect for 3-3-2 vertices

- Massless spin-3 $\oplus$ massless spin-2

$$N = 4, 6(, 8)$$

- Partially massless spin-3 $\oplus$ massless spin-2 (Joung, Lopez, Taronna 12)

$$N = 2, 4, 4, 6, 6$$

- Massive spin-3 $\oplus$ massless spin-2 (Metsaev 06, 12)

$$N = 3, 4, 4, 5, 5, 6, 6(, 7, 8)$$

Deformations of curvatures

- We have to consider the most general quadratic deformations for all curvatures and require that the deformed curvatures transform covariantly.
- For spin 3 curvatures they correspond to the minimal substitution rules. Variations that do not vanish on-shell:

$$\begin{aligned}
\delta\hat{\mathcal{R}}_{\mu\nu}{}^{ab,cd} &\approx c_0[R_{\mu\nu}{}^{e(a}\zeta^{b)e,cd} + R_{\mu\nu}{}^{e(c}\zeta^{d)e,ab} \\
&\quad -\hat{\eta}^{e(a}\mathcal{R}_{\mu\nu}{}^{b)e,cd} - \hat{\eta}^{e(c}\mathcal{R}_{\mu\nu}{}^{d)e,ab}] \\
\delta\hat{\mathcal{R}}_{\mu\nu}{}^{ab,c} &\approx c_0[R_{\mu\nu}{}^{d(a}\eta^{b)d,c} + R_{\mu\nu}{}^{dc}\eta^{ab,d} + \mathcal{R}_{\mu\nu}{}^{ab,cd}\hat{\xi}^d]
\end{aligned}$$

- For the gravitational curvature we obtain:

$$\delta\hat{R}_{\mu\nu}{}^{ab} \approx b_1\mathcal{R}_{\mu\nu}{}^{de,c[a}\zeta^{b]c,de}$$

Interacting Lagrangian

- is the sum of the free Lagrangians where curvatures are repalced by the deformed ones plus abelian vertex:

$$\mathcal{L} = \left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} [a_1 \hat{\mathcal{R}}_{\mu\nu}{}^{ae,bf} \hat{\mathcal{R}}_{\alpha\beta}^{ce,df} + a_2 \hat{\mathcal{R}}_{\mu\nu}{}^{ae.b} \hat{\mathcal{R}}_{\alpha\beta}^{cw,d} + a_0 \hat{R}_{\mu\nu}{}^{ab} \hat{R}_{\alpha\beta}{}^{cd}]$$
$$+ c_1 \left\{ {}^{\mu\nu\alpha\beta\gamma}_{abcde} \right\} \mathcal{R}_{\mu\nu}{}^{af,bg} \mathcal{R}_{\alpha\beta}{}^{cf,dg} h_\gamma{}^e$$

- Invariance fixes the relation

$$b_1 \Lambda \sim c_0$$

- Both non-abelian and abelian vertices contain terms up to six derivatives but we can adjust coefficients so that all six derivatives terms vanish on-shell (Vasiliev 11).

Deformations of curvatures

- We have one-forms only $\Rightarrow$ the procedure goes exactly as in the massless case.
- For the partially massless spin-3 it again corrsponds to the minimal substitution rules. Transformations that do not vanish on-shell look like:

$$\begin{aligned} \delta\hat{\mathcal{R}}_{\mu\nu}{}^{ab,c} &\approx c_0[R_{\mu\nu}{}^{d(a}\eta^{b)d,c} + R_{\mu\nu}{}^{dc}\eta^{ab,d} \\ \delta\hat{\mathcal{R}}_{\mu\nu}{}^{ab} &\approx -R_{\mu\nu}{}^{c[a}\eta^{b]c} - m\mathcal{R}_{\mu\nu}{}^{c[a,b]}\hat{\xi}^c] \end{aligned}$$

- For the gravitational curvature we obtain:

$$\delta\hat{R}_{\mu\nu}{}^{ab} \approx b_0\mathcal{R}_{\mu\nu}{}^{cd,[a}\eta^{cd,|b]} + 2b_0\mathcal{R}_{\mu\nu}{}^{[a|c,d}\eta^{b]c,d}$$

Interacting Lagrangian

- has the same structure:

$$
\begin{aligned}
\mathcal{L} &= \left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} [a_2 \hat{\mathcal{R}}_{\mu\nu}{}^{ae,b} \hat{\mathcal{R}}_{\alpha\beta}{}^{ce,d} + a_3 \hat{\mathcal{R}}_{\mu\nu}{}^{ab} \hat{\mathcal{R}}_{\alpha\beta}{}^{cd} + a_0 \hat{R}_{\mu\nu}{}^{ab} \hat{R}_{\alpha\beta}{}^{cd}] \\
& + c_1 \left\{ {}^{\mu\nu\alpha\beta\gamma}_{abcde} \right\} \mathcal{R}_{\mu\nu}{}^{af,b} \mathcal{R}_{\alpha\beta}{}^{cf,d} h_\gamma{}^e
\end{aligned}
$$

- Invariance fixes relation between two coupling constants:

$$c_0 \sim b_0$$

- Both non-abelian and abelian vertices contain terms with up to four derivatives.

Deformations of curvatures

- For the partially massless curvatures we again get minimal substitutions:

$$\delta\hat{\mathcal{R}}_{\mu\nu}{}^{ab,cd} \approx R_{\mu\nu}{}^{e(a}\zeta^{b)e,cd} + R_{\mu\nu}{}^{e(c}\zeta^{d)e,ab} - \frac{3m}{2d}[\hat{\xi}^{(a}\mathcal{W}_{[\mu,\nu]}{}^{b),cd} + \dots$$
$$\delta\hat{\mathcal{R}}_{\mu\nu}{}^{ab,c} \approx R_{\mu\nu}{}^{d(a}\eta^{b)d,c} + R_{\mu\nu}{}^{dc}\eta^{ab,d} - \mathcal{R}_{\mu\nu}{}^{ab,cd}\hat{\xi}^{d}$$

- The most general solution for the gravitational curvature and torsion we get six free parameters:

$$b_0, b_1, b_2, b_3, b_4, b_5$$

- At the same time we have five possible field redefinitions:

$$\omega_\mu{}^{ab} \Rightarrow \omega_\mu{}^{ab} + \kappa_1\Sigma_\mu{}^{de,c[a}W^{b]c,de} + \kappa_2 W^{ac,bd}\Omega_\mu{}^{cd}$$
$$h_\mu{}^a \Rightarrow h_\mu{}^a + \kappa_3 W^{ab,cd}\Omega_\mu{}^{cd,b} + \kappa_4 W^{ab,cd}W_\mu{}^{b,cd} + \kappa_5 e_\mu{}^a W^2$$

Interacting Lagrangian

- has the same general structure:

$$\begin{aligned}\mathcal{L} &= \left\{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \right\} [a_1 \hat{\mathcal{R}}_{\mu\nu}{}^{ae,bf} \hat{\mathcal{R}}_{\alpha\beta}{}^{ce,df} + a_2 \hat{\mathcal{R}}_{\mu\nu}{}^{ae,b} \hat{\mathcal{R}}_{\alpha\beta}{}^{ce,d} + a_3 \hat{\mathcal{R}}_{\mu\nu}{}^{ab} \hat{\mathcal{R}}_{\alpha\beta}{}^{cd}] \\ &+ a_4 \left\{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \right\} \hat{\mathcal{R}}_{\mu\nu}{}^{ad,e} \hat{\mathcal{W}}_{\alpha}{}^{bd,ce} + a_5 \left\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \right\} \hat{\mathcal{W}}_{\mu}{}^{ac,de} \hat{\mathcal{W}}_{\nu}{}^{bc,de} \\ &+ a_0 \left\{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \right\} \hat{R}_{\mu\nu}{}^{ab} \hat{R}_{\alpha\beta}{}^{cd} \\ &+ c_1 \left\{ \begin{smallmatrix} \mu\nu\alpha\beta\gamma \\ abcde \end{smallmatrix} \right\} \mathcal{R}_{\mu\nu}{}^{af,bg} \mathcal{R}_{\alpha\beta}{}^{cf,dg} h_{\gamma}{}^{e} + c_2 \left\{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \right\} \mathcal{W}_{\mu}{}^{ad,ef} \mathcal{W}_{\nu}{}^{bd,ef} h_{\alpha}{}^{c}\end{aligned}$$

- The number of independent vertices depends on which field redefinitions are considered as admissible.

Final remarks

- In the Skvortsov-Vasiliev case to reproduce vertices with more than four derivatives we still have to include some zero-forms.
- If we consider the genaral massive case and take partially massless limit we naturally end with the non-minimal version.
- The general formalism can be applied to the massive case. From one hand the most general deformations produce a lot of higher derivatives vertices for lower spin components. From the other hand we will face a lot of possible field redefinitions.
- Technically the most important open question is the admissibility of these redefinitions.