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Higher covariant derivative regularization in supersymmetric theories

Regularization of theories with symmetries

Higher spin theories are an interesting direction of modern research. These theories are invariant under some gauge symmetry transformations. There are also supersymmetric higher spin theories. (Super)string theories can be considered as (supersymmetric) higher spin theories.

Absence of ultraviolet divergencies is a very attractive feature of superstring theories. Possibly, some other higher spin theories also have such a feature due to large symmetries. So, in prospect, it would be interesting to investigate quantum corrections in various higher spin theories.

For calculating quantum corrections one should regularize a theory. It is highly desirable that the regularization does not break symmetries of the theory.

However, the most popular dimensional technique is not convenient in supersymmetric theories.

Dimensional technique in supersymmetric theories

Dimensional regularization breaks the supersymmetry and is not convenient for calculations in supersymmetric theories. That is why supersymmetric theories are mostly regularized by the dimensional reduction. However, the dimensional reduction is not self-consistent.

W.Siegel, Phys.Lett. **B 84** (1979) 193; **B 94** (1980) 37.

Removing of the inconsistencies leads to the loss of explicit supersymmetry:

L.V.Avdeev, G.A.Chochia, A.A.Vladimirov, Phys.Lett. **B 105** (1981) 272.

As a consequence, supersymmetry can be broken by quantum corrections in higher loops.

L.V.Avdeev, Phys.Lett. **B 117** (1982) 317;
L.V.Avdeev, A.A.Vladimirov, Nucl.Phys. **B 219** (1983) 262.

Therefore, the dimensional technique is not convenient in supersymmetric theories and it is desirable to use another regularization.

Higher covariant derivative regularization

The higher covariant derivative regularization is a consistent regularization, which does not break supersymmetry.

A.A.Slavnov, Nucl.Phys., **B 31** (1971) 301; Theor.Math.Phys. **13** (1972) 1064.

In order to regularize a theory by higher derivatives it is necessary to add a term with higher degrees of covariant derivatives. Then divergences remain only in the one-loop approximation. These remaining divergences are regularized by inserting the Pauli–Villars determinants.

A.A.Slavnov, Theor.Math.Phys. **33** (1977) 977.

The higher covariant derivative regularization can be generalized to the $\mathcal{N}=1$ supersymmetric case

V.K.Krivoshchekov, Theor.Math.Phys. **36** (1978) 745;
P.West, Nucl.Phys. **B 268** (1986) 113.

Also it can be constructed for $\mathcal{N}=2$ supersymmetric theories

V.K.Krivoshchekov, Phys.Lett. **B 149** (1984) 128;
I.L.Buchbinder, N.G.Pletnev, K.S., Phys.Lett. **B 751** (2015) 434.

Revealing structure of quantum corrections using the higher covariant derivative

Higher covariant derivative regularization allows not only to calculate quantum corrections in manifestly gauge and supersymmetric way. It also enables one to reveal some interesting features of quantum corrections, which are not manifest with other regularization.

In particular, we demonstrate how the higher derivative regularization allows naturally to obtain the exact NSVZ β-function in $\mathcal{N}=1$ supersymmetric gauge theories.

$$\beta(\alpha) = -\frac{\alpha^2\Big(3C_2 - T(R) + C(R)_i{}^j\gamma_j{}^i(\alpha)/r\Big)}{2\pi(1-C_2\alpha/2\pi)}, \qquad \text{where}$$

$$\mathrm{tr}\,(T^A T^B) \equiv T(R)\,\delta^{AB}; \qquad (T^A)_i{}^k (T^A)_k{}^j \equiv C(R)_i{}^j;$$

$$f^{ACD} f^{BCD} \equiv C_2\delta^{AB}; \qquad r \equiv \delta_{AA}.$$

NSVZ β-function in $\mathcal{N}=1$ supersymmetric theories

The NSVZ β-function was obtained from different arguments: instantons, anomalies etc.

V.Novikov, M.A.Shifman, A.Vainshtein, V.I.Zakharov, Nucl.Phys. **B 229** (1983) 381; Phys.Lett. **B 166** (1985) 329; M.A.Shifman, A.I.Vainshtein, Nucl.Phys. **B 277** (1986) 456; D.R.T.Jones, Phys.Lett. **B 123** (1983) 45.

Using the dimensional reduction and $\overline{\text{DR}}$-scheme a β-function of $\mathcal{N}=1$ supersymmetric theories was calculated up to the four-loop approximation:

L.V.Avdeev, O.V.Tarasov, Phys.Lett. **B 112** (1982) 356; I.Jack, D.R.T.Jones, C.G.North, Phys.Lett. **B 386** (1996) 138; Nucl.Phys. **B 486** (1997) 479; R.V.Harlander, D.R.T.Jones, P.Kant, L.Mihaila, M.Steinhauser, JHEP **0612** (2006) 024.

The result coincides with the NSVZ β-function only in one- and two-loop approximations. In the higher loops it is necessary to make a special tuning of the coupling constant. At present, there is no general prescription how to construct this finite renormalization in all orders.

NSVZ β-function for $\mathcal{N}=1$ SQED with N_f flavors

A simple particular case of the $\mathcal{N}=1$ supersymmetric Yang–Mills theory is the $\mathcal{N}=1$ supersymmetric electrodynamics (SQED) with N_f flavors, which (in the massless case) is described by the action

$$S = \frac{1}{4e_0^2}\mbox{Re}\int d^4x\, d^2\theta\, W^a W_a + \sum_{i=1}^{N_f} \frac{1}{4}\int d^4x\, d^4\theta\, \Big(\phi_i^* e^{2V}\phi_i + \widetilde\phi_i^* e^{-2V}\widetilde\phi_i\Big),$$

where V is a real gauge superfield, ϕ_i and $\widetilde\phi_i$ with $i=1,\ldots,N_f$ are chiral matter superfields, and $W_a = \bar D^2 D_a V/4$. This case corresponds to

$$C_2 = 0;\qquad C(R) = I;\qquad T(R) = 2N_f\qquad r=1,$$

where I is the $2N_f\times 2N_f$ unit matrix. Therefore, for $\mathcal{N}=1$ SQED with N_f flavors the NSVZ β-function has the form

$$\beta(\alpha) = \frac{\alpha^2 N_f}{\pi}\Big(1-\gamma(\alpha)\Big).$$

M.A.Shifman, A.I.Vainshtein, V.I.Zakharov, JETP Lett. **42** (1985) 224; Phys.Lett. **B 166** (1986) 334.

$\mathcal{N}=1$ SQED with N_f flavors, regularized by higher derivatives

In order to regularize the theory by higher derivatives it is necessary to add the higher derivative term to the action:

$$S_{\text{reg}} = \frac{1}{4e_0^2}\text{Re}\int d^4x\, d^2\theta\, W^a R(\partial^2/\Lambda^2) W_a$$
$$+\sum_{i=1}^{N_f}\frac{1}{4}\int d^4x\, d^4\theta\,\Big(\phi_i^* e^{2V}\phi_i + \widetilde{\phi}_i^* e^{-2V}\widetilde{\phi}_i\Big),$$

where $R(\partial^2/\Lambda^2)$ is a regulator, e.g. $R = 1 + \partial^{2n}/\Lambda^{2n}$.

Adding the higher derivative term allows to remove all divergences beyond the one-loop approximation. In order to remove one-loop divergencies we insert in the generating functional the Pauli–Villars determinants:

$$Z[J] = \int D\mu \prod_I \Big(\det PV(V, M_I)\Big)^{N_f c_I} \exp\Big\{iS_{\text{reg}} + iS_{\text{gf}} + \text{Sources}\Big\},$$

$$\sum_I c_I = 1;\ \ \sum_I c_I M_I^2 = 0;\ \ M_I = a_I\Lambda,\ \ \text{where } a_I \neq a_I(e_0).$$

Renormalization

$$\Gamma^{(2)} = \int \frac{d^4p}{(2\pi)^4}\, d^4\theta \Big(-\frac{1}{16\pi} V(-p)\, \partial^2\Pi_{1/2} V(p)\, d^{-1}(\alpha_0, \Lambda/p)$$
$$+\frac{1}{4}\sum_{i=1}^{N_f}\Big(\phi_i^*(-p,\theta)\phi_i(p,\theta) + \widetilde{\phi}_i^*(-p,\theta)\widetilde{\phi}_i(p,\theta)\Big)\, G(\alpha_0, \Lambda/p)\Big).$$

where $\partial^2\Pi_{1/2}$ is a supersymmetric transversal projection operator.

Then we defined the renormalized coupling constant $\alpha(\alpha_0, \Lambda/\mu)$, requiring that the inverse invariant charge $d^{-1}(\alpha_0(\alpha, \Lambda/\mu), \Lambda/p)$ is finite in the limit $\Lambda \to \infty$. The renormalization constant Z_3 is defined by

$$\frac{1}{\alpha_0} \equiv \frac{Z_3(\alpha, \Lambda/\mu)}{\alpha}.$$

The renormalization constant Z is constructed, requiring that the renormalized two-point Green function ZG is finite in the limit $\Lambda \to \infty$:

$$G_{\text{ren}}(\alpha, \mu/p) = \lim_{\Lambda\to\infty} Z(\alpha, \Lambda/\mu) G(\alpha_0, \Lambda/p).$$

The renormalization group functions defined in terms of the bare coupling constant

In most original papers

V.Novikov, M.A.Shifman, A.Vainshtein, V.I.Zakharov, Nucl.Phys. **B 229** (1983) 381; Phys.Lett. **B 166** (1985) 329; M.A.Shifman, A.I.Vainshtein, V.I.Zakharov, JETP Lett. **42** (1985) 224; Phys.Lett. **B 166** (1986) 334.

the NSVZ β-function was derived for the renormalization group functions defined in terms of the bare coupling constant

$$\beta\Big(\alpha_0(\alpha,\Lambda/\mu)\Big) \equiv \frac{d\alpha_0(\alpha,\Lambda/\mu)}{d\ln\Lambda}\Big|_{\alpha=\text{const}};$$

$$\gamma_i{}^j\Big(\alpha_0(\alpha,\Lambda/\mu)\Big) \equiv -\frac{d\ln Z_i{}^j(\alpha,\Lambda/\mu)}{d\ln\Lambda}\Big|_{\alpha=\text{const}}$$

These renormalization group functions

1. are scheme independent for a fixed regularization;

2. depend on the regularization;

2. in all loops satisfy the NSVZ relation in the case of $\mathcal{N}=1$ SQED with N_f flavors, regularized by higher derivatives.

The renormalization group functions defined in terms of the bare coupling constant

The above RG functions do not depend on the renormalization prescription, because they can be expressed via unrenormalized Green functions:

$$0=\lim_{p\to 0}\frac{d d^{-1}(\alpha_0,\Lambda/p)}{d\ln\Lambda}\Big|_{\alpha=\text{const}}=\lim_{p\to 0}\Big(\frac{\partial d^{-1}(\alpha_0,\Lambda/p)}{\partial\alpha_0}\beta(\alpha_0)-\frac{\partial d^{-1}(\alpha_0,\Lambda/p)}{\partial\ln p}\Big)$$

where in the last equality α_0 and p are considered as independent variables. Similarly, differentiating

$$\ln G(\alpha_0,\Lambda/q)=\ln G_{\text{ren}}(\alpha,\mu/q)-\ln Z(\alpha,\Lambda/\mu)$$
$$+(\text{terms vanishing in the limit } q\to 0)$$

with respect to $\ln\Lambda$ at a fixed value of α, in the limit $q\to 0$ we obtain

$$\gamma(\alpha_0)=\lim_{q\to 0}\Big(\frac{\partial\ln G(\alpha_0,\Lambda/q)}{\partial\alpha_0}\beta(\alpha_0)-\frac{\partial\ln G(\alpha_0,\Lambda/q)}{\partial\ln q}\Big).$$

Therefore, $\beta(\alpha_0)$ and $\gamma(\alpha_0)$ do not depend on an arbitrariness of choosing the renormalization constants.

NSVZ relation with the HD regularization

With the higher covariant derivative regularization loop integrals giving a β-function defined in terms of the bare coupling constant are integrals of total derivatives

A.Soloshenko, K.S., hep-th/0304083.

and even integrals of double total derivatives

A.V.Smilga, A.I.Vainshtein, Nucl.Phys. **B 704** (2005) 445.

This allows to calculate one of the loop integrals analytically and obtain the NSVZ relation for the RG functions defined in terms of the bare coupling constant. In the Abelian case this has been done in all loops

K.S., Nucl.Phys. **B 852** (2011) 71; JHEP **1408** (2014) 096.

$$\frac{\beta(\alpha_0)}{\alpha_0^2} = \frac{d}{d\ln\Lambda}\Big(d^{-1}(\alpha_0,\Lambda/p) - \alpha_0^{-1}\Big)\Big|_{p=0}$$
$$= \frac{N_f}{\pi}\Big(1 - \frac{d}{d\ln\Lambda}\ln G(\alpha_0,\Lambda/q)\Big|_{q=0}\Big) = \frac{N_f}{\pi}\Big(1-\gamma(\alpha_0)\Big).$$

Three-loop calculation for $\mathcal{N}=1$ SQED

$$\frac{\beta(\alpha_0)}{\alpha_0^2} = 2\pi N_f \frac{d}{d\ln\Lambda}\Bigg\{ \sum_I c_I \int \frac{d^4q}{(2\pi)^4} \frac{\partial}{\partial q^\mu}\frac{\partial}{\partial q_\mu} \frac{\ln(q^2+M^2)}{q^2} + 4\pi \int \frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{e^2}{k^2 R_k^2}$$

$$\times \frac{\partial}{\partial q^\mu}\frac{\partial}{\partial q_\mu}\Bigg(\frac{1}{q^2(k+q)^2} - \sum_I c_I \frac{1}{(q^2+M_I^2)((k+q)^2+M_I^2)}\Bigg)\Bigg[R_k\Bigg(1+\frac{e^2 N_f}{4\pi^2}\ln\frac{\Lambda}{\mu}\Bigg)$$

$$-2e^2 N_f\Bigg(\int \frac{d^4t}{(2\pi)^4}\frac{1}{t^2(k+t)^2} - \sum_J c_J \int \frac{d^4t}{(2\pi)^4}\frac{1}{(t^2+M_J^2)((k+t)^2+M_J^2)}\Bigg)\Bigg]$$

$$+4\pi \int \frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{d^4l}{(2\pi)^4}\frac{e^4}{k^2R_k l^2 R_l}\frac{\partial}{\partial q^\mu}\frac{\partial}{\partial q_\mu}\Bigg\{\Bigg(-\frac{2k^2}{q^2(q+k)^2(q+l)^2(q+k+l)^2}$$

$$+\frac{2}{q^2(q+k)^2(q+l)^2}\Bigg) - \sum_I c_I\Bigg(-\frac{2(k^2+M_I^2)}{(q^2+M_I^2)((q+k)^2+M_I^2)((q+l)^2+M_I^2)}$$

$$\times\frac{1}{((q+k+l)^2+M_I^2)} + \frac{2}{(q^2+M_I^2)((q+k)^2+M_I^2)((q+l)^2+M_I^2)} - \frac{1}{(q^2+M_I^2)^2}$$

$$\times\frac{4M_I^2}{((q+k)^2+M_I^2)((q+l)^2+M_I^2)}\Bigg)\Bigg\}$$

Structure of the β-function with the dimensional reduction

In the case of using the higher derivative regularization

$$\frac{d}{d\ln\Lambda}\Big(d^{-1}-\alpha_0^{-1}\Big)\Big|_{p=0} = \frac{d}{d\ln\Lambda}\Big(\mbox{One-loop} - 16\pi^3 N_f \int \frac{d^4q}{(2\pi)^4}\delta^4(q)\ln G\Big).$$

The corresponding equality obtained with the dimensional reduction in the three-loop approximation has the form

S.S.Aleshin, A.L.Kataev, K.S., arXiv:1511.05675.

$$d^{-1} - \alpha_0^{-1} = \mbox{One-loop} - 8\pi N_f \Lambda^\varepsilon \frac{\varepsilon}{1-\varepsilon}\int \frac{d^dq}{(2\pi)^d}\frac{1}{q^2(q+p)^2}(\ln G)_{1-\mbox{loop}}$$

$$-8\pi N_f \Lambda^\varepsilon \frac{2\varepsilon}{1-3\varepsilon/2}\int \frac{d^dq}{(2\pi)^d}\frac{1}{q^2(q+p)^2}(\ln G)_{2-\mbox{loops},N_f} + \mbox{finite terms}$$

$$+O(N_f\alpha_0^2) + O(\alpha_0^3).$$

and does not give the NSVZ relation in $\overline{\mbox{DR}}$-scheme.

The RG functions defined in terms of the renormalized coupling constant

RG function defined in terms of the bare coupling constant are scheme independent for a fixed regularization. However, RG functions are usually defined by a different way, in terms of the renormalized coupling constant:

$$\widetilde{\beta}\Big(\alpha(\alpha_0,\Lambda/\mu)\Big) \equiv \frac{d\alpha(\alpha_0,\Lambda/\mu)}{d\ln\mu}\Big|_{\alpha_0=\text{const}};$$

$$\widetilde{\gamma}_i{}^j\Big(\alpha(\alpha_0,\Lambda/\mu)\Big) \equiv \frac{d\ln Z_i{}^j(\alpha(\alpha_0,\Lambda/\mu),\Lambda/\mu)}{d\ln\mu}\Big|_{\alpha_0=\text{const}}.$$

These RG functions are scheme-dependent. They coincide with the RG functions defined in terms of the bare coupling constant, if the boundary conditions

$$Z_3(\alpha,x_0)=1; \qquad Z_i{}^j(\alpha,x_0)=1$$

are imposed on the renormalization constants, where x_0 is an arbitrary fixed value of $\ln\Lambda/\mu$.

A.L.Kataev and K.S., Nucl.Phys. **B 875** (2013) 459; Phys.Lett. **B 730** (2014) 184; Theor.Math.Phys. **181** (2014) 1531.

The NSVZ-scheme with the higher derivatives

$$\widetilde{\gamma}\left(\alpha(\alpha_0,x)\right) = -\frac{d\ln Z\left(\alpha(\alpha_0,x),x\right)}{dx}$$
$$= -\frac{\partial\ln Z(\alpha,x)}{\partial\alpha}\cdot\frac{\partial\alpha(\alpha_0,x)}{\partial x} - \frac{\partial\ln Z\left(\alpha(\alpha_0,x),x\right)}{\partial x},$$

where the total derivative with respect to $x = \ln\Lambda/\mu$ also acts on x inside α. Calculating these expressions at the point $x = x_0$ and taking into account that $\partial\ln Z(\alpha,x_0)/\partial\alpha = 0$ we obtain

$$\widetilde{\gamma}(\alpha_0) = \gamma(\alpha_0).$$

The equality for the β-functions can be proved similarly.

The RG functions $\widetilde{\beta}$ and $\widetilde{\gamma}$ (defined in terms of the renormalized coupling constant) are scheme-dependent. They satisfy the NSVZ relation only in a certain subtraction scheme, called the NSVZ scheme, which is evidently fixed in all loops by the boundary conditions

$$(Z_3)_{\mbox{NSVZ}}(\alpha_{\mbox{NSVZ}},x_0) = 1; \qquad Z_{\mbox{NSVZ}}(\alpha_{\mbox{NSVZ}},x_0) = 1,$$

if the theory is regularized by higher derivatives.

The scheme dependence in the three-loop approximation

The (three-loop) renormalized coupling constant for $\mathcal{N}=1$ SQED can be calculated in the case $R_k = 1 + k^{2n}/\Lambda^{2n}$:

$$\frac{1}{\alpha_0} = \frac{1}{\alpha} - \frac{N_f}{\pi}\Big(\ln\frac{\Lambda}{\mu} + b_1\Big) - \frac{\alpha N_f}{\pi^2}\Big(\ln\frac{\Lambda}{\mu} + b_2\Big) - \frac{\alpha^2 N_f}{\pi^3}\Big(\frac{N_f}{2}\ln^2\frac{\Lambda}{\mu}$$
$$- \ln\frac{\Lambda}{\mu}\Big(N_f\sum_{I=1}^{n} c_I \ln a_I + N_f + \frac{1}{2} - N_f b_1\Big) + b_3\Big) + O(\alpha^3),$$

where b_i are arbitrary finite constants.

Similarly, the renormalization constant Z (in the two-loop approximation) for the matter superfields is not also uniquely defined:

$$Z = 1 + \frac{\alpha}{\pi}\Big(\ln\frac{\Lambda}{\mu} + g_1\Big) + \frac{\alpha^2(N_f+1)}{2\pi^2}\ln^2\frac{\Lambda}{\mu}$$
$$-\frac{\alpha^2}{\pi^2}\ln\frac{\Lambda}{\mu}\Big(N_f\sum_{I=1}^{n} c_I\ln a_I - N_f b_1 + N_f + \frac{1}{2} - g_1\Big) + \frac{\alpha^2 g_2}{\pi^2} + O(\alpha^3),$$

where g_i are other arbitrary finite constants.

The subtraction scheme is fixed by fixing values of the constants b_i and g_i.

The scheme dependence in the three-loop approximation

The RG functions defined in terms of the bare coupling constant are

$$\frac{\beta(\alpha_0)}{\alpha_0^2} = \frac{N_f}{\pi} + \frac{\alpha_0 N_f}{\pi^2} - \frac{\alpha_0^2 N_f}{\pi^3}\Big(N_f \sum_{I=1}^{n} c_I \ln a_I + N_f + \frac{1}{2}\Big) + O(\alpha_0^3);$$

$$\gamma(\alpha_0) = -\frac{\alpha_0}{\pi} + \frac{\alpha_0^2}{\pi^2}\Big(N_f \sum_{I=1}^{n} c_I \ln a_I + N_f + \frac{1}{2}\Big) + O(\alpha_0^3).$$

They do not depend on the finite constants b_i and g_i (i.e. they are scheme-independent) and satisfy the NSVZ relation.

The RG functions defined in terms of the renormalized coupling constant are

$$\frac{\widetilde{\beta}(\alpha)}{\alpha^2} = \frac{N_f}{\pi} + \frac{\alpha N_f}{\pi^2} - \frac{\alpha^2 N_f}{\pi^3}\Big(N_f \sum_{I=1}^{n} c_I \ln a_I + N_f + \frac{1}{2} + N_f(b_2 - b_1)\Big) + O(\alpha^3)$$

$$\widetilde{\gamma}(\alpha) = -\frac{\alpha}{\pi} + \frac{\alpha^2}{\pi^2}\Big(N_f + \frac{1}{2} + N_f \sum_{I=1}^{n} c_I \ln a_I - N_f b_1 + N_f g_1\Big) + O(\alpha^3)$$

and depend on a subtraction scheme.

The NSVZ scheme in the three-loop approximation

The NSVZ scheme is determined by the conditions

$$\alpha_0(\alpha_{\textsf{NSVZ}}, x_0) = \alpha_{\textsf{NSVZ}}; \qquad Z_{\textsf{NSVZ}}(\alpha_{\textsf{NSVZ}}, x_0) = 1$$

For simplicity we set $g_1 = 0$ (this constant can be excluded by a redefinition of μ). In this case $x_0 = 0$ and the above conditions (for the NSVZ scheme) give

$$g_2 = b_1 = b_2 = b_3 = 0.$$

In this case in the considered approximations

$$\frac{\widetilde\beta(\alpha)}{\alpha^2} = \frac{N_f}{\pi} + \frac{\alpha N_f}{\pi^2} - \frac{\alpha^2 N_f}{\pi^3}\Big(N_f\sum_{I=1}^n c_I \ln a_I + N_f + \frac{1}{2}\Big) + O(\alpha^3) = \frac{\beta(\alpha)}{\alpha^2};$$

$$\widetilde\gamma(\alpha) = \frac{d\ln Z}{d\ln\mu} = -\frac{\alpha}{\pi} + \frac{\alpha^2}{\pi^2}\Big(N_f + \frac{1}{2} + N_f\sum_{I=1}^n c_I \ln a_I\Big) + O(\alpha^3) = \gamma(\alpha).$$

As a consequence, in this scheme the NSVZ relation is satisfied.

RG function for $\mathcal{N}=1$ SQED in different subtraction schemes

NSVZ-scheme with the higher derivatives

$$\widetilde{\gamma}_{\text{NSVZ}}(\alpha) = -\frac{\alpha}{\pi} + \frac{\alpha^2}{\pi^2}\Big(\frac{1}{2} + N_f \sum_{I=1}^{n} c_I \ln a_I + N_f\Big) + O(\alpha^3);$$

$$\widetilde{\beta}_{\text{NSVZ}}(\alpha) = \frac{\alpha^2 N_f}{\pi}\Big(1 + \frac{\alpha}{\pi} - \frac{\alpha^2}{\pi^2}\Big(\frac{1}{2} + N_f \sum_{I=1}^{n} c_I \ln a_I + N_f\Big) + O(\alpha^3)\Big).$$

MOM-scheme (The results with the dimensional reduction and with the higher derivative regularization coincide.)

$$\widetilde{\gamma}_{\text{MOM}}(\alpha) = -\frac{\alpha}{\pi} + \frac{\alpha^2(1+N_f)}{2\pi^2} + O(\alpha^3);$$

$$\widetilde{\beta}_{\text{MOM}}(\alpha) = \frac{\alpha^2 N_f}{\pi}\Big(1 + \frac{\alpha}{\pi} - \frac{\alpha^2}{2\pi^2}\Big(1 + 3N_f\,(1-\zeta(3))\Big) + O(\alpha^3)\Big).$$

$\overline{\text{DR}}$-scheme

$$\widetilde{\gamma}_{\overline{\text{DR}}}(\alpha) = -\frac{\alpha}{\pi} + \frac{\alpha^2(2+2N_f)}{4\pi^2} + O(\alpha^3);$$

$$\widetilde{\beta}_{\overline{\text{DR}}}(\alpha) = \frac{\alpha^2 N_f}{\pi}\Big(1 + \frac{\alpha}{\pi} - \frac{\alpha^2(2+3N_f)}{4\pi^2} + O(\alpha^3)\Big).$$

Simple non-Abelian example: exact expression for the Adler D-function in $\mathcal{N}=1$ SQCD

M.A.Shifman and K.S., Phys.Rev.Lett. **114** (2015) 051601; Phys.Rev. **D 91** (2015) 105008.

We consider $\mathcal{N}=1$ (massless) SQCD interacting with the Abelian gauge field:

$$S = \frac{1}{2g_0^2}\mbox{tr}\,\mbox{Re}\int d^4x\, d^2\theta\, W^a W_a + \frac{1}{4e_0^2}\mbox{Re}\int d^4x\, d^2\theta\, \boldsymbol{W}^a \boldsymbol{W}_a$$
$$+\sum_{f=1}^{N_f}\frac{1}{4}\int d^4x\, d^4\theta\Big(\Phi_f^+ e^{2q_f\boldsymbol{V}+2V}\Phi_f + \widetilde\Phi_f^+ e^{-2q_f\boldsymbol{V}-2V^t}\widetilde\Phi_f\Big).$$

where the gauge superfield strengths are given by

$$W_a \equiv \frac{1}{8}\bar D^2\big(e^{-2V} D_a e^{2V}\big); \qquad \boldsymbol{W}_a = \frac{1}{4}\bar D^2 D_a \boldsymbol{V}.$$

We assume that the gauge group is $SU(N_c)\times U(1)$, and matter superfields belong to the (anti)fundamental representation. The considered theory contains two coupling constants:

$$\alpha_s = \frac{g^2}{4\pi} \qquad \mbox{and} \qquad \alpha = \frac{e^2}{4\pi}.$$

The Adler D-function

We consider quantum corrections to the electromagnetic coupling contant α, which appear due to the quark loop with internal gluon and quark lines. The diagrams containing internal photon lines are omitted. (Thus, the electromagnetic field $\boldsymbol{V}$ is considered as an external field.)

Due to the Ward identity the two-point Green function of the superfield $\boldsymbol{V}$ is transversal:

$$\Delta\Gamma^{(2)} = -\frac{1}{16\pi}\int \frac{d^4p}{(2\pi)^4}\, d^4\theta\, \boldsymbol{V}\, \partial^2\Pi_{1/2}\boldsymbol{V}\, \Big(d^{-1}(\alpha_0, \alpha_{0s}, \Lambda/p) - \alpha_0^{-1}\Big).$$

We calculate the Adler function, which is defined in terms of the bare coupling constant by the equation

$$D(\alpha_{0s}) = \frac{3\pi}{2}\frac{d}{d\ln\Lambda}\Big(d^{-1}(\alpha_0, \alpha_{0s}, \Lambda/p) - \alpha_0^{-1}\Big)\Big|_{p=0} = \frac{3\pi}{2\alpha_0^2}\frac{d\alpha_0}{d\ln\Lambda}.$$

Thus, it depends on regularization, but is independent of a subtraction scheme.

The higher covariant derivative regularization

We add to the action the higher derivative term, e.g.,

$$S_\Lambda = \frac{1}{2g_0^2}\,\mathrm{tr}\,\mathrm{Re}\int d^4x\, d^2\theta\,(e^{\Omega}W^a e^{-\Omega})\Big[R\Big(-\frac{\bar\nabla^2\nabla^2}{16\Lambda^2}\Big)-1\Big](e^{\Omega}W_a e^{-\Omega}).$$

The covariant derivatives have the form

$$\nabla_a = e^{-\Omega^+}D_a e^{\Omega^+};\qquad \bar\nabla_{\dot a} = e^{\Omega}\bar D_{\dot a}e^{-\Omega},\qquad \text{where}\qquad e^{2V}=e^{\Omega^+}e^{\Omega},$$

Λ is a dimensionful parameter, and $R-1$ is a regulator, such as $R(0)-1=0$ and $R(x)\to\infty$ for $x\to\infty$, for example, $R(x)=1+x^n$.

Remaining one-loop (sub)divergences are regularized by inserting the Pauli–Villars determinants into the generating functional:

$$\Gamma[\boldsymbol{V}] = -i\ln\int DV\,D\Phi\,D\widetilde\Phi\prod_{I=1}^{m}\det(V,\boldsymbol{V},M_I)^{c_I}\exp\Big(i(S+S_\Lambda+S_{\mathrm{gf}}+S_{\mathrm{ghosts}})\Big),$$

where $M_I = a_I\Lambda$ and a_I do not depend on α_{0s}.

Exact expression for the Adler function

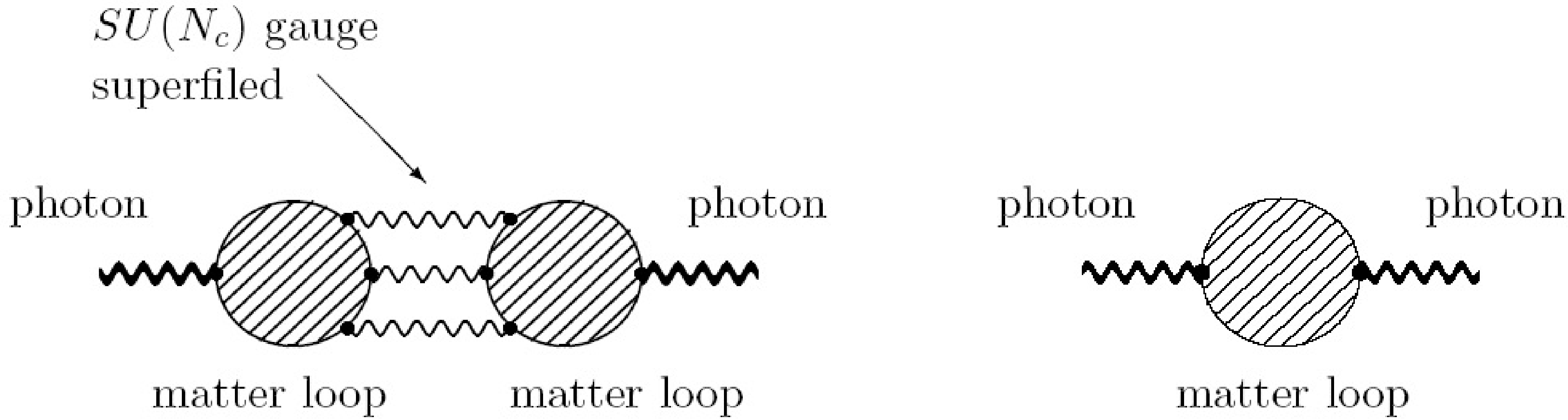
It is possible to derive the following NSVZ-like exact expression for the Adler function for the considered theory

$$D(\alpha_{0s}) = \frac{3}{2}\sum_f q_f^2 \cdot N_c\Big(1-\gamma(\alpha_{0s})\Big).$$

Note that, in general, the Adler D-function consists of two contributions

$$D(\alpha_{0s}) = \sum_f q_f^2\, D_1(\alpha_{0s}) + \Big(\sum_f q_f\Big)^2 D_2(\alpha_{0s}),$$

which correspond to two different types of diagrams:



Conclusion

- ✓ The higher covariant derivative regularization is a consistent regularization which allows to calculate quantum corrections in a manifestly gauge and supersymmetric way. That is why it is more convenient for supersymmetric theories than the dimensional regularization (reduction). Possibly, in prospect, it can be used for the supersymmetric higher spin theories.
- ✓ The higher covariant derivative regularization allows to reveal some features of quantum corrections which are not manifest with other regularization. For example, in supersymmetric gauge theories it allows to reduce integrals for the β-function to integrals of δ-singularities.
- ✓ (At least in the Abelian case) the NSVZ relation is obtained for the RG functions defined in terms of the bare coupling constant in the case of using the higher derivative regularization. For the standard definition of the RG functions it is possible to construct a simple prescription giving the NSVZ scheme in all orders. It seems that all this results are also valid in the non-Abelian case.

Thank you for the attention!