Fermionic higher-spin triplets in AdS

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(based on paper in preparation with A. Agugliaro and F. Azzurli)

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String Theory as a theory of interacting massive higher spin fields

- String excitations give rise to an infinite number of fields of increasing spin and mass

  $M_s^2 \sim T(s-1)$ (in open string theory)

- Infinite tower of higher spin string states plays a crucial role in ensuring a smooth ultraviolet behavior (or even uv finiteness) of superstring theory, thus making it a consistent theory of quantum gravity.

- Can String Theory be a spontaneously broken phase of an underlying gauge theory of massless higher spin fields?

  T -> 0

Gross '88,…, Sundborg '00, Lindstrom & Zabzine '03, Bonelli '03, …, Sagnotti & Taronna '10,...

String Theory as a theory of interacting massive higher spin fields

- One can try to deduce the structure of higher spin field interactions from the action of String Field Theory

$$S_{\text{open string}} = <\Phi|Q|\Phi> + |\Phi>^3$$

but due to the complexity of the problem, only terms in the Lagrangian describing a system of free massless higher spin fields have been obtained in this way so far

- This system of massless fields was called triplet (by Francia & Sagnotti)

$$\phi_{m_1 \cdots m_s}(x), \quad C_{m_1 \cdots m_{s-1}}(x), \quad D_{m_1 \cdots m_{s-2}}(x)$$

These fields and their equations were first obtained by S. Ouvry and J. Stern, and by A. Bengtsson in 1986

Higher-spin triplets

Ouvry & Stern '86, Bengtsson '86; Henneaux & Teitelboim '88; Pashnev '89, …,
Francia & Sagnotti '02; Sagnotti & Tsulaia '03; Barnich, Bonelli & Grigoriev '05;
Buchbinder, Fotopoulos, Petkou & Tsulaia '06; Buchbinder, Galajinsky & Krykhtin '07;
Sorokin & Vasiliev '08; Campoleoni, Francia & Sagnotti '08, Francia '10, Fotopoulos & Tsulaia '09,…,
Bekaert, Boulanger & Francia '15

"Triplets" – fields transforming under a reducible representation of SO(1,D-1)
Spectrum of physical states of (simplest) HS triplets:
s, s-2, s-4, ..., 1 or 0 – in the case of bosons
s, s-1, s-2, ..., ½ - in the case of fermions

- Basically, these systems are obtained by picking up a single Regge trajectory of an (open) string theory $M_s^2 \sim T(s-1)$, $s=0,1,\ldots,\infty$
  - taking $T \to 0$, $M_s^2 \to 0$
  - cutting the Regge tragectory at $s=s_0$

Consider, for instance a free open bosonic string in flat space–time, whose worldsheet is parametrized by a 'spatial' coordinate $\sigma \in [0, \pi]$ and a 'time' coordinate τ. String dynamics is described by the coordinates

$$X^m(\tau, \sigma) = x^m + 2\alpha' p^m \tau + i\sqrt{2\alpha'} \sum_{n \neq 0}^{\infty} \frac{1}{n} a_n^m e^{-in\tau} cos(n\sigma)$$

and momenta

$$P^m(\tau, \sigma) = p^m + \frac{1}{\sqrt{2\alpha'}} \sum_{n \neq 0}^{\infty} a_n^m e^{-in\tau} cos(n\sigma),$$

where x^m and p^m are the center of mass variables and a_n^m are the string oscillator modes satisfying (upon quantization) the commutation relations $[p^m, x^p] = -i\eta^{mp}$, $[a_n^m, a_l^p] = n\delta_{n+l}\eta^{mp}$.

String dynamics is subject to the Virasoro constraints

$$L_k = \frac{1}{2} \sum_{n=-\infty}^{+\infty} a_{k-n}^m a_{m\,n} = \sqrt{2\alpha'}\, p^m\, a_{m\,k} + \frac{1}{2} \sum_{n \neq k,0} a_{k-n}^m a_{m\,n}, \quad k \neq 0,$$

$$L_0 = 2\alpha' p^m p_m + \sum_{n>0} a_{-n}^m a_{m\,n}.$$

The latter produces the mass shell condition for the string states

$$M^2 = -p^m p_m = \frac{1}{2\alpha'} \sum_{n>0} a_{-n}^m a_{m\,n}\,.$$

We observe that in the tensionless limit $\alpha' \to \infty$ all string states become massless, while the properly rescaled Virasoro constraints become at most linear in the oscillator modes

$$l_0 = \frac{1}{2\alpha'} L_0|_{\alpha' \to \infty} = p^m p_m, \qquad l_k = \frac{1}{\sqrt{2\alpha'}} L_k|_{\alpha' \to \infty} = p_m\, a_k^m$$

and satisfy a simple algebra without any central charge

$$[l_0, l_k] = 0, \qquad [l_j, l_k] = \delta_{j+k}\, l_0\,.$$

Thus in the tensionless limit the quantum consistency of string theory does not require any critical dimension for the string to live in. Note that at $\alpha' \to \infty$ the string coordinate blows up and is not well defined, while the oscillator modes remain appropriate variables for carrying out the quantization of the theory.

The corresponding nilpotent BRST charge takes the form

$$Q = \sum_{n=-\infty}^{+\infty} \left(c_{-n}\, l_n - \frac{n}{2} b_0\, c_{-n} c_n\right),$$

where c_n and b_n are the ghosts and anti–ghosts associated with the constraint algebra

Massless higher spin triplets from String Theory

- In the tensionless limit T —> 0, the string BRST operator is:

$$Q = c_0 p_m p^m + \sum_{k\neq 0} (c_{-k} p_m a_k^m - \frac{k}{2} b_0 c_{-k} c_k), \qquad k = \pm 1, \pm 2, \ldots, \pm\infty$$

p_m - string center-of-mass momentum $(m = 0,1,\ldots, D-1)$

a_k^m - string oscillator operators ($|\mathrm{k}| = 1,2,\ldots,\infty$ labels the Regge trajectories)

c_0, c_k and b_0, b_k - string reparametrization ghosts and anti-ghosts

- String Field contains an *infinite* number of HS fields

$$|\Phi> = \sum \phi_{mnp\cdots q\cdots}(x)\, a^m a^n \cdots a^p \cdots a^q c \cdots c b \cdots b\,|0>$$

Massless higher spin triplets from String Theory

- Reduce the infinite number to a finite *independent* set of fields by picking up string states corresponding to a single Regge trajectory cut at a level of spin s and satisfying the string field equation of motion

$$Q\,|\,\Phi>=0, \qquad \delta\,|\,\Phi>=Q\,|\,\Lambda>, \qquad Q^2=0$$

BRST gauge symmetry

$$|\,\Phi>=\phi_{m_1\cdots m_s}(x)\,a_{-1}^{m_1}\cdots a_{-1}^{m_s}\,|\,0>+C_{m_1\cdots m_{s-1}}(x)\,a_{-1}^{m_1}\cdots a_{-1}^{m_{s-1}}c_0 b_{-1}\,|\,0>$$
$$+D_{m_1\cdots m_{s-2}}(x)\,a_{-1}^{m_1}\cdots a_{-1}^{m_{s-2}}c_{-1}b_{-1}\,|\,0>$$

Symmetric tensor fields $\phi_s(x)$, $C_{s-1}(x)$ and $D_{s-2}(x)$ form the simplest bosonic HS triplet *(they are not subject to tracelessness conditions)*

Bosonic HS triplet equations of motion and gauge symmetries

$$\partial^2 \phi_{m_1 \cdots m_s} = s \partial_{(m_s} C_{m_1 \cdots m_{s-1})}, \qquad \partial^2 \equiv \partial_n \partial^n$$

$$C_{m_1 \cdots m_{s-1}} = \partial^n \phi_{n m_1 \cdots m_{s-1}} - (s-1) \partial_{(m_{s-1}} D_{m_1 \cdots m_{s-2})}$$ - auxiliary field

$$\partial^2 D_{m_1 \cdots m_{s-2}} = s \partial^n C_{n m_1 \cdots m_{s-2}}$$

spectrum of physical states:

massless particles of spin

s, s-2, s-4, ..., 1 or 0

Guage transformations:

$$\delta \phi_{m_1 \cdots m_s} = s \partial_{(m_s} \Lambda_{m_1 \cdots m_{s-1})}(x),$$

$$\delta C_{m_1 \cdots m_{s-1}} = \partial^2 \Lambda_{m_1 \cdots m_{s-1}},$$

$$\delta D_{m_1 \cdots m_{s-2}} = \partial^n \Lambda_{n m_1 \cdots m_{s-2}}$$ - auxiliary field

`Geometrical' meaning of HS triplet fields

What is the geometrical nature of the triplet fields?

$\phi_{m_1 \cdots m_s}(x)$ - is a metric-like gauge field, analogous to graviton (it is not constraint by the traceless condition *in contrast to the Fronsdal fields*)

$g_{mn}(x), \quad \delta g_{mn} = \partial_m \xi_n + \partial_n \xi_m$ - linearized diffeomorphisms

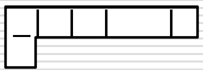
What about auxiliary fields $C_{s\text{-}1}(x)$ and $D_{s\text{-}2}(x)$? Can they be related to other geometrical quantities, such as a generalized Christoffel symbol, i.e. a HS connection?

HS triplets in the frame-like formulation

(D.S. & M. Vasiliev, arXiv:0807.0206)

- $\phi_s(x)$, $C_{s-1}(x)$ and $D_{s-2}(x)$ are components of the (*unconstrained*) higher-spin vielbein and spin connection

$$e^{a_1\cdots a_{s-1}} = dx^m e_m^{a_1\cdots a_{s-1}}(x), \qquad \omega^{a_1\cdots a_{s-1},b} = dx^m \omega_m^{a_1\cdots a_{s-1},b}(x)$$



$$\omega^{(a_1\cdots a_{s-1},b)} = 0, \quad \omega^{a_1\cdots a_{s-1},b}\eta_{a_1 b} = 0; \quad de^{a_1\cdots a_{s-1}} - dx^m \omega^{a_1\cdots a_{s-1},b}\eta_{mb} = 0$$ - torsion free condition

Higher-spin gauge transformations:

$$\delta e^{a_1\cdots a_{s-1}} = d\xi^{a_1\cdots a_{s-1}}(x) - (s-1)\,dx^m \xi^{a_1\cdots a_{s-1},b}(x)\,\eta_{mb}, \quad \delta\omega^{a_1\cdots a_{s-1},b} = d\xi^{a_1\cdots a_{s-1},b}(x) - (s-2)\,dx^m \xi^{a_1\cdots a_{s-1},bc}(x)\,\eta_{mc}$$

Relation to the triplet fields:

$$\phi^{a_1\cdots a_s} = s\, e_m^{\,(a_1\cdots a_{s-1}}\eta^{a_{s-1})m}, \quad D^{a_1\cdots a_{s-2}} = e_m^{a_1\cdots a_{s-2}a_{s-1}}\delta^m_{a_{s-1}},$$

$$C^{a_1\cdots a_{s-1}} = (s-1)\,\omega_m^{a_1\cdots a_{s-1},m} + \partial^m e_m^{a_1\cdots a_{s-1}}$$

The frame-like action for HS triplets

- The action is constructed by analogy with that for gravity

$$S=\frac{1}{2}\int e^a e^b R^{cd}\varepsilon_{abcd} \Rightarrow \int e^a\left(de^b-\frac{1}{2}e^f\omega^b{}_f\right)\omega^{cd}\varepsilon_{abcd}$$

$D=4$, but can be written for any D

Note that e^a and ω^{ab} are regarded as independent fields

- The (linearized) HS triplet action in Minkowski space

$$S=\int dx^a\left(de^{n_1\cdots n_{s-2}b}-\frac{s-1}{2}dx_m\omega^{n_1\cdots n_{s-2}b,m}\right)\omega_{n_1\cdots n_{s-2}}{}^{c,d}\varepsilon_{abcd}$$

- The HS triplet equations of motion

$$\frac{\delta S}{\delta\omega}\rightarrow de^{a_1\cdots a_{s-1}}-dx^m\omega^{a_1\cdots a_{s-1},b}\eta_{mb}=0, \qquad \frac{\delta S}{\delta e}\rightarrow \partial\,\omega=0$$

- dynamical equation *

Fermionic higher-spin triplets (originate from RNS strings) *(Sagnotti & Tsulaia '03)*

$\Psi^{\alpha}_{m_1\dots m_{s-\frac{1}{2}}}, \quad \chi^{\alpha}_{m_1\dots m_{s-\frac{3}{2}}}, \quad \lambda^{\alpha}_{m_1\dots m_{s-\frac{5}{2}}}$ propagate states with spins s, s-1,…,1/2

Equations of motion in flat space-time

$$\gamma^n \partial_n \Psi_{m_1\dots m_{s-\frac{1}{2}}} = (s-\tfrac{1}{2})\partial_{(m_1}\chi_{m_2\dots m_{s-\frac{1}{2}})}$$

$$\partial^n \Psi_{nm_2\dots m_{s-\frac{1}{2}}} - (s-\tfrac{3}{2})\partial_{(m_2}\lambda_{m_3\dots m_{s-\frac{1}{2}})} = \gamma^n \partial_n \chi_{m_2\dots m_{s-\frac{1}{2}}}$$

$$\gamma^n \partial_n \lambda_{m_1\dots m_{s-\frac{5}{2}}} = (s-\tfrac{5}{2})\partial^n \chi_{nm_1\dots m_{s-\frac{5}{2}}}$$

Gauge transformations

$$\delta\Psi_{m_1\dots m_{s-\frac{1}{2}}} = (s-\tfrac{1}{2})\partial_{(m_1}\xi_{m_2\dots m_{s-\frac{1}{2}})}$$

$$\delta\chi_{m_1\dots m_{s-\frac{3}{2}}} = \gamma^n \partial_n \xi_{m_1\dots m_{s-\frac{3}{2}}}$$

$$\delta\lambda_{m_1\dots m_{s-\frac{5}{2}}} = \partial^n \xi_{nm_1\dots m_{s-\frac{5}{2}}}$$

Generalizing this fermionic systems to AdS encountered a problem with finding their eom and a Lagrangian description

(*Sagnotti & Tsulaia '03; Buchbinder, Galajinsky & Krykhtin '07)*

Fermionic triplets in AdS from their frame-like description *(D.S. & Vasiliev'08, Agugliaro, Azzurli & D.S. '15)*

In the frame-like formulation the fermionic triplets are encoded in the single unconstrained 1-form tensor-spinor $\psi^{a_1 \ldots a_{s-3/2}} = dx^m \psi_m^{a_1 \ldots a_{s-3/2}}$

$$\Psi^{a_1 \ldots a_{s-1/2}} = e^{m(a_1} \psi_m^{a_2 \ldots a_{s-1/2})}, \quad \chi^{a_1 \ldots a_{s-3/2}} = \gamma^m \psi_m^{a_1 \ldots a_{s-3/2}}, \quad \lambda^{a_1 \ldots a_{s-5/2}} = e_b^m \psi_m^{b a_1 \ldots a_{s-5/2}}$$

Gauge transformations:

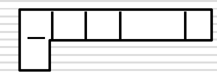
$$\delta\psi^{a_1 \ldots a_{s-3/2}} = D\xi^{a_1 \ldots a_{s-3/2}} - e^b \xi^{a_1 \ldots a_{s-3/2},c} \eta_{bc}, \qquad D\psi = (d + \tfrac{1}{4}\omega^{ab}\gamma_{ab} + \tfrac{i\sqrt{-\Lambda}}{2} e^a \gamma_a)\psi$$

$$\gamma_b \xi^{a_1 \ldots a_{s-3/2},b} = i\sqrt{-\Lambda}\,\gamma^{(a_1}{}_b\, \xi^{a_2 \ldots a_{s-3/2})b}, \qquad \xi^{a_1 \ldots a_{s-3/2},b} \eta_{a_1 b} = 0$$

Frame-like action for reducible HS fermionic field in AdS

It is useful to introduce an auxiliary fermionic HS connection

$$\omega^{a_1\dots a_{s-3/2},b} = dx^m \omega_m^{a_1\dots a_{s-3/2},b}, \qquad \delta\omega^{a_1\dots a_{s-3/2},b} = D\xi^{a_1\dots a_{s-3/2},b} - e^d \xi^{a_1\dots a_{s-3/2},bc}\eta_{dc}$$



$$\gamma_b \omega^{a_1\dots a_{s-3/2},b} = i\sqrt{-\Lambda}\,\gamma^{(a_1}{}_b\psi^{a_2\dots a_{s-3/2})b}, \qquad \omega^{a_1\dots a_{s-3/2},b}\eta_{a_1 b} = 0$$

and gauge invariant 2-form torsion: $T^{a_1\dots a_{s-3/2}} = D\psi^{a_1\dots a_{s-3/2}} - e^b\omega^{a_1\dots a_{s-3/2},c}\eta_{bc}$

Action:

$$S = i\int_{AdS_D} e^{a_1}\dots e^{a_{D-3}}\varepsilon_{a_1\dots a_{D-3}bcd}\left(\bar\psi_{n_1\cdots n_{s-3/2}}\gamma^{bcd}T^{n_1\cdots n_{s-3/2}} - 6\left(s-\frac{3}{2}\right)\bar\psi_{n_1\cdots n_{s-5/2}}{}^{b}\gamma^{c}T^{n_1\cdots n_{s-5/2}d}\right)$$

$$+\tfrac{1}{2}\int d^D x\sqrt{-g}\,\bar\rho(\not{D} + 2i\sqrt{-\Lambda})\rho$$

spin 1/2

the relative coefficients are fixed by the requirement that
only the gamma-trace part of the HS connection enters the action

$$\Psi^{a_1 \ldots a_{s-1/2}} = e^{m\,(a_1}\psi_m^{a_2 \ldots a_{s-1/2})}, \quad \chi^{a_1 \ldots a_{s-3/2}} = \gamma^m \psi_m^{a_1 \ldots a_{s-3/2}}, \quad \lambda^{a_1 \ldots a_{s-5/2}} = e_b^m \psi_m^{b a_1 \ldots a_{s-5/2}}$$

Metric-like action for fermionic triplets in AdS

(Agugliaro, Azzurli & D.S. '15)

$$\begin{aligned} S^m_{AdS} = \int d^D x \sqrt{g} \Big[& i\bar{\chi}^{b(r-1)} \not{\nabla} \chi_{b(r-1)} - i\bar{\chi}^{b(r-1)} \nabla \cdot \Psi_{b(r-1)} + i\nabla \cdot \bar{\Psi}^{b(r-1)} \chi_{b(r-1)} + \frac{i}{r} \bar{\Psi}^{b(r)} \not{\nabla} \Psi_{b(r)} \\ & + i(r-1)\left(-\bar{\lambda}^{b(r-2)} \not{\nabla} \lambda_{b(r-2)} - \nabla \cdot \bar{\chi}^{b(r-2)} \lambda_{b(r-2)} + \bar{\lambda}^{b(r-2)} \nabla \cdot \chi_{b(r-2)} \right) \\ & + \sqrt{-\Lambda} \Big(\frac{D+2r}{2} \bar{\chi}^{b(r-1)} \chi_{b(r-1)} - \frac{D+2r-4}{2r} \bar{\Psi}^{b(r)} \Psi_{b(r)} + (r-1) \frac{D+2r-8}{2} \bar{\lambda}^{b(r-2)} \lambda_{b(r-2)} \\ & + \frac{3}{2}(r-1) \bar{\chi}^{b(r-2)} \lambda_{b(r-2)} + \frac{3}{2}(r-1) \bar{\lambda}^{b(r-2)} \not{\chi}_{b(r-2)} - \frac{3}{2} \bar{\not{\Psi}}^{b(r-1)} \chi_{b(r-1)} - \frac{3}{2} \bar{\chi}^{b(r-1)} \not{\Psi}_{b(r-1)} \\ & + \bar{\not{\Psi}}^{b(r-1)} \not{\Psi}_{b(r-1)} - (r-1) \bar{\not{\chi}}^{b(r-2)} \not{\chi}_{b(r-2)} - (r-1)(r-2) \bar{\not{\chi}}^{b(r-3)} \not{\chi}_{b(r-3)} \Big) \Big] \end{aligned}$$

$\nabla = d + \omega$ - conventional covariant derivative and r=s-1/2

Gauge symmetry

$$\begin{aligned} \delta\Psi^{a(r)} &= \nabla^a \xi^{a(r-1)} + \frac{i}{2}\sqrt{-\Lambda}\gamma^a \xi^{a(r-1)} \\ \delta\chi^{a(r-1)} &= \not{\nabla}\xi^{a(r-1)} + i\frac{D+2r-2}{2}\sqrt{-\Lambda}\xi^{a(r-1)} - i\sqrt{-\Lambda}\gamma^a \not{\xi}^{a(r-2)} \\ \delta\lambda^{a(r-2)} &= \nabla \cdot \xi^{a(r-1)} + \frac{i}{2}\sqrt{-\Lambda}\not{\xi}^{a(r-2)}. \end{aligned}$$

Equations of motion for spin 3/2 - 1/2 doublet

$$\left(\gamma^b\nabla_b + i\frac{D-2}{2}\sqrt{-\Lambda}\right)\psi_a - i\sqrt{-\Lambda}\gamma_a\gamma^b\psi_b = (\nabla_a - \frac{3i}{2}\sqrt{-\Lambda}\,\gamma_a)\,\chi$$

$$(\nabla_a - \frac{3i}{2}\sqrt{-\Lambda}\,\gamma_a)\psi^a = \left(\gamma^a\nabla_a - i\frac{D+2}{2}\sqrt{-\Lambda}\right)\chi$$

↓

$$\left(\gamma^a\nabla_a - i\frac{D-4}{2}\sqrt{-\Lambda}\right)\underline{(\chi - \gamma^b\psi_b)} = 0$$

propagating spin 1/2

gauge symmetry

$$\delta\psi_a = (\nabla_a + \frac{i}{2}\sqrt{-\Lambda}\,\gamma_a)\,\xi$$

$$\delta\chi = \left(\gamma^a\nabla_a + i\frac{D}{2}\sqrt{-\Lambda}\right)\xi$$

Sagnotti & Tsulaia '03:

$$\left(\gamma^b\nabla_b + i\frac{D-2}{2}\sqrt{-\Lambda}\right)\psi_a - \frac{i\sqrt{-\Lambda}}{2}\gamma_a\gamma^b\psi_b = \nabla_a\,\chi$$

$$(\nabla_a - i\frac{D-2}{2}\sqrt{-\Lambda}\,\gamma_a)\psi^a = \gamma^a\nabla_a\,\chi$$

↓

$\chi = \gamma^a\psi_a$ - pure gauge

Conclusion and outlook

- Triplet systems of fields of spin s, s-2,…,1 or 0 appears in a truncated free action for the String Field Theory in flat space

- In the frame-like formulation, these fields are endowed with a geometrical meaning of higher-spin vielbeins and connections transforming under higher-spin local symmetries.

- Starting from the frame-like formulation for the fermionic higher-spin triplets (describing physical states of spin s-1,…,1/2) we have resolved at long last the problem of their metric-like Lagrangian description in AdS spaces.

Outlook

- To understand whether and how the triplets in AdS may arise from the quantization of strings in AdS at T->0 *(Bonelli '03)*

- Proceed with studying interactions of HS triplet fields along lines put forward by *Fotopoulos & Tsualia '09* (current exchanges, cubic & quartic vertices, etc.)
  - reducibility can make things simpler:
    single triplet vertex contains a # of vertices of irreducible HS fields

- Minimal Vasiliev Theory is a theory of a single s=2∞ "triplet", can it be extended to an interacting theory of infinite sets of "triplets (as in string theory)?