Quartic Interactions in Higher-Spin Gravity from CFT

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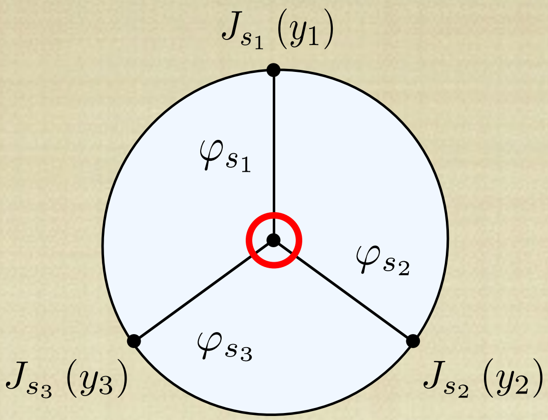
Imperial College London

Moscow, November 24, 2015

arXiv: 1412.0016 and 1508.04292
[with X. Bekaert, J. Erdmenger and C. Sleight]

Cubic vertices



$$= \quad \langle J_{s_1}(y_1) J_{s_2}(y_2) J_{s_3}(y_3) \rangle$$

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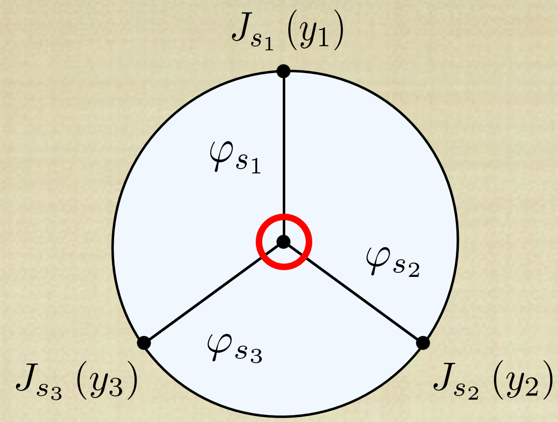

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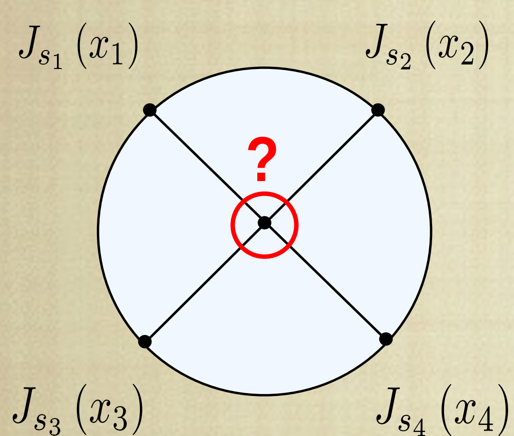
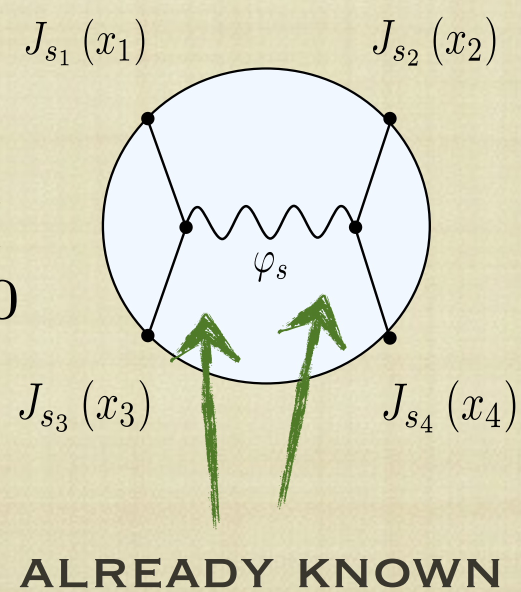


$$+ \sum_{s=0}^{\infty}$$



$+$ u- and t-channels

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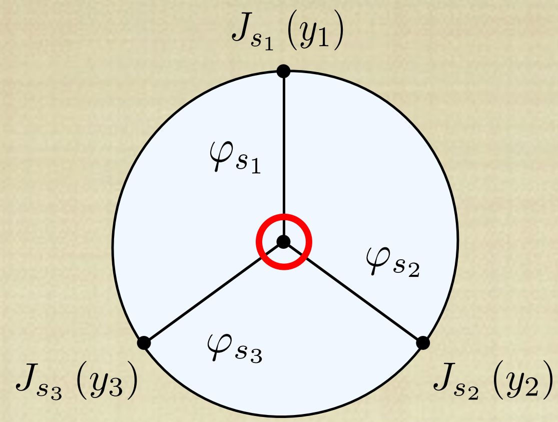

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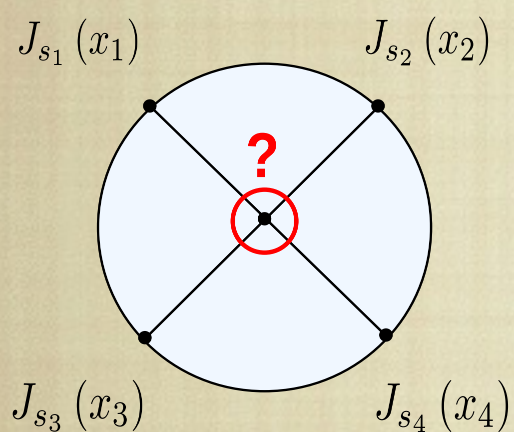

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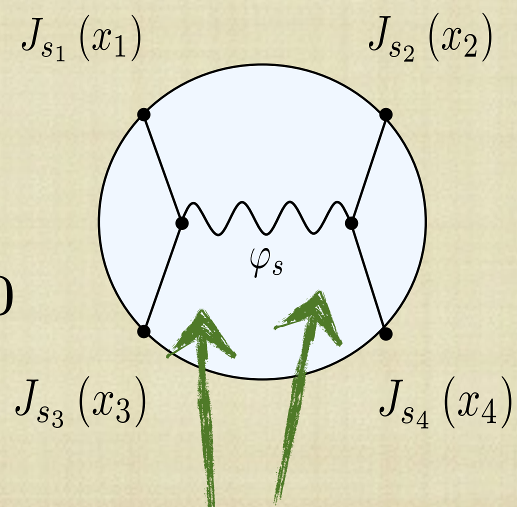

$+$ u- and t-channels

ALREADY KNOWN

$$= \langle J_{s_1}(y_1) J_{s_2}(y_2) J_{s_3}(y_3) J_{s_4}(y_4) \rangle$$

TO START: QUARTIC VERTEX FOR SCALAR FIELDS

Motivation. HS interactions

Holographic reconstruction vs Noether procedure

- Noether procedure after a cubic level becomes technically tedious

See, however, [Vasiliev'90],[Metsaev'91],[Polyakov'10], [Taronna'11],[Dempster,Tsulaia'12], [Buchbinder,Krykhtin'15],[Polyakov'15]

- How far can one go reconstructing hs interactions from holography?
- Recent developments in computations of amplitudes in AdS and CFT (in the context of bootstrap, Mellin amplitudes)

[Mack,Penedones,Costa,Paulos,Fitzpatrick,Kaplan,Simmons-Duffin,..]

- The power of these techniques is not completely understood

Motivation. HS interactions

Locality & Noether procedure:

with locality - no solutions **?** without locality - infinitely many solutions

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The way out: weaken the notion of locality

Requirements for weak locality:

- Make solutions of the Noether procedure possible
- Rule out pathologically non-local behaviour

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Proposal: the vertex is weakly local iff the associated amplitude is an entire function

$$\phi^2 \frac{1}{\Box}\phi^2 \to \frac{1}{s}$$

not weakly local

$$\phi^2 \frac{1}{\Box - \Lambda}\phi^2 \to \frac{1}{s-\Lambda}$$

not weakly local

$$\phi^2 e^{\Box}\phi^2 \to e^{s}$$

weakly local

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not weakly local	not weakly local	weakly local

Locality in HS was recently discussed in [Vasiliev'15],[Skvortsov,Taronna'15]

Motivation. Holographic reconstruction

CFT -> AdS

- Can this be done consistently for any tree level n-point functions?
- What one should require from CFT to expect a local bulk dual (sub-AdS radius scale)?

[Gary,Giddings,Penedones'09],
[Heemskerk,Penedones,Polchinski,Sully'09],
[El-Showk,Papadodimas'11],
[Fitzpatrick,Kaplan,Poland,Simmons-Duffin'12],
[Maldacena,Simmons-Duffin,Zhiboedov'15] and many other

- Assume agreement at tree level. Does this imply agreement for all loops?

Higher spin holography

Bulk: **Minimal Higher spin gauge theory in 4d**

$$S = \int \sqrt{g} d^4x \nabla^{\mu_1} \varphi^{\mu_2 \dots \mu_{s+1}} \nabla_{\mu_1} \varphi_{\mu_2 \dots \mu_{s+1}} + \dots$$

$$\delta \varphi_{\mu_1 \dots \mu_s} = \nabla_{\mu_1} \xi_{\mu_2 \dots \mu_s} + \dots \qquad s = 0,\ 2,\ 4,\ \dots\ \infty$$

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Boundary: **Free O(N) vector model in 3d**

$$S = \int d^3x \partial^\mu \phi^a \partial_\mu \phi_a$$

$$J_{\mu_1 \dots \mu_s} = \phi^a \partial_{\mu_1} \dots \partial_{\mu_s} \phi_a + \dots, \qquad \partial^{\mu_1} J_{\mu_1 \dots \mu_s} = 0$$

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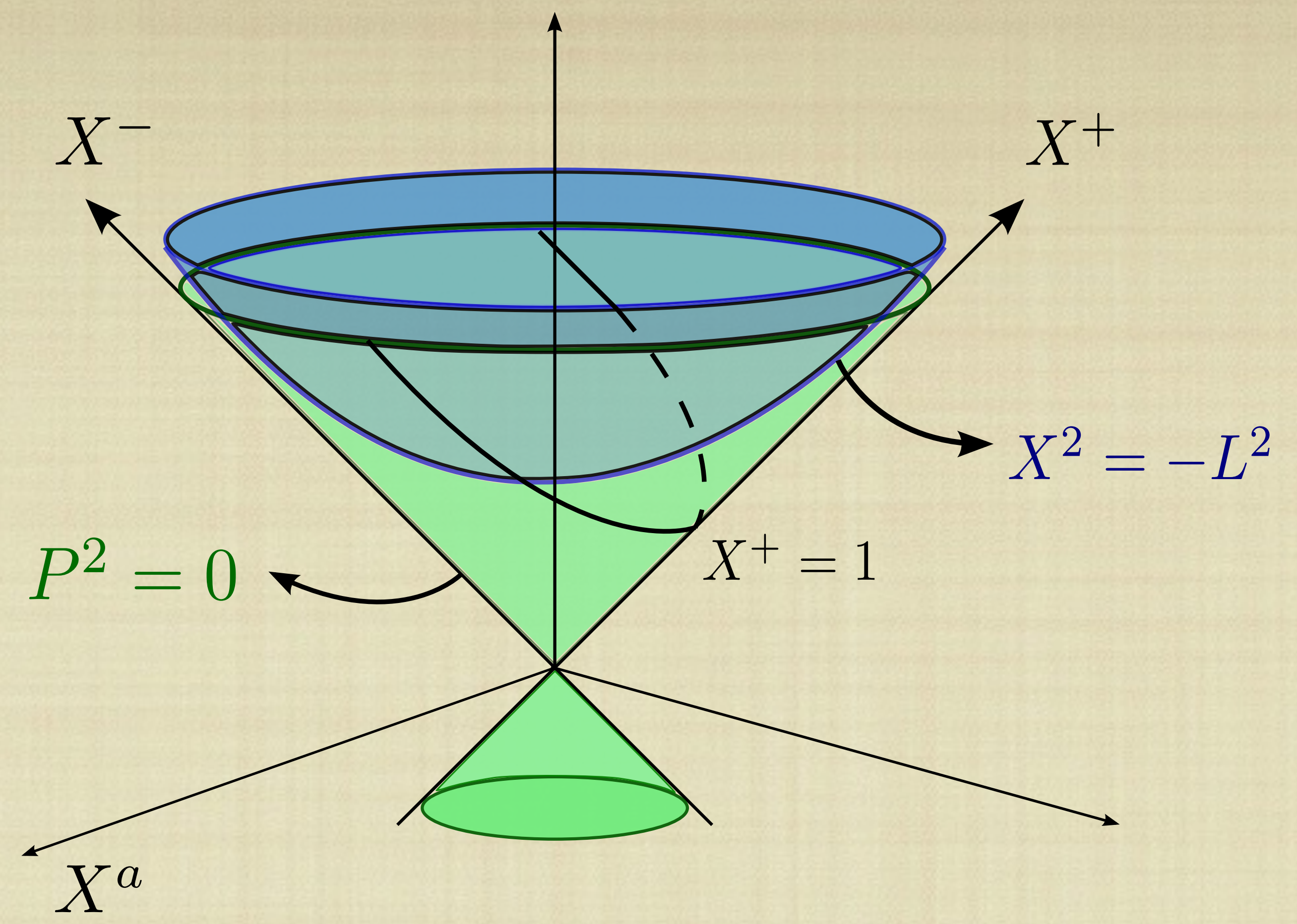
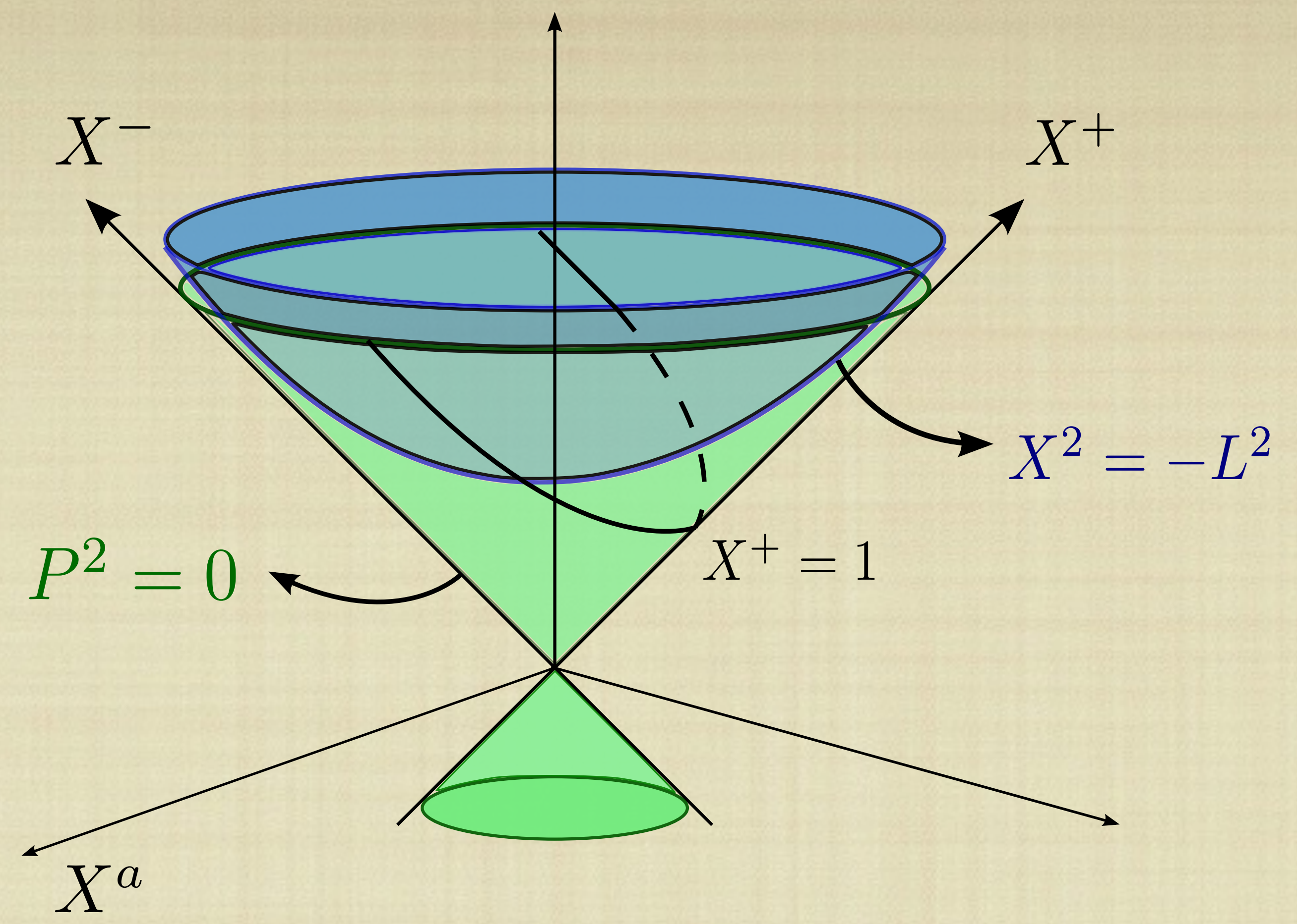
Duality: $\varphi_{\mu_1 \dots \mu_s} \quad \leftrightarrow \quad J_{\mu_1 \dots \mu_s}$



X^-
X^+
$X^2 = -L^2$
$X^+ = 1$
$P^2 = 0$
X^a



Ambient formalism for AdS

$$
\begin{aligned}
AdS_{d+1} &\rightarrow \mathbb{R}^{d+2}, \quad g = \mathrm{diag}(+,+,-,\ldots,-) \\
AdS_{d+1}\ \text{bulk} &\rightarrow X^2 = 1 \\
\text{boundary} &\rightarrow P^2 = 0, \quad P \sim \alpha P \\
W \text{ and } Z &\rightarrow \text{auxiliary vectors}
\end{aligned}
$$

3-pt functions

CFT:

$$\langle J^0(x_1)J^0(x_2)J^s_{\mu_1\ldots\mu_s}(x_3)\rangle = C_{00s}\frac{\left(\frac{x_{13}}{r_{13}}-\frac{x_{23}}{r_{23}}\right)_{\mu_1}\ldots\left(\frac{x_{13}}{r_{13}}-\frac{x_{23}}{r_{23}}\right)_{\mu_s}}{r_{12}^{\frac{\Delta_1+\Delta_2-\Delta_3+s}{2}}r_{13}^{\frac{\Delta_3+\Delta_1-\Delta_2-s}{2}}r_{23}^{\frac{\Delta_3+\Delta_2-\Delta_1-s}{2}}}$$

$$J^0(x) =: \phi^a(x)\phi_a(x) :$$

$$J^s_{\mu(s)}(x) =: \phi^a(x)\partial_{\mu_1}\ldots\partial_{\mu_s}\phi_a(x) : +\ldots$$

$$\Delta_1 = \Delta_2 = d-2$$

$$\Delta_3 = d+s-2$$

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$$\Delta_1=\Delta_2=d-2$$

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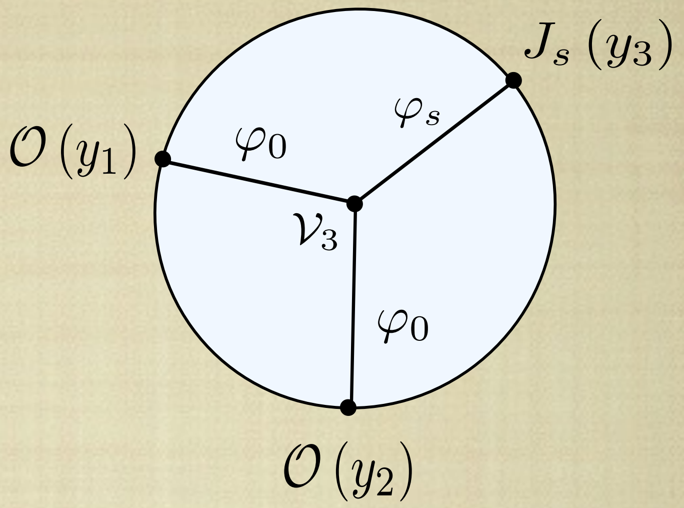
Doing Wick contractions

$$C_{00s}=\frac{2^{s/2+3/2}}{\sqrt{s!}\sqrt{N}}\frac{(d/2-1)_s}{\sqrt{(d+s-3)_s}}$$

[Diaz,Dorn'06]

3-pt Witten diagram

Bulk:



$$g_{00s}\int_{AdS} d^{d+1}x\sqrt{g}\varphi^{\mu_1\ldots\mu_s}J_{\mu_1\ldots\mu_s}$$

$$\sim 2^s g_{00s}\int_{AdS} d^{d+1}x\sqrt{g}\varphi^{\mu_1\ldots\mu_s}\varphi\nabla_{\mu_1}\ldots\nabla_{\mu_s}\varphi$$

3-pt Witten diagram

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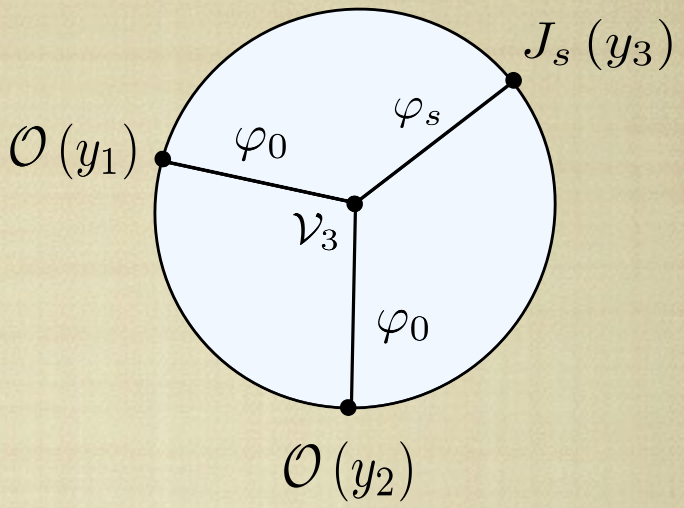

Higher-spin bulk-to-boundary propagator

$$\Pi_s(X,W;P,Z)\sim\frac{(2(Z\cdot X)(P\cdot W)-2(W\cdot Z)(P\cdot X))^s}{(-2X\cdot P)^{\Delta_s+s}}-\text{traces}$$

[Mikhailov'02]

3-pt Witten diagram

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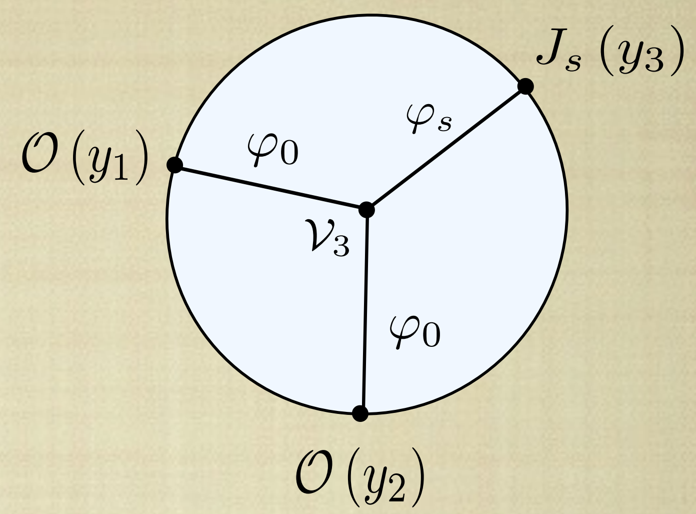

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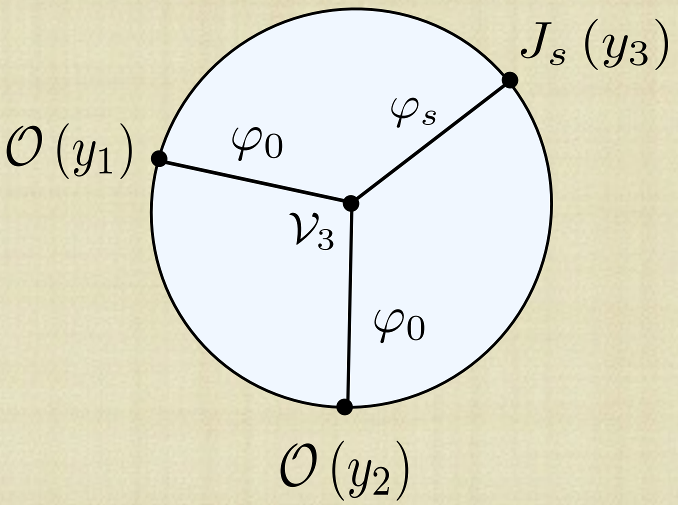
[Mikhailov'02]

Can be reduced to an analogous computation for the scalar

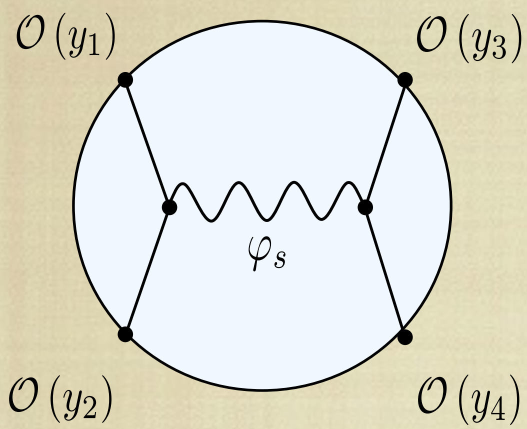
[Paulos'11][Costa,Goncalves,Penedones'14]

Matching bulk & CFT

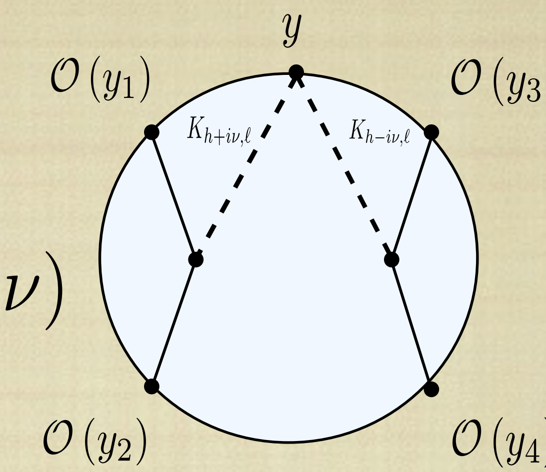
$$g_{00s} = \frac{2^{4-s/2}}{\sqrt{N}\Gamma(s)}$$



Exchanges. Split representation



$$= \sum_{\ell=0}^{s} \int_{\partial \mathrm{AdS}} d^d y \int_{-\infty}^{\infty} d\nu \;\; b_\ell^{s.ch}(\nu)$$



$$F_{\nu,s}(u,v) = \int_{\partial AdS} d^d y \langle \mathcal{O}_0(y_1)\mathcal{O}_0(y_2)\mathcal{O}_{h+i\nu,s}(y)\rangle\langle\mathcal{O}_{h-i\nu,s}(y)\mathcal{O}(y_3)\mathcal{O}(y_4)\rangle$$

$$F_{\nu,s}(u,v) = \kappa(\nu,s)G_{h+i\nu}(u,v) + \kappa(-\nu,s)G_{h-i\nu,s}(u,v)$$

Produces the conformal block decomposition!

Acting along these lines we found a conformal block decomposition for higher spin exchanges!

[Bekaert,Erdmenger,P.,Sleight'14]

HS exchange in d=4

Simplification: currents are traceless

$$\mathcal{A}(P_1,P_2;P_3,P_4)$$

$$= \frac{g_{00s}^2}{(P_1 \cdot P_2)(P_3 \cdot P_4)} \int_{-\infty}^{+\infty} d\nu \, \frac{2^{-2i\nu+2s-3}\Gamma(i\nu+\frac{1}{2})\Gamma^2\left(\frac{1}{4}(2s+2i\nu+1)\right)}{\pi^{5/2}\Gamma(i\nu)(2i\nu+2s+1)}$$

$$\times \frac{\Gamma^2\left(\frac{1}{4}(2s-2i\nu+1)\right)}{(\nu^2+(s-\frac{1}{2})^2)} G_{h+i\nu,s}(u,v)$$

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$$\times\frac{\Gamma^2\left(\frac{1}{4}(2s-2i\nu+1)\right)}{(\nu^2+(s-\frac{1}{2})^2)}G_{h+i\nu,s}(u,v)$$

closing the contour in the lower half plane we pick the poles at:

$$\nu=-i(2n+s+1/2)\quad\Rightarrow\quad \Delta_s=h+i\nu=2\Delta+n+s=2d+2n+s-4$$

Double-trace operators

$$\mathcal{O}_{n,s}=:\Box^n(\phi^a\phi_a)\partial_{\mu_1}\ldots\partial_{\mu_s}(\phi^b\phi_b):+\ldots$$

$$\Delta=\Delta(\varphi_0)=\Delta(\phi^a\phi_a)$$

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closing the contour in the lower half plane we pick the poles at:

$$\nu = -i(2n+s+1/2) \quad \Rightarrow \quad \Delta_s = h + i\nu = 2\Delta + n + s = 2d + 2n + s - 4$$

Double-trace operators

$$\mathcal{O}_{n,s} =: \Box^n(\phi^a\phi_a)\partial_{\mu_1}\ldots\partial_{\mu_s}(\phi^b\phi_b) : + \ldots$$

$$\Delta = \Delta(\varphi_0) = \Delta(\phi^a\phi_a)$$

$$\nu = -i(s-1/2) \quad \Rightarrow \quad \Delta_s = h + i\nu = \Delta + s = d + s - 2$$

Single-trace operators (conserved currents)

$$\mathcal{J}_s =: \phi^a\partial_{\mu_1}\ldots\partial_{\mu_s}\phi_a : + \ldots$$

Quartic vertices

The basis: $\mathcal{V}_{n,s} = J_{\mu_1 \dots \mu_s} \Box^n (J^{\mu_1 \dots \mu_s}), \quad s = 2k, \quad k \geq n \geq 0$

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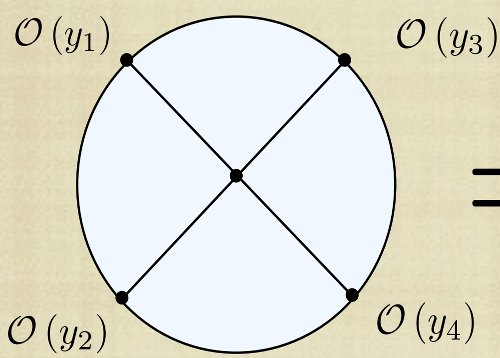

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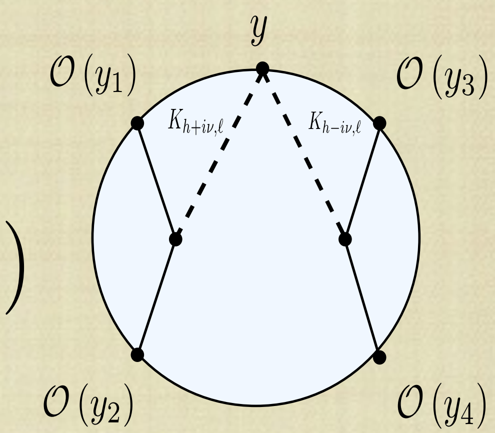

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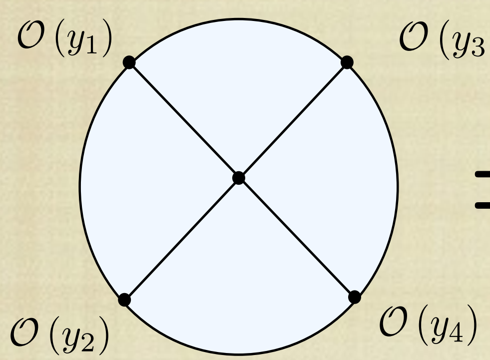

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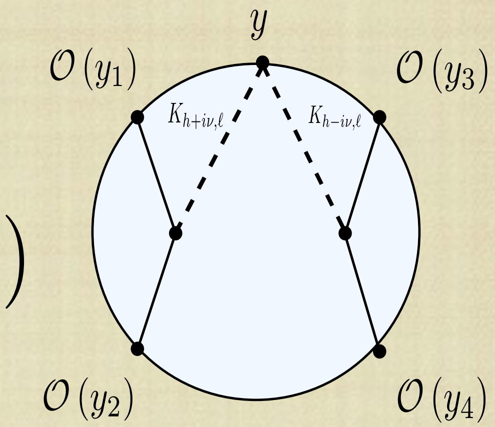

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$$\times (\nu^2+s+\frac{9}{4})^n\Gamma^2(\frac{1}{4}(2s-2i\nu+1))G_{h+i\nu,s}(u,v)$$

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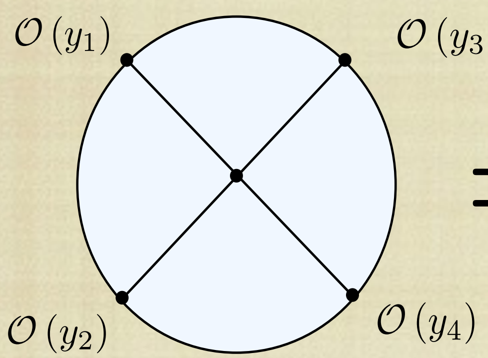

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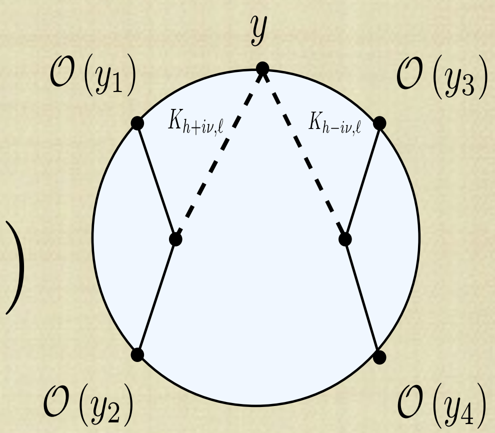

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$$\times (\nu^2 + s + \frac{9}{4})^n \Gamma^2(\frac{1}{4}(2s - 2i\nu + 1)) G_{h+i\nu,s}(u,v)$$

DOUBLE TRACE POLES

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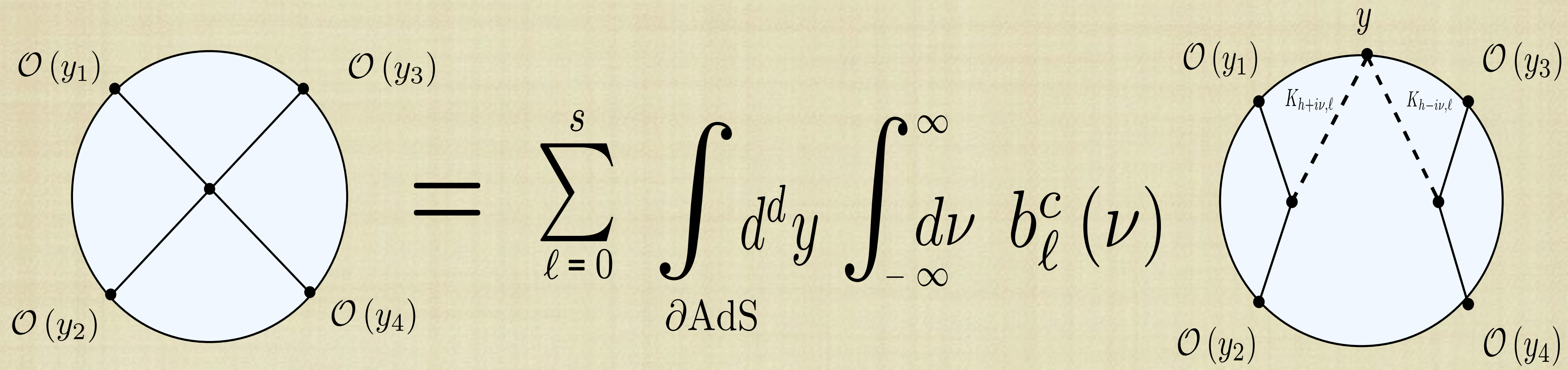

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$$\times(\nu^2+s+\frac{9}{4})^n\Gamma^2(\frac{1}{4}(2s-2i\nu+1))G_{h+i\nu,s}(u,v)$$

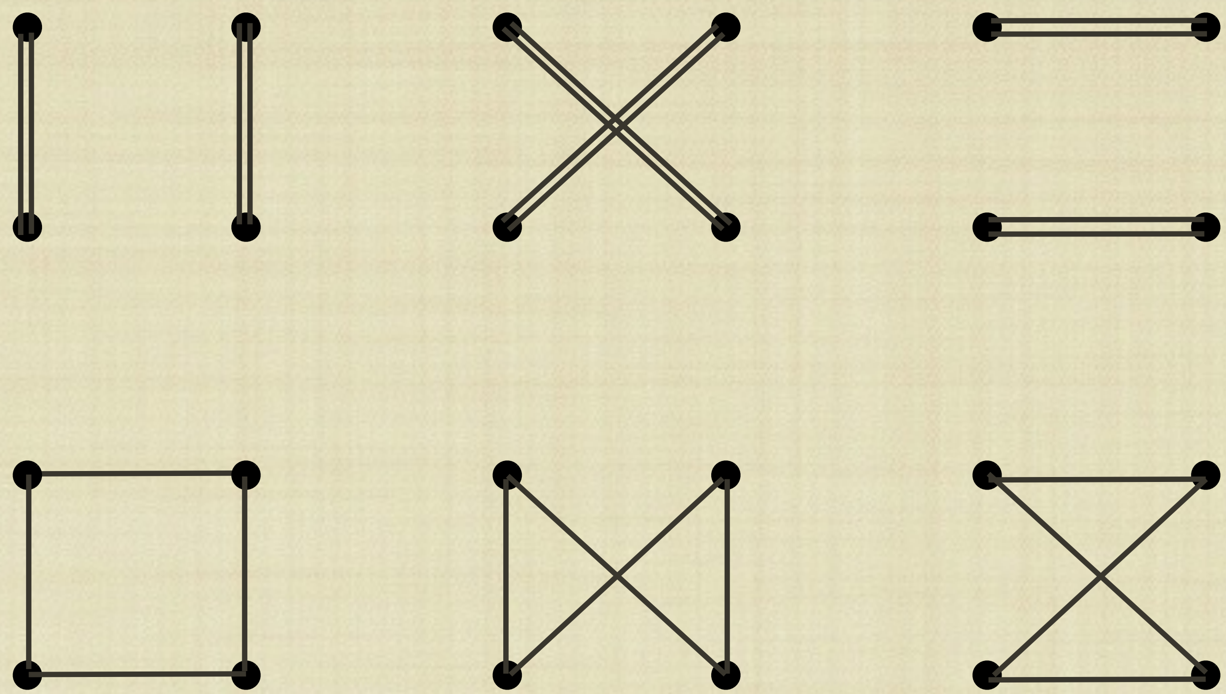
The only difference with exchanges

Double trace poles

CFT

4-pt function via Wick contractions

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle \quad =$$



$\sim O(N^0)$

$=$

$\sim O(1/N)$

$\mathcal{O} = \phi^a \phi_a$

$$= \langle \phi(x)\phi(y)\rangle = \frac{1}{|x-y|^{2\delta}}$$

OPE:

$$\mathcal{O}\mathcal{O} \sim 1 + \sum_s \mathcal{J}_s + \sum_{n,s} \mathcal{O}_{n,s}$$

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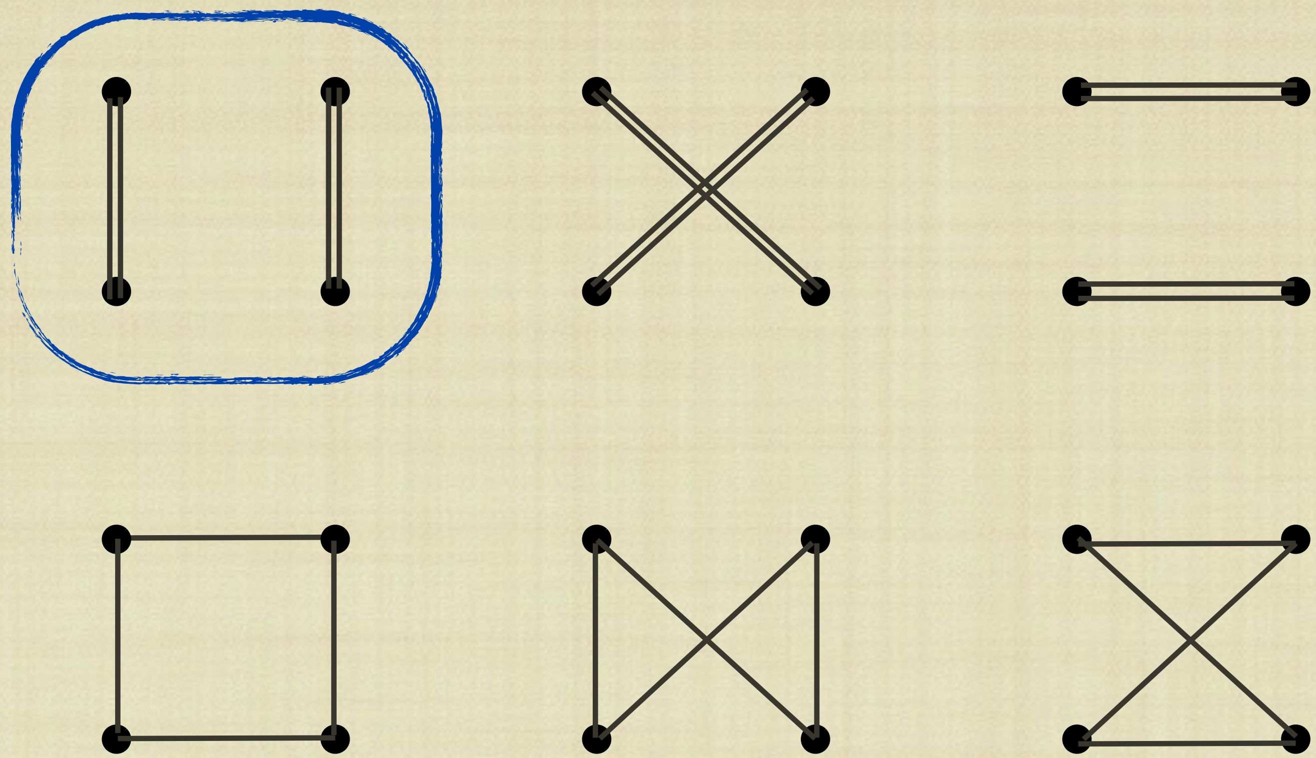

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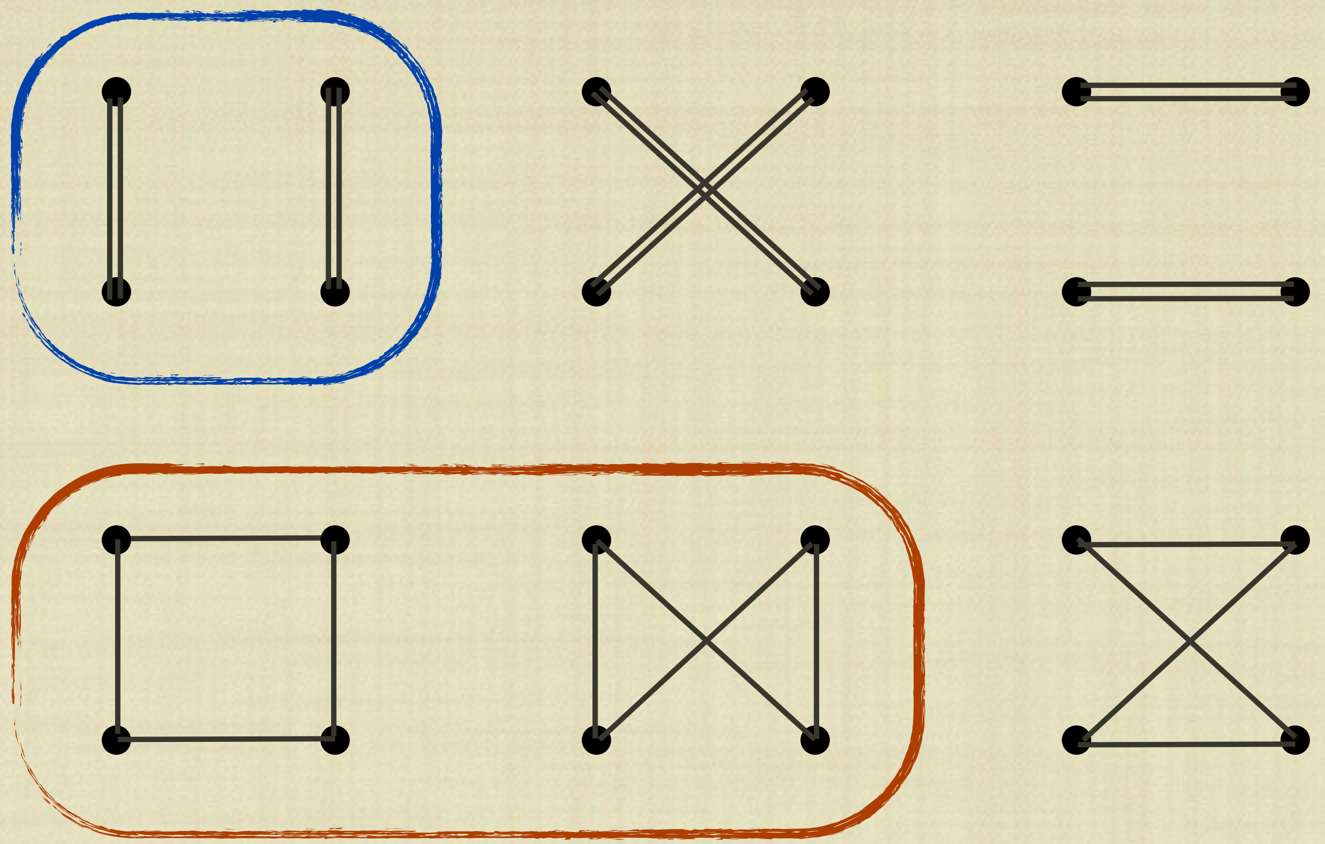

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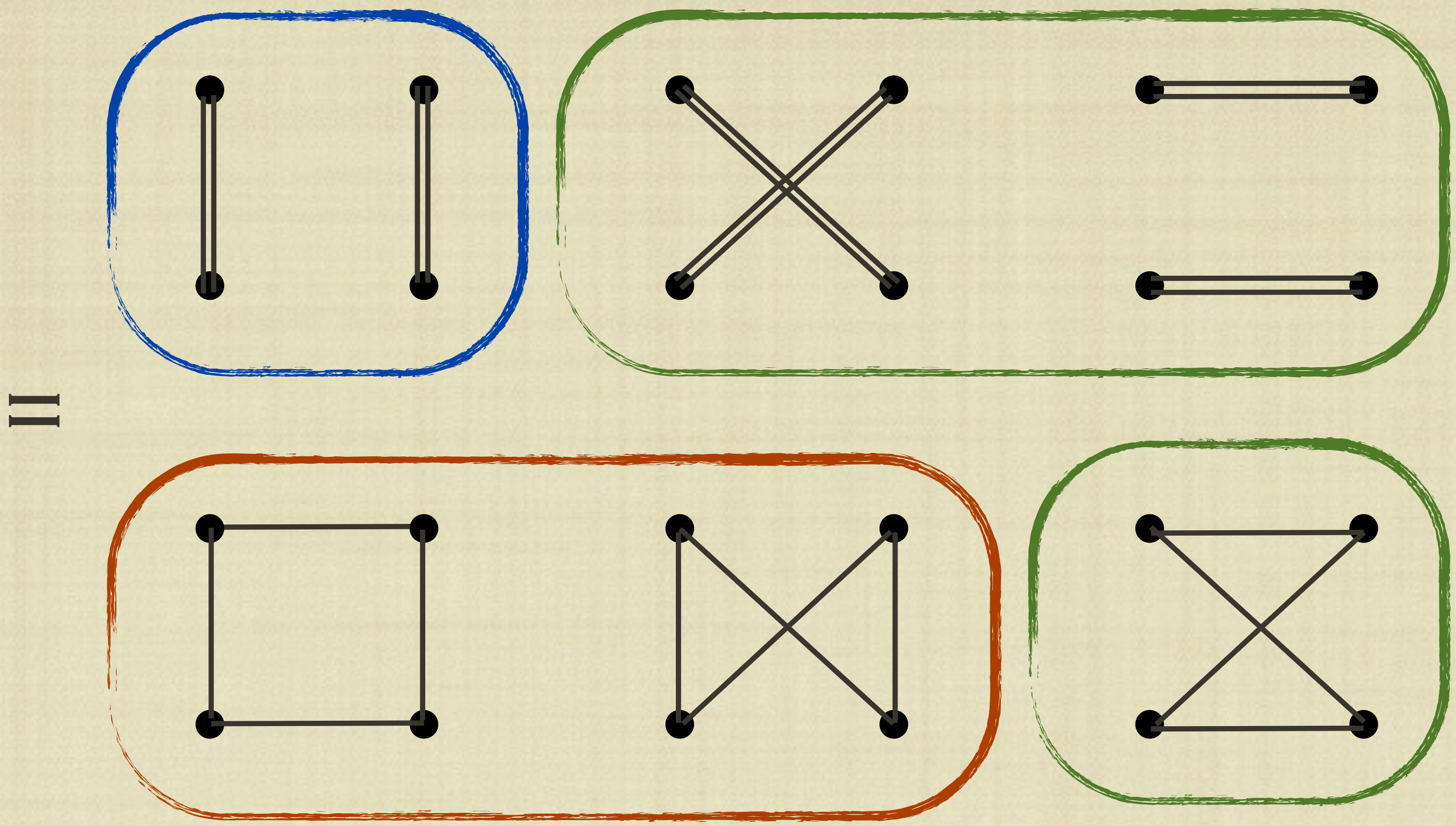

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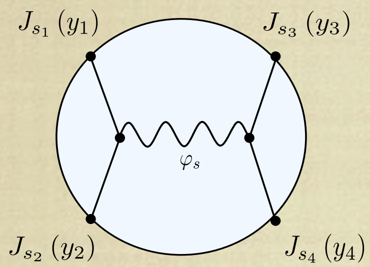
Single channel problem

$$\mathcal{V}_4 = \sum_{n,s} a_{n,s} J_{\mu_1 \ldots \mu_s} \Box^n J^{\mu_1 \ldots \mu_s}$$

$$\sum_{s=0}^{\infty}$$



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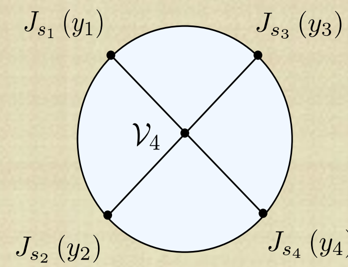

$=$ “ $\frac{1}{3}\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O} \rangle$ ”

Use representation $\int d\nu\, \alpha(\nu)\, G_{h+i\nu,s}(u,v)$

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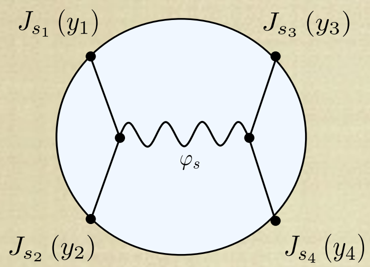

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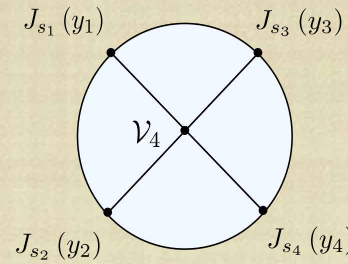

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$$\frac{\Gamma^2\left(\frac{1}{4}(2s-2i\nu+1)\right)}{\nu^2+(s-\frac{1}{2})^2}(\ldots)$$

$$\left[\;\ldots\;\right]$$

$$\sum_{n=0}^{\infty} a_{n,s}(-1)^n \left(\nu^2+s+\frac{9}{4}\right)^n \Gamma^2\left(\frac{1}{4}(2s-2i\nu+1)\right)(\ldots)$$

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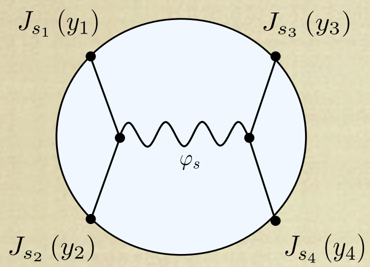

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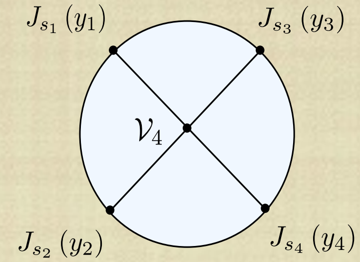

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$$p_s(\nu)$$

Solve for $p_s(\nu)$. Taylor series coefficients at $\nu^2 = -s - 9/4$ define $a_{n,s}$

The vertex

$$\mathcal{V}_4 = \sum_{n,s} a_{n,s} J_{\mu_1 \dots \mu_s} \Box^n J^{\mu_1 \dots \mu_s} \qquad s = 2k$$

$$\sum_m a_{m,s} (-1)^m (\nu^2 + s + \frac{9}{4})^m = \frac{2^{8-s}}{N} \frac{1}{\nu^2 + (s - \frac{1}{2})^2} \left[\frac{\pi}{\Gamma^2\left(\frac{2s-2i\nu+1}{4}\right)\Gamma^2\left(\frac{2s+2i\nu+1}{4}\right)} - \frac{1}{\Gamma^2(s)} \right]$$

$$-\frac{1}{N} \frac{(-1)^{\frac{s}{2}} \pi^{\frac{3}{2}} 2^{s+5} \Gamma\left(s + \frac{3}{2}\right) \Gamma\left(\frac{s}{2} + \frac{1}{2}\right)}{\sqrt{2}\Gamma\left(\frac{s}{2} + 1\right) \Gamma(s+1) \Gamma\left(\frac{3}{4} - \frac{i\nu}{2}\right) \Gamma\left(\frac{3}{4} + \frac{i\nu}{2}\right) \Gamma\left(s + \frac{1}{2} + i\nu\right) \Gamma\left(s + \frac{1}{2} - i\nu\right)}$$

The vertex

$$\mathcal{V}_4 = \sum_{n,s} a_{n,s} J_{\mu_1\ldots\mu_s} \Box^n J^{\mu_1\ldots\mu_s} \qquad s = 2k \qquad k \geq n$$

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The set of vertices used is excessive

Locality vs ∂^∞

In flat space:

Mandelstam variables

$$\mathcal{V}_4 \quad \to \quad \mathcal{A}(s, t, u)$$

Poles in $\mathcal{A}$ $\leftrightarrow$ exchanges

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Definition:

Weak Locality	=	Amplitude is an entire function

Locality vs ∂^∞

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$$\mathcal{V}_4 \quad \rightarrow \quad \mathcal{A}(s,t,u)$$

Poles in $\mathcal{A}$ $\leftrightarrow$ exchanges

Definition:

Weak Locality = Amplitude is an entire function

$$\phi^2 \frac{1}{\Box}\phi^2 \rightarrow \frac{1}{s}$$

not weakly local

$$\phi^2 \frac{1}{\Box - \Lambda}\phi^2 \rightarrow \frac{1}{s-\Lambda}$$

not weakly local

$$\phi^2 e^{\Box}\phi^2 \rightarrow e^s$$

weakly local

Leaves room for local infinite derivative interactions

Brief intro to Mellin amplitudes

Defintion:

$$\{\mathcal{M}f\}(s) = \varphi(s) = \int_0^\infty x^{s-1} f(x) dx \qquad \{\mathcal{M}^{-1}\varphi\}(x) = f(x) = \frac{1}{2\pi i}\int x^{-s}\varphi(s) ds$$

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Roughly, we just apply it to conformal cross-ratios:

$$\langle \mathcal{O}_1(P_1)\ldots\mathcal{O}_n(P_n)\rangle \propto \int d\gamma \mathcal{M}(\gamma_{ij}) \prod_{i<j}^{n} \frac{\Gamma(\gamma_{ij})}{(-2P_i\cdot P_j)^{\gamma_{ij}}}$$

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Mellin Amplitudes for Witten diagrams	$\approx$	**Scattering Amplitudes for Feynman diagrams**

[Mack'09],[Penedones'10],[Paulos'11],
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For example, $\quad \mathcal{V}_n(\nabla,\phi) \quad \to \quad \mathcal{V}_n(\partial,\phi)$

$\mathcal{M}(p_i\cdot p_j) \approx \mathcal{A}(p_i\cdot p_j)$ up to subleading terms in $p_i\cdot p_j$

Weak Locality in AdS

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Definition:

Weak Locality in AdS	=	**Mellin amplitude is an entire function**

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Criterion of weak locality:

Weak Locality in AdS	=	No single trace conformal blocks

Holographic vertex

$\mathcal{V}_4$ is weakly local

because contact diagram does not contain single trace conformal blocks

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because coefficients in the conformal block decomposition are finite

$\mathcal{V}_4$ is non-zero?

Conclusion

- Holographic quartic vertex is computed
  - HS propagators
  - Currents in AdS, improvements
  - Bulk amplitudes for 4pt exchanges and contact diagrams
  - OPE on the CFT side
- Locality of the holographic vertex

Outlook

- More explicit result. Is it zero?
- Improve techniques. Mellin amplitudes?
- Vertices for fields with spin, higher vertices
- Holographic reconstruction. General statements about bulk locality
- Holographic reconstruction. Agreement for all loops?

Complete HS massless propagator

Fronsdal equation with a source

$$\left(1 - 1/4u_1^2\partial_{u_1} \cdot \partial_{u_1}\right) \mathcal{F}_s(x_1, u_1, \nabla_1)\Pi_s(x_1, u_1, x_2, u_2) = \\ -\{\{(u_1 \cdot u_2)^2\}\}\delta(x_1, x_2) + (u_2 \cdot \nabla_2)\Lambda(x_1, u_1, x_2, u_2)$$

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$$\Pi_s = \sum_{j=0}^{[s/2]} \int d\nu g_{s,j}(\nu)(g_{AA})^j(g'_{AA})^j\Omega_{\nu,s-2j}$$

$$g_{s,0}(\nu) = \frac{1}{(h+s-2)^2+\nu^2}$$

$$g_{s,j}(\nu) = \frac{(1/2)_{j-1}}{2^{2j+3}\cdot j!}\frac{(s-2j+1)_{2j}}{(h+s-j)_j(h+s-j-3/2)_j}\times \\ \frac{(h/2+s/2-j+i\nu/2)_{j-1}(h/2+s/2-j-i\nu/2)_{j-1}}{(h/2+s/2-j+1/2+i\nu/2)_j(h/2+s/2-j+1/2-i\nu/2)_j}$$

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$\Delta = d + s - 2$

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$$(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)}$$

$$h = \frac{d}{2}$$

$$[(h+s-1)^2+\nu^2][(h+s-3)^2+\nu^2]\ldots$$

$$\Delta = d+s-1,\ d+s-3,\ \ldots$$

[Bekaert, Erdmenger, P., Sleight'14]

HS exchange in general dimensions

$$\mathcal{A}_s(y_1, y_2, y_3, y_4) = \sum_{k=0}^{[s/2]} \int d\nu \, b_{s-2k}(\nu) F_{\nu, s-2k}(u, v)$$

$$b_{s-2k}(\nu) = (g_{00s})^2 \, \frac{4^{s-2k} g_{s,k} \tau_{s,k}^2 \Gamma^2\left(\frac{3-2h-2(s-2k)}{2}\right) \Gamma^2(1-h-(s-2k))}{\pi^{3h+1} 2^{4h+6(s-2k)+3} \Gamma^2(2-2h-2(s-2k)) \Gamma^4(\Delta+1-h)}$$

$$\tau_{s,k}(\nu) = \sum_{m=0}^{k} \frac{2^{2k} \cdot k!}{m!(k-m)!} (\Delta - h - k + m + 1/2)_{k-m} \times \left(\frac{h+s-2m+1+i\nu}{2}\right)_m \left(\frac{h+s-2m+1-i\nu}{2}\right)_m$$

[Bekaert,Erdmenger,P.,Sleight'14]

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$$\Delta = \Delta(\varphi_0)$$

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[Bekaert, Erdmenger, P., Sleight'14]

Direct method

$$\mathcal{O}_{n,s} \to \mathcal{O}\partial_{\mu_1}\ldots\partial_{\mu_s}\Box^n\mathcal{O} + \ldots$$

?

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?

Problem:

$$K_\mu \mathcal{O}_1 = 0, \quad K_\mu \mathcal{O}_2 = 0, \quad \Delta(\mathcal{O}_1) = \Delta_1, \quad \Delta(\mathcal{O}_2) = \Delta_2$$

Find

$$\mathcal{O}_{n,s} \to \sum_{s_1,b_1,b_2} a_{s,n}(s_1,s_2;b_1,b_2,b_{12})\partial_{a(s_1)}\Box^{b_1}\partial^{m(b_{12})}\mathcal{O}_1\partial_{a(s_2)}\Box^{b_2}\partial_{m(b_{12})}\mathcal{O}_2$$

such that $\quad K_\mu \mathcal{O}_{n,s} = 0$

Direct method

$$\mathcal{O}_{n,s} \to \mathcal{O}\partial_{\mu_1}\dots\partial_{\mu_s}\Box^n\mathcal{O} + \dots$$

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such that $K_\mu \mathcal{O}_{n,s} = 0$

$$a_{s,n}(s_1,s_2;b_1,b_2,b_{12}) = \frac{s!}{s_1!s_2!}\frac{(\Delta_1+s_1+2b_1+b_{12})_{s_2}}{(\Delta_2+2b_2+b_{12})_{s_2}(\Delta_1+s_1+n)_{b_1-b_2}(\Delta_2+s_2+n)_{b_2-b_1}}$$
$$\times\frac{(-1)^{s_2+b_1+b_2}n!}{2^{b_1+b_2}b_1!b_2!(n-b_1-b_2)!}\frac{(\Delta_1+s+n)_{b_1}(\Delta_2+s+n-b_1)_{b_2}}{(\Delta_1+1-h)_{b_1}(\Delta_2+1-h)_{b_2}}$$
$$\times\sum_{k=0}^{b_2}\frac{b_2!}{k!(b_2-k)!}\frac{(b_1-k+1)_k(\Delta_1+b_1-h-k+1)_k}{(\Delta_2+s+n-b_1)_k(\Delta_1+s+n+b_1-k)_k}.$$

Agrees with partial results in the literature

[Mikhailov'02][Penedones'10][Fitzpatrick,Kaplan'11]

OPE coefficients

From $\langle \mathcal{O}\mathcal{O}\mathcal{O}_{n,s} \rangle$ and $\langle \mathcal{O}_{n,s}\mathcal{O}_{n,s} \rangle$

$$C^2_{\mathcal{O}\mathcal{O}\mathcal{O}_{n,s}} = \left(1 + \frac{4}{N}(-1)^n \frac{\Gamma(s)}{\Gamma(\frac{s}{2})} \frac{\left(\frac{\Delta}{2}\right)_{n+\frac{s}{2}}}{\left(\frac{\Delta+1}{2}\right)_{\frac{s}{2}} (\Delta)_{n+\frac{s}{2}}}\right)$$
$$\times \frac{\left[(-1)^s + 1\right] 2^s \left(\frac{\Delta}{2}\right)^2_n (\Delta)^2_{s+n}}{s!n! \left(s + \frac{d}{2}\right)_n (2\Delta + n - d + 1)_n (2\Delta + 2n + s - 1)_s (2\Delta + n + s - \frac{d}{2})_n}$$

Agrees with

$d = 4$ result

[Dolan,Osborn'00]

$O(N^0)$ part

[Fitzpatrick,Kaplan'11]