Higher Spin Theory
and Holography-3
November, 2015

BRST, AdS/CFT, and conformal fields

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Plan

1) **Introduction**

2) **Modified Lorentz and de Donder gauges and (global) BRST Lagrangian for AdS fields**

3) **Computation of BRST two-point functions from BRST Lagrangian of AdS fields**

4) **BRST Lagrangian of conformal fields from AdS/CFT**

General setup of gravity/gauge theory duality

$\mathbf{S_{AdS}(\Phi)}$ **type IIB superstring field action**

$$\Phi = \phi, \quad \phi^{A}, \quad \phi^{AB}, \ldots \phi^{A_1 \ldots A_s}$$

fields in AdS(5) $\times$ S(5)

Dirichlet problem

$$\frac{\delta S_{AdS}}{\delta \Phi} = 0$$

$$\Phi(x,z) \sim z^{d-\Delta} \Phi_{\mathsf{sh}}(x)$$

Use solution corresponding $\mathbf{\Phi}_{\text{sh}}$

$$S_{AdS}(\mathbf{\Phi}) \equiv S_{\text{eff}}(\mathbf{\Phi}_{\text{sh}})$$

$$= \frac{\delta^n S_{\text{eff}}}{\delta \Phi_{\text{sh}}(x_1) \ldots \delta \Phi_{\text{sh}}(x_n)}$$

$$\langle \Phi_{\text{cur}}(x_1) \ldots \Phi_{\text{cur}}(x_n) \rangle$$

correlation functions from AdS

Long-term motivation

Computation of $S_{\text{eff}}(\Phi_{\text{sh}})$

for superstring theory

Supestring - theory of gauge fields

massive (higher) + massless(low)

Method for analysis of gauge theories ?

among others

Light-cone gauge approach

BRST approach

Light-cone gauge approach to closed supestring

$$\mathcal{L} = \Phi(\Box - M^2)\Phi + \Phi^3$$

(Green Schwarz)

1) **simple Lagrangian**

2) **convenient for tree level**

3) **hard for loop computation**

BRST approach to closed string

$$\mathcal{L} = \Phi(\Box - M^2)\Phi + \Phi^3 + \Phi^4 + \ldots$$

1) **more complicated Lagrangian**

2) **Lorentz covariance** $\rightarrow$ **computation of tree and loop diagrams is simpler**

BRST

Find helpful gauge conditions

de Donder like gauge ?

Goal

Find convenient gauges and compute

$$\mathrm{S}_{\mathrm{eff}}(\Phi_{\mathrm{sh}})$$

for **arbitrary spin** fields

scalar

$$S = \int d^d x dz \mathcal{L}$$

$$\mathcal{L} = \frac{1}{2}\sqrt{g}(g^{\mu\nu}\partial_\mu \Phi \partial_\nu \Phi + m^2 \Phi^2)$$

$$\Phi = z^{\frac{d-1}{2}} \phi$$

scalar

$$\mathcal{L} = \frac{1}{2}|\partial^{\mathbf{a}}\phi|^{\mathbf{2}} + \frac{1}{2}|\mathcal{T}_\nu\phi|^{\mathbf{2}}$$

$$\mathcal{T}_\nu \equiv \partial_z + \frac{\nu}{z}$$

$$\nu = \sqrt{m^2 + \frac{d^2}{4}}$$

scalar

Solution to Dirichlet problem

$$\left(\Box + \partial_{\mathbf{z}}^{\mathbf{2}} - \frac{\nu^2 - \frac{1}{4}}{z^2}\right)\phi = \mathbf{0}$$

$$\phi(\mathbf{x}, \mathbf{z}) \quad \xrightarrow{z \to 0} \quad \mathbf{z}^{-\nu + \frac{1}{2}} \; \phi_{\mathsf{sh}}(\mathbf{x})$$

$$\phi(\mathbf{x}, \mathbf{z}) \quad \xrightarrow{z \to \infty} \quad 0$$

scalar

Solution to Dirichlet problem

$$\phi(\mathbf{x}, \mathbf{z}) = \int d^d y \, \mathrm{G}_\nu(\mathbf{x} - \mathbf{y}, \mathbf{z}) \phi_{\text{sh}}(\mathbf{y})$$

$$\mathrm{G}_\nu(\mathbf{x}, \mathbf{z}) = \frac{z^{\nu + \frac{1}{2}}}{(z^2 + |x|^2)^{\nu + \frac{d}{2}}}$$

scalar

Effective action

$$\mathbf{S}_{\mathrm{eff}} = \mathbf{c}_0 \int d^d x_1 d^d x_2 \, \boldsymbol{\Gamma}_{\mathbf{12}}$$

$$\mathbf{c}_0 = \nu, \qquad \nu = \sqrt{m^2 + \frac{d^2}{4}}$$

$$\boldsymbol{\Gamma}_{\mathbf{12}} = \frac{\phi_{\mathrm{sh}}(x_1)\phi_{\mathrm{sh}}(x_2)}{|x_{12}|^{2\nu+d}}$$

$$|x_{12}|^2 = (x_1 - x_2)^a (x_1 - x_2)^a$$

spin-1

$$\mathcal{L} = -\frac{1}{4} F^{AB} F^{AB}$$

$$F^{AB} = D^A \phi^B - D^B \phi^A$$

spin-1

$$\mathcal{L} = \frac{1}{2}\phi^A \Box_{AdS} \phi^A + \frac{1}{2} L^2$$

$$L = D^A \phi^A$$

$$\Box_{AdS} \phi^A = 0$$

$$L = 0$$

Coupled EOM

$$ds^2 = \frac{1}{z^2}(dx^a dx^a + dz dz)$$

$\mathbf{x^a}$ boundary flat coordinates

$\mathbf{z}$ radial coordinate

spin-1

bulk $\mathbf{so(d,1)} \longrightarrow$ boundary $\mathbf{so(d-1,1)}$

$$\phi^{\mathbf{A}} \quad = \quad \phi^{\mathbf{a}} \oplus \phi^{\mathbf{z}}$$

$$\phi \equiv \phi^{z}$$

spin-1

standard Lorentz gauge

$$D^A \phi^A = 0$$

leads to coupled equations

$$\left(\Box + \partial_z^2 - \frac{\nu_1^2 - \frac{1}{4}}{z^2} \right) \phi^{\mathbf{a}} + \partial^a \phi = 0$$

$$\left(\Box + \partial_z^2 - \frac{\nu_0^2 - \frac{1}{4}}{z^2} \right) \phi + \partial^a \phi^{\mathbf{a}} = 0$$

spin-1

Modified Lorentz gauge

$$\mathbf{L} = 0$$

$$\mathbf{L} \equiv \mathbf{D^A} \phi^{\mathbf{A}} + \frac{2}{R}\phi$$

RRM, 1999

Polchinski and Strassler 2001

gives **decoupled equations**

spin-1

Decoupled equations

$$\left(\Box + \partial_z^2 - \frac{\nu_1^2 - \frac{1}{4}}{z^2}\right) \phi^{\mathbf{a}} = 0$$

$$\left(\Box + \partial_z^2 - \frac{\nu_0^2 - \frac{1}{4}}{z^2}\right) \phi = 0$$

$$\nu_1 = \frac{d-2}{2}, \qquad \nu_0 = \frac{d-4}{2}$$

spin-1

Solution to Dirichlet problem

$$\phi^a(x,z) = \int d^d y \, \mathbf{G}_{\nu_1}(\mathbf{x}-\mathbf{y},\mathbf{z}) \;\; \phi^a_{\mathsf{sh}}(y)$$

$$\phi(x,z) = \int d^d y \, \mathbf{G}_{\nu_0}(\mathbf{x}-\mathbf{y},\mathbf{z}) \;\; \phi_{\mathsf{sh}}(y)$$

$$\mathbf{G}_\nu(\mathbf{x},\mathbf{z}) \equiv \frac{z^{\nu+1/2}}{(z^2+|x|^2)^{\nu+\frac{d}{2}}}$$

Modified Lorentz gauge for bulk AdS fields

$$\mathcal{L} = \frac{1}{2}\phi^a \Box_{\nu_1} \phi^a + \frac{1}{2}\phi \Box_{\nu_0} \phi + \frac{1}{2}\mathbf{L}^2$$

$$\mathbf{L} \equiv \mathbf{D}^{\mathbf{A}} \phi^{\mathbf{A}} + \mathbf{2}\phi$$

$$\Box_\nu \equiv \Box + \partial_z^2 - \frac{\nu^2 - \frac{1}{4}}{z^2}$$

Faddeev-Popov procedure

$$\mathcal{L}_{tot} = \mathcal{L} + \mathcal{L}_{qu}$$

$$\mathcal{L}_{qu} = -b\mathbf{L} + \bar{c}\,\Box_{\nu_1} c + \frac{1}{2}\xi b^2$$

$\xi = 0$ **Landau gauge**

$\xi = 1$ **Feynman gauge**

BRST

$$\mathsf{s}\phi^a = \partial^a c \qquad \mathsf{s}\phi = \mathcal{T}_{\nu_1} c$$

$$\mathsf{s}c = 0$$

$$\mathsf{s}\bar{c} = b$$

$$\mathsf{s}b = 0$$

anti-BRST

$$\bar{\mathsf{s}}\phi^a = \partial^a \bar{c} \qquad \bar{\mathsf{s}}\phi = \mathcal{T}_{\nu_1} \bar{c}$$

$$\bar{\mathsf{s}}c = -b$$

$$\bar{\mathsf{s}}\bar{c} = 0$$

$$\bar{\mathsf{s}}b = 0$$

$\mathcal{T}_\nu \equiv \partial_{\mathbf{z}} + \frac{\nu}{z}$

$$s^2 = 0 \qquad \bar{s}^2 = 0 \qquad s\bar{s} + \bar{s}s = 0$$

OFF-SHELL

Feynman gauge $\xi = 1$

Integrate out field $\mathbf{b}$

$$\mathcal{L} = \frac{1}{2}\phi^a \Box_{\nu_1} \phi^a + \frac{1}{2}\phi \Box_{\nu_0} \phi + \bar{c}\, \Box_{\nu_1} c$$

BRST

$$\mathsf{s}\phi^a = \partial^a c \qquad \mathsf{s}\phi = \mathcal{T}_{\nu_1} c$$

$$\mathsf{s}c = 0$$

$$\mathsf{s}\bar{c} = \mathbf{L}$$

anti-BRST

$$\bar{\mathsf{s}}\phi^a = \partial^a \bar{c} \qquad \bar{\mathsf{s}}\phi = \mathcal{T}_{\nu_1} \bar{c}$$

$$\bar{\mathsf{s}}c = -\mathbf{L}$$

$$\bar{\mathsf{s}}\bar{c} = 0$$

$$s^2 = 0 \qquad \bar{s}^2 = 0 \qquad s\bar{s} + \bar{s}s = 0$$

ON-SHELL for c and $\bar{c}$

spin-1

Decoupled equations

$$\left(\Box + \partial_z^2 - \frac{\nu_1^2 - \frac{1}{4}}{z^2}\right)\phi^{\mathbf{a}} = 0$$

$$\left(\Box + \partial_z^2 - \frac{\nu_0^2 - \frac{1}{4}}{z^2}\right)\phi = 0$$

$$\nu_1 = \tfrac{d-2}{2}, \qquad \nu_0 = \tfrac{d-4}{2}$$

spin-1

$$\left(\Box + \partial_z^2 - \frac{\nu_1^2 - \frac{1}{4}}{z^2}\right)\mathbf{c} = 0$$

$$\left(\Box + \partial_z^2 - \frac{\nu_1^2 - \frac{1}{4}}{z^2}\right)\bar{\mathbf{c}} = 0$$

$$\nu_1 = \frac{d-2}{2}$$

spin-1

Solution to Dirichlet problem

$$\phi^a(x,z) = \int d^d y \mathbf{G}_{\nu_1}(\mathbf{x}-\mathbf{y},\mathbf{z}) \ \phi^a(y)$$

$$\phi(x,z) = \int d^d y \mathbf{G}_{\nu_0}(\mathbf{x}-\mathbf{y},\mathbf{z}) \ \phi(y)$$

$$c(x,z) = \int d^d y \mathbf{G}_{\nu_1}(\mathbf{x}-\mathbf{y},\mathbf{z}) \ c(y)$$

$$\bar{c}(x,z) = \int d^d y \mathbf{G}_{\nu_1}(\mathbf{x}-\mathbf{y},\mathbf{z}) \ \bar{c}(y)$$

$$\mathbf{G}_\nu(\mathbf{x},\mathbf{z}) \equiv \frac{z^{\nu+1/2}}{(z^2+|x|^2)^{\nu+\frac{d}{2}}}$$

spin-1: Effective action

$$S_{\text{eff}} = \int d^d x_1 d^d x_2 \mathbf{\Gamma_{12}}$$

$$\Gamma_{12} = \frac{\phi^a(x_1)\phi^a(x_2)}{|x_{12}|^{2(d-1)}}$$

$$+ \frac{\phi(x_1)\phi(x_2)}{|x_{12}|^{2(d-2)}}$$

$$+ \frac{\bar{c}(x_1)c(x_2)}{|x_{12}|^{2(d-1)}}$$

BRST

$$\mathsf{s}\phi^a = \partial^a c \qquad \mathsf{s}\phi = -\Box c$$

$$\mathsf{s}c = 0$$

$$\mathsf{s}\bar{c} = \partial^a \phi^a + \phi$$

anti-BRST

$$\bar{\mathsf{s}}\phi^a = \partial^a \bar{c} \qquad \bar{\mathsf{s}}\phi = -\Box \bar{c}$$

$$\bar{\mathsf{s}}c = -\partial^a \phi^a - \phi$$

$$\bar{\mathsf{s}}\bar{c} = 0$$

$$\mathsf{s}^2 = 0 \qquad \bar{\mathsf{s}}^2 = 0 \qquad \mathsf{s}\bar{\mathsf{s}} + \bar{\mathsf{s}}\mathsf{s} = 0$$

ON-SHELL for c and $\bar{c}$

regularization

$d - [d] = -2\varepsilon\,,$ $\qquad [d]$ – **integer**

$$\frac{1}{|x|^{2\nu+d}} \overset{\varepsilon \to 0}{\approx} \frac{1}{\varepsilon} \Box^{\nu_{\mathrm{int}}} \delta^{(d)}(x)$$

$$S_{\mathrm{eff}} \overset{\varepsilon \approx 0}{\simeq} \frac{1}{\varepsilon} \int d^d x \, \mathcal{L}_{conf} \, ,$$

$$\mathcal{L}_{conf} = \frac{1}{2} \phi^a \Box^{k+1} \phi^a + \frac{1}{2} \phi \Box^k \phi + \bar{c} \, \Box^{k+1} c$$

$$k \equiv \frac{d-4}{2}$$

$$b = \phi + \partial^a \phi^a$$

$$\mathcal{L}_{tot} = \mathcal{L} + \mathcal{L}_{qu}$$

$$\mathcal{L} = -\frac{1}{4} F^{ab} \Box^k F^{ab} , \qquad F^{ab} \equiv \partial^a \phi^b - \partial^b \phi^a$$

$$\mathcal{L}_{qu} = -b \Box^k \partial^a \phi^a + \frac{1}{2} b \Box^k b + \overline{c} \Box^{k+1} c$$

$$\mathsf{s}\phi^a = \partial^a c$$

$$\mathsf{s}b = 0$$

$$\mathsf{s}c = 0$$

$$\mathsf{s}\bar{c} = b$$

$$\bar{\mathsf{s}}\phi^a = \partial^a \bar{c}$$

$$\bar{\mathsf{s}}b = 0$$

$$\bar{\mathsf{s}}c = -b$$

$$\bar{\mathsf{s}}\,\bar{c} = 0$$

Arbitrary spin-s AdS field

$$\Phi^{A_1 \dots A_s}$$

Impose modified de Donder gauge

$$\mathbf{D}^{\mathbf{A}_1\mathbf{A}_2\ldots\mathbf{A}_s} = 0$$

$$\mathbf{D}^{\mathbf{A}_1\mathbf{A}_2\ldots\mathbf{A}_s}$$

$$\equiv \mathbf{D}^{\mathbf{A}}\boldsymbol{\Phi}^{\mathbf{A}\mathbf{A}_2\ldots\mathbf{A}_s} - \frac{1}{2}\mathbf{D}^{\mathbf{A}_2}\boldsymbol{\Phi}^{\mathbf{A}\mathbf{A}\mathbf{A}_3\ldots\mathbf{A}_s}$$

$$+ 2\boldsymbol{\Phi}^{\mathbf{z}\mathbf{A}_2\ldots\mathbf{A}_s} - \eta^{\mathbf{z}\mathbf{A}_2}\boldsymbol{\Phi}^{\mathbf{A}\mathbf{A}\mathbf{A}_3\ldots\mathbf{A}_s}$$

leads to coupled EOM

Decompose

$$\mathbf{so(d,1)} \longrightarrow \mathbf{so(d-1,1)}$$

$$\Phi^{A_1\ldots A_s} = \Phi^{a_1\ldots a_s}$$

$$\Phi^{a_1\ldots a_{s-1}}$$

$$\cdots\cdots$$

$$\Phi^{a_1 a_2}$$

$$\Phi^{a_1}$$

$$\Phi$$

$$\phi^{a_1\ldots a_s} = \Phi^{a_1\ldots a_s} + \ldots$$

$$\phi^{a_1\ldots a_{s-1}} = \Phi^{a_1\ldots a_{s-1}} + \ldots$$

$$\ldots\ldots$$

$$\phi^{a_1 a_2} = \Phi^{a_1 a_2} + \ldots$$

$$\phi^{a} = \Phi^{a_1}$$

$$\phi = \Phi$$

Decoupled equations

$$(\Box + \partial_z^2 - \frac{{\nu_{s'}}^2 - \frac{1}{4}}{z^2})\phi^{a_1\ldots a_{s'}} = 0$$

$$\nu_{s'} = s' + \frac{d-4}{2}$$

$$\phi^{a_1\ldots a_{s'}}(x, z) = \int d^d y G_{\nu_{s'}}(x-y, z)\phi_{sh}^{a_1\ldots a_{s'}}(y)$$

$$
\begin{aligned}
L \;=\; & \phi^{a_1\dots a_s}\Box_{\nu_s}\phi^{a_1\dots a_s} \\
+ \; & \phi^{a_1\dots a_{s-1}}\Box_{\nu_{s-1}}\phi^{a_1\dots a_{s-1}} \\
+ \; & \dots\dots\dots\dots \\
+ \; & \phi\Box_{\nu_0}\phi \\
\\
+ \; & D^{a_1\dots a_{s-1}}D^{a_1\dots a_{s-1}} \\
+ \; & D^{a_1\dots a_{s-2}}D^{a_1\dots a_{s-2}} \\
+ \; & \dots\dots\dots\dots \\
+ \; & DD
\end{aligned}
$$

$$
D^{a_1\dots a_k} = \partial^a\phi^{aa_1\dots a_k} + g_k\mathcal{T}_{..}\phi^{a_1\dots a_k} + f_k\mathcal{T}_{..}\phi^{aaa_1\dots a_k}
$$

$$
\mathcal{T}_\nu = \partial_z + \frac{\nu}{z}
$$

0-

Apply Faddeev-Popov procedure

Faddeev-Popov fields

$$c^{a_1 \cdots a_{s-1}}, \qquad c^{a_1 \cdots a_{s-2}}, \ldots, c$$

$$\bar{c}^{a_1 \cdots a_{s-1}}, \qquad \bar{c}^{a_1 \cdots a_{s-2}}, \ldots, \bar{c}$$

Nakanishi-Lautrup fields

$$b^{a_1 \cdots a_{s-1}}, \qquad b^{a_1 \cdots a_{s-2}}, \ldots, b$$

$$L_{tot} = L + \mathbf{L}_{\mathbf{qu}}$$

$$\begin{aligned}
\mathbf{L}_{\mathbf{qu}} &= \bar{c}^{a_1 \dots a_{s-1}} \Box_{\nu_s} c^{a_1 \dots a_{s-1}} \\
&+ \bar{c}^{a_1 \dots a_{s-2}} \Box_{\nu_{s-1}} c^{a_1 \dots a_{s-2}} \\
&+ \dots\dots\dots\dots \\
&+ \bar{c}\,\Box_{\nu_1} c
\end{aligned}$$

$$\begin{aligned}
&+ b^{a_1 \dots a_{s-1}} D^{a_1 \dots a_{s-1}} \\
&+ b^{a_1 \dots a_{s-2}} D^{a_1 \dots a_{s-2}} \\
&+ \dots\dots\dots\dots \\
&+ bD
\end{aligned}$$

$$\begin{aligned}
&+ \frac{1}{2} \xi b^{a_1 \dots a_{s-1}} b^{a_1 \dots a_{s-1}} \\
&+ \frac{1}{2} \xi b^{a_1 \dots a_{s-2}} b^{a_1 \dots a_{s-2}} \\
&+ \dots\dots\dots\dots \\
&+ \frac{1}{2} \xi bb
\end{aligned}$$

Use Feynman gauge $\xi = 1$

Integrate out Nakanishi-Lautrup fields

$$
\begin{aligned}
L_{tot} &= \phi^{a_1 \dots a_s} \Box_{\nu_s} \phi^{a_1 \dots a_s} \\
&+ \phi^{a_1 \dots a_{s-1}} \Box_{\nu_{s-1}} \phi^{a_1 \dots a_{s-1}} \\
&+ \dots\dots\dots\dots \\
&+ \phi \Box_{\nu_0} \phi \\
&+ \bar{\mathbf{c}}^{\mathbf{a_1 \dots a_{s-1}}} \Box_{\nu_s} \mathbf{c}^{\mathbf{a_1 \dots a_{s-1}}} \\
&+ \bar{\mathbf{c}}^{\mathbf{a_1 \dots a_{s-2}}} \Box_{\nu_{s-1}} \mathbf{c}^{\mathbf{a_1 \dots a_{s-2}}} \\
&+ \dots\dots\dots\dots \\
&+ \bar{\mathbf{c}} \, \Box_{\nu_1} \mathbf{c}
\end{aligned}
$$

$$S_{\text{eff}} = \int dx_1^d dx_2^d \mathbf{\Gamma_{12}}$$

$$\mathbf{\Gamma_{12}} = \frac{\phi^{\mathbf{a_1 \ldots a_s}}(\mathbf{x_1})\phi^{\mathbf{a_1 \ldots a_s}}(\mathbf{x_2})}{|x_{12}|^{2(s+d-2)}}$$

$$+ \quad \frac{\phi^{\mathbf{a_1 \ldots a_{s-1}}}(\mathbf{x_1})\phi^{\mathbf{a_1 \ldots a_{s-1}}}(\mathbf{x_2})}{|x_{12}|^{2(s+d-3)}}$$

$$+ \quad \ldots\ldots\ldots\ldots$$

$$+ \quad \frac{\phi(\mathbf{x_1})\phi(\mathbf{x_2})}{|x_{12}|^{2(d-2)}}$$

$$+ \frac{\mathbf{\bar{c}^{a_1 \ldots a_{s-1}}}(\mathbf{x_1})\mathbf{c^{a_1 \ldots a_{s-1}}}(\mathbf{x_2})}{|x_{12}|^{2(s+d-2)}}$$

$$+ \quad \frac{\mathbf{\bar{c}^{a_1 \ldots a_{s-2}}}(\mathbf{x_1})\mathbf{c^{a_1 \ldots a_{s-2}}}(\mathbf{x_2})}{|x_{12}|^{2(s+d-3)}}$$

$$+ \quad \ldots\ldots\ldots\ldots$$

$$+ \quad \frac{\mathbf{\bar{c}}(\mathbf{x_1})\mathbf{c}(\mathbf{x_2})}{|x_{12}|^{2(d-2)}}$$

$$S_{conf} = \int d^d x \mathcal{L}_{conf}$$

$$\begin{aligned}
\mathcal{L}_{\mathbf{conf}} = {} & \phi^{\mathbf{a_1 \ldots a_s}} \Box^{\nu_{\mathbf{s}}} \phi^{\mathbf{a_1 \ldots a_s}} \\
& + \phi^{\mathbf{a_1 \ldots a_{s-1}}} \Box^{\nu_{\mathbf{s-1}}} \phi^{\mathbf{a_1 \ldots a_{s-1}}} \\
& + \quad \ldots\ldots\ldots\ldots \\
& + \phi \Box^{\nu_{\mathbf{0}}} \phi \\
& + \bar{c}^{a_1 \ldots a_{s-1}} \Box^{\nu_s} c^{a_1 \ldots a_{s-1}} \\
& + \bar{c}^{a_1 \ldots a_{s-2}} \Box^{\nu_{s-1}} c^{a_1 \ldots a_{s-2}} \\
& + \quad \ldots\ldots\ldots\ldots \\
& + \bar{c} \Box^{\nu_1} c
\end{aligned}$$