Instanton solutions to 3D HS gravity coupled to matter

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Summary

- The 3D Prokushkin-Vasiliev equations
  - Oscillator algebras .
  - Full equations (bosonic) and maximally-symmetric vacua.
- Solving the equations
  - Gauge function method.
  - A 3D Lorentz-invariant solution. Comparison with 4D.
- Attempts at a spacetime and holographic interpretation
  - Interpretation as a CDL instanton and comparison with gravity+scalar
  - Holographic interpretation (at varying λ) .
- Conclusions and Outlook

Kinematics

- Master-fields living on *correspondence space*, locally $\mathcal{X} \times \mathcal{Z} \times \mathcal{Y} \times \mathcal{A}$:

$$\widehat{W} = dx^\mu \widehat{W}_\mu(y,z;\psi_{1,2},k,\rho|x) \ ,$$
→ gauge fields of all spins + auxiliary

$$\widehat{B} = \widehat{B}(y,z;\psi_{1,2},k,\rho|x) \ ,$$
→ matter fields and their derivatives

$$\widehat{S} = dz^\alpha \widehat{S}_\alpha(y,z;\psi_{1,2},k,\rho|x) \ .$$
→ z-space connection, no extra local dof

- Commuting oscillators (y_α, z_α) → $\mathfrak{sp}(2)$ doublets

$$[y_\alpha, y_\beta]_\star = 2i\epsilon_{\alpha\beta} \ , \qquad [z_\alpha, z_\beta]_\star = -2i\epsilon_{\alpha\beta} \ , \qquad [y_\alpha, z_\beta]_\star = 0$$

- Star-product:

$$(\widehat{f}\star\widehat{g})(z,y) = \frac{1}{(2\pi)^2}\int d^2u d^2v \ \exp(iu_\alpha v^\alpha)\, \widehat{f}(z+u,y+u)\,\widehat{g}(z-v,y+v)$$

- Inner kleinian operator $\widehat{\kappa}$:

$$\widehat{\kappa} = e^{iy^\alpha z_\alpha} \ , \qquad \widehat{\kappa}\star\widehat{f}(z,y) = \widehat{f}(-z,-y)\star\widehat{\kappa} \ , \qquad \widehat{\kappa}\star\widehat{\kappa} = 1$$

Kinematics

- Two pairs of Clifford algebras generated by $\psi_{1,2}$ and (k,ρ):

$$\{\psi_i, \psi_j\} = 2\delta_{ij}, \qquad i = 1,2 \ ,$$

$$k^2 = \rho^2 = 1 \ , \quad \{\rho, k\} = 0 \ , \quad \{k, y_\alpha\} = \{k, z_\alpha\} = 0 \ , \quad [\rho, y_\alpha] = [\rho, z_\alpha] = 0$$

- z-oscillators → auxiliary, NC coordinates. Equations fix the evolution along these twistor directions in such a way that it gives rise to consistent interactions to all orders among physical fields. The latter are contained in the (z-independent) initial conditions for the z-evolution,

$$W = \widehat{W}_{|z=0} \ , \quad C = \widehat{B}_{|z=0} \ .$$

- y-oscillators + ψ_1 → realize the global symmetry algebra = symmetry algebra of the most symmetric vacuum AdS_3 = 3D HS algebra, corresponding to a specific infinite-dimensional extension of the AdS_3-isometry algebra $\mathfrak{so}(2,2)$,

$$\mathfrak{so}(2,2) \simeq \mathfrak{sp}(2) \oplus \mathfrak{sp}(2) \ , \qquad M_{\alpha\beta} = \frac{1}{4i}\{y_\alpha, y_\beta\}_\star \equiv \frac{1}{2i} y_\alpha y_\beta \ , \quad P_{\alpha\beta} = M_{\alpha\beta}\psi_1$$

- Restrict to bosonic subalgebra. k → central element, can project on k = ± 1 .

3D HS algebras

- There is a one-parameter family of oscillators realizing $\mathfrak{so}(2,2)$ → *deformed* oscillators $$[\tilde{y}_\alpha,\tilde{y}_\beta]_\star = 2i\epsilon_{\alpha\beta}(1+\nu k)\ ,\qquad M_{\alpha\beta} = -\frac{i}{4}\{\tilde{y}_\alpha,\tilde{y}_\beta\}_\star$$ *(Wigner, Vasiliev)*
- Correspondigly, the bosonic HS algebra splits into $$\mathfrak{hs}(2;\nu)_\pm \oplus \mathfrak{hs}(2;\nu)_\pm\ ,\qquad \mathfrak{hs}(2;\nu)_\pm \to \frac{\mathcal{U}[\mathfrak{sp}(2)]}{\mathcal{I}[C_2+\frac{3\pm 2\nu-\nu^2}{4}]}$$ *(Vasiliev, Feigin)*
- ν = vacuum value of B, related to the mass of the scalar field and to the 't Hooft coupling λ of the conjectured boundary dual theory, *(Gaberdiel-Gopakumar)* $$\mathfrak{hs}(2;\nu)_\pm = \mathfrak{hs}[\lambda]\ ,\qquad \lambda = \frac{1\mp\nu}{2}$$
- The ψ_2-dependence of the master-fields activates an additional sector of the theory. It implements a twist operation $\psi_1 \to -\psi_1$ changing the sign of the translation generator.

$$C(\tilde{y};\psi_{1,2}) = C^{tw}(\tilde{y};\psi_1) + C^{phys}(\tilde{y};\psi_1)\psi_2\ ,$$
$$\omega(\tilde{y};\psi_{1,2}) = \omega^{phys}(\tilde{y};\psi_1) + \omega^{tw}(\tilde{y};\psi_1)\psi_2$$

constant + Killing tensors (C^{tw})

physical scalar, $(\Box - m^2)\phi = 0$ (C^{phys})

physical HS fields (ω^{phys})

twisted one-forms (ω^{tw})

3D Vasiliev equations

- Full equations:

$$
\begin{aligned}
d\widehat{W} + \widehat{W} \star \widehat{W} &= 0 \\
d\widehat{B} + [\widehat{W}, \widehat{B}]_\star &= 0 \\
d\widehat{S}_\alpha + [\widehat{W}, \widehat{S}_\alpha]_\star &= 0 \\
[\widehat{S}_\alpha, \widehat{B}]_\star &= 0 \\
[\widehat{S}_\alpha, \widehat{S}_\beta]_\star &= -2i\epsilon_{\alpha\beta}(1 + \widehat{B} \star k\widehat{\kappa})
\end{aligned}
$$

(Vasiliev, Prokushkin-Vasiliev)

- AdS_3 vacuum: *(Vasiliev)*

$$
\begin{aligned}
\widehat{B}_0 &= B_0 = \nu \, , \\
\widehat{S}_{0\alpha} &= \rho z_\alpha \left[1 + \nu \int_{-1}^{1} ds(1-s) \left(F^-(\nu \ln|s|) e^{\frac{i}{2}(1+s)u} + F^+(\nu \ln|s|) e^{\frac{i}{2}(1-s)u} k\right)\right] =: \tilde{z}_\alpha \, , \\
\widehat{W}_0 &= W_0 = e_0 + \omega_0 \, , \qquad \omega_0 = \frac{1}{4i}(\omega_0^{\alpha\beta} + e_0^{\alpha\beta}\psi_1)\{\tilde{y}_\alpha, \tilde{y}_\beta\}_\star \, ,
\end{aligned}
$$

$$
F^\pm(x) \equiv \tfrac{1}{8}\left({}_1F_1\left[1/2, 2, x\right] \pm {}_1F_1\left[1/2, 2, -x\right]\right) \, , \qquad u := y^\alpha z_\alpha
$$

$$
[\tilde{z}_\alpha, \tilde{z}_\beta]_\star = -2i\epsilon_{\alpha\beta}(1 + \nu k \kappa) \, , \qquad [\tilde{y}_\alpha, \tilde{y}_\beta]_\star = 2i\epsilon_{\alpha\beta}(1 + \nu k) \, , \qquad [\tilde{y}_\alpha, \tilde{z}_\beta]_\star = 0
$$

- We shall look for solutions that deviate from AdS bckgrd, separating out the latter as

$$
\widehat{B} = \nu + \widehat{\Phi}
$$

Exact solutions: gauge function method

- $\mathcal{X}$ x $\mathcal{Y}$ x $\mathcal{Z}$-space eqns:

- $\mathcal{Y}$ x $\mathcal{Z}$-space eqns:

$$\begin{aligned} \widehat{W} &= \widehat{L}^{-1}\star d\widehat{L}\ , \\ \widehat{\Phi} &= \widehat{L}^{-1}\star\widehat{\Phi}'\star\widehat{L}, \qquad d\widehat{\Phi}'=0\ , \\ \widehat{S}_\alpha &= \widehat{L}^{-1}\star\widehat{S}'_\alpha\star\widehat{L}, \qquad d\widehat{S}'_\alpha=0\ , \\ [\widehat{S}'_\alpha,\widehat{\Phi}']_\star &= 0\ , \\ [\widehat{S}'_\alpha,\widehat{S}'_\beta]_\star &= -2i\epsilon_{\alpha\beta}(1+\nu k\widehat{\kappa}+\widehat{\Phi}'\star k\widehat{\kappa}) \end{aligned}$$

- Solve locally all equations with at least one spacetime component via some gauge function, chosen as an AdS_3 coset element

$$L(\tilde{y};\psi_1|x) = \exp_\star\left(-\frac{i}{8}\frac{\operatorname{arctanh}\sqrt{x^2}}{\sqrt{x^2}}x^{\alpha\beta}\{\tilde{y}_\alpha,\tilde{y}_\beta\}_\star\psi_1\right)$$

- *AdS_3*:

$$\begin{aligned} \widehat{B}'_0 &= \nu\ , \\ \widehat{S}'_{0\alpha} &= \rho z_\alpha\left[1+\nu\int_{-1}^{1}ds(1-s)\left(F^-(\nu\ln|s|)e^{\frac{i}{2}(1+s)u}+F^+(\nu\ln|s|)e^{\frac{i}{2}(1-s)u}k\right)\right] =: \tilde{z}_\alpha\ . \end{aligned}$$

- Solve the twistor-space equations, then "dress" all fields with *x*-dependence by performing star-products with the gauge function.

Gauge fields sector

- We want to interpret the coefficients of the master fields as space-time tensors → it should be possible to extract Lorentz tensors (and a Lorentz connection) out of the gauge fields generating function.

- But, in general, the expansion coefficients in *W* are not Lorentz tensors!

- The proper Lorentz generator, at the full level, is

$$\boxed{\widehat{M}_{\alpha\beta} \;=\; -\frac{i}{2}(y_\alpha y_\beta - z_\alpha z_\beta) - \frac{i}{4}\{\widehat{S}_\alpha, \widehat{S}_\beta\}_\star \;=:\; \widehat{M}^{(0)}_{\alpha\beta} + \widehat{M}^{(S)}_{\alpha\beta}}$$

under which the physical-field generating functions transform as

$$\begin{aligned} \delta_{\widehat{\epsilon}_L}\Phi &= -[\widehat{\epsilon}_0, \Phi]_\star \ , \\ \delta_{\widehat{\epsilon}_L}W &= [W, \widehat{\epsilon}_0]_\star + \frac{1}{2}d\Lambda^{\alpha\beta}\widehat{M}^L_{\alpha\beta\,|z=0} \end{aligned}$$

- *W* decomposes into a proper Lorentz connection and Lorentz tensors as

$$W \;=\; \frac{1}{2}\omega^{\alpha\beta}\left(M_{\alpha\beta} - \frac{i}{4}\{\widehat{S}_\alpha, \widehat{S}_\beta\}_{\star\,|z=0}\right) + e\psi_1 + E^{HS}\psi_1 + \Omega^{HS}$$

(Vasiliev)

A Lorentz-invariant solution

- All the ingredients in the twistor-space ν-vacuum solutions commute with ψ_2 → we can generalize it as

$$\widehat{B}' = \nu + \mu\psi_2 \; ,$$
$$\widehat{S}'_\alpha = \rho z_\alpha \left[1 + (\nu + \mu\psi_2)\int_{-1}^{1} ds(1-s)\left(F^-((\nu+\mu\psi_2)\ln|s|)e^{\frac{i}{2}(1+s)u} + F^+((\nu+\mu\psi_2)\ln|s|)e^{\frac{i}{2}(1-s)u}k\right)\right]$$

- However, the presence of ψ_2 implies that the twistor-space solution no longer commutes with the gauge function, breaking the $\mathfrak{so}(2,2)$ isometry down to $\mathfrak{so}(2,1)$.

- For $\nu = 0$ and in stereographic coordinates the zero-form master-field reads

$$\boxed{\widehat{\Phi} = \Phi = \mu L^{-1}\psi_2 \star L = \mu L^{-1} \star L^{-1}\psi_2 = \mu\psi_2\sqrt{1-x^2}\exp\left(-\frac{i}{2}x^{\alpha\beta}y_\alpha y_\beta \psi_1\right)}$$

[AdS_3 metric in stereographic coordinates: $ds^2 = \dfrac{4dx^2}{(1-x^2)^2}$]

- From Φ at $y = 0$ we can read off the AdS_3-massless scalar ($m^2 = -3/4$)

$$\boxed{\phi(x) = \mu\sqrt{1-x^2}}$$

A Lorentz-invariant solution

- The gauge fields are to be retrieved from the splitting

$$W = -\frac{i}{4}\omega_0^{\alpha\beta} y_\alpha y_\beta - \frac{i}{4} e_0^{\alpha\beta} y_\alpha y_\beta \psi_1 = -\frac{i}{4}\omega^{\alpha\beta}\left(y_\alpha y_\beta + \frac{1}{2}\{\widehat{S}_\alpha, \widehat{S}_\beta\}_{\star\,|z=0}\right) - \frac{i}{4} e^{\alpha\beta} y_\alpha y_\beta \psi_1 + E^{HS}\psi_1 + \Omega^{HS}$$

- Since S_α contains ψ_2 it gives rise to both a physical sector, where only the spin-2 field is activated, and a twisted sector, to which all spins contribute:

$$\frac{1}{2}\{\widehat{S}_\alpha, \widehat{S}_\beta\}_{\star\,|z=0} = A\, y_\alpha y_\beta + B\, a_\alpha{}^\gamma y_\gamma a_\beta{}^\delta y_\delta + C\, a_{(\alpha}{}^\gamma y_{\beta)} y_\gamma \psi_1 - k\psi_2 \left(\int_{-1}^{1} ds\, \left[F_1\, a_\alpha{}^\gamma y_\gamma\, a_\beta{}^\delta y_\delta + F_2\, \psi_1 a_{\alpha\beta}\right] e^{\frac{i}{1+s^2 a^2}\frac{(1+s)^2}{2} a^{\alpha\beta} y_\alpha y_\beta \psi_1} + \int_{-1}^{1} ds \int_{-1}^{1} d\tilde{s}\, \left[F_3\, y_\alpha y_\beta + F_4\, a_\alpha{}^\gamma y_\gamma\, a_\beta{}^\delta y_\delta\right] e^{\frac{i}{1+s^2\tilde{s}^2 a^2}\frac{(1-s\tilde{s})^2}{2} a^{\alpha\beta} y_\alpha y_\beta \psi_1}\right)$$

- Solving for e and ω → a conformally rescaled AdS_3 metric,

$$ds^2 = \frac{4\Omega^2\, d(g_1 x)^2}{(1-g_1^2 x^2)^2}$$

$$g_1 = \exp\int_1^{x^2} \frac{1-\eta(t)}{2\eta(t)t} dt\,, \qquad \Omega = \frac{1-g_1^2 x^2}{g_1(1-x^2)}\eta\,, \qquad \eta = 1 + \frac{C x^2 f}{1+\sqrt{1-x^2}}$$

A Lorentz-invariant solution

- The results in the physical sector are analogous to those obtained in 4D by Sezgin and Sundell (algebraic structures are fixed by Lorentz invariance, only the embedding of translations in the bckgrd isometry group is different, and causes some complication).

$$\phi(x) \;=\; \mu\sqrt{1-x^2}\;, \quad ds^2 \;=\; \frac{4\Omega^2\, d(g_1 x)^2}{(1-g_1^2x^2)^2}$$

$$\phi(x) \;=\; \nu(1-x^2)\;, \qquad ds^2 \;=\; \frac{4\Omega^2 d\tilde{x}^2}{(1-\tilde{x}^2)^2}$$

(Sezgin-Sundell)

- The solution is asymptotically AdS, with Ω → 1 at the boundary, i.e., same AdS radius.

$$\Omega \;\sim\; 1+\frac{\pi^2\mu^2}{64}h+\mathcal{O}(h^2,\mu^4)\;, \qquad h \;:=\; \sqrt{1-x^2}$$

- A proper physical interpretation in the bulk as well as a holographic one await a HS generalization of geometry (and, in strict relation, a proper understanding of restrictions to specific functional classes, of asymptotic charges and small vs. large gauge transformations).

- Nevertheless, we now attempt a description in terms of standard spin-2 geometry, using this exact solution (the simplest non-trivial one, in some respect) as a lab to try and explore its possible duals and show some effects of the differences between gravity+scalar theories and HSGRA (non-localities, etc.).

Interpretation as a CDL instanton

- In coordinates

$$x^a = \tanh\frac{\rho}{2}\, n^a \ , \quad n^a n_a = 1 \ , \qquad x^2 > 0 \ ,$$
$$x^a = \tan\frac{T}{2}\, m^a \ , \quad m^a m_a = -1 \ , \qquad x^2 < 0 \ ,$$

the solution reads

$$\begin{aligned} ds^2 &= d\rho^2 + \eta(\rho)^2 \sinh^2\rho(-d\tau^2 + \cosh^2\tau d\varphi^2) \\ \phi &= \mu\,\mathrm{sech}\frac{\rho}{2} \ , \qquad x^2 > 0 \ , \\ ds^2 &= -dT^2 + \eta(T)^2 \sin^2 T(d\beta^2 + \sinh^2\beta d\varphi^2) \\ \phi &= \frac{\mu}{\cos\frac{T}{2}} \ , \qquad x^2 < 0 \end{aligned}$$

- This has the form of (the continuation to the minkowskian signature of) a Coleman-De Luccia (CDL) instanton, manifestly O(2,1)-symmetric, describing a bubble of true vacuum inside *AdS*, subject to a big crunch for $T = \pi$ (a genuine singularity from the spin-2 point of view).

Comparison with 2-derivative gravity+scalar

- It is interesting to compare the near-boundary ($x^2 \to 1$, $\rho \to \infty$) behaviour of this solution with that of similar ones in a 2-derivative gravity+scalar theory.

$$\begin{aligned} ds^2 &= d\tilde{\rho}^2 + G^2(\tilde{\rho}) ds^2_{dS_2} \\ G^2(\tilde{\rho}) &\sim \frac{e^{2\tilde{\rho}}}{4} + \frac{\pi^2\mu^2}{64} e^{\frac{3\tilde{\rho}}{2}} + \mathcal{O}\left(e^{\tilde{\rho}}, \mu^4\right) \\ \phi &\sim 2\mu\left(1 - \frac{\mu^2}{8}\right) e^{-\frac{\tilde{\rho}}{2}} - 2\mu\left(1 - \frac{3\mu^2}{8}\right) e^{-\frac{3\tilde{\rho}}{2}} + \mathcal{O}\left((e^{-\frac{5\tilde{\rho}}{4}}, \mu^4\right) \end{aligned}$$

- ϕ has the standard fall-off of an AdS_3-massless scalar (m^2 = -3/4 = $\Delta(\Delta$-2))

$$\phi \sim \beta e^{-\Delta_- \tilde{\rho}} + \alpha e^{-\Delta_+ \tilde{\rho}} \ , \qquad \Delta_- = 1/2 \ , \quad \Delta_+ = 3/2$$

- However, starting from a canonically normalized scalar field with m^2 = -3/4 and coupling it to gravity via a standard 2-derivative action, and imposing O(2,1)-symmetry, one gets a class of solutions with

$$G^2(\tilde{\rho}) \sim \frac{e^{2\tilde{\rho}}}{4} - \frac{\epsilon\beta^2\Delta_-}{2^{2(1-\Delta_-)}} e^{2(1-\Delta_-)\tilde{\rho}} + \mathcal{O}\left((e^{\tilde{\rho}})^{max(0,2-4\Delta_-)}\right)$$

→ the subleading term in our solution grows too fast near the boundary!

Comparison with 2-derivative gravity+scalar

- If one insists in engineering a 2-derivative action admitting a solution analogous to ours, one finds that it is only possible to produce our asymptotic behaviour with a NON-canonically normalized scalar field,

$$
\begin{aligned}
S &= \int d^3x\sqrt{-g}\left(R+2-\frac{1}{2}K(\phi)\partial_\mu\phi\partial^\mu\phi - V(\phi)\right) \\
K &\sim -\frac{\pi^2\mu\left(1+\frac{\mu^2}{4}\right)}{32\phi}+\mathcal{O}(\phi^0,\mu^3) \\
V &\sim \frac{7\pi^2\mu}{256}\left(1+\frac{\mu^2}{4}\right)\phi+\mathcal{O}(\phi^2,\mu^3)
\end{aligned}
$$

- We can field-redefine to a canonically normalized scalar, $\phi = \frac{8\left(1-\frac{\mu^2}{4}\right)}{\pi^2\mu}\chi^2+\mathcal{O}(\chi^3,\mu)$
  but now the latter has $m^2 = -7/16$, or $\Delta_- = 1/4$, which
  Indeed retrieves the near-boundary behaviour of the metric that we found.

- Hence, the two-derivative action should not be taken seriously as a "simplified model" of PV theory: not surprising, since the PV system is known to describe higher derivative interactions with arbitrarily high number of derivatives.

Holographic interpretation

- We shall now attempt to find a dual CFT configuration to our solution. To first order in μ, the bckgrd remains global AdS while the scalar profile solves the free KG equation. We will use this to propose a dual picture for our solutions, under the assumption that the standard AdS-CFT dictionary for free scalar fields remains valid.

- We can explore the duality for the general theory governed by $\mathfrak{hs}[\lambda]$, with $0 \leq \lambda \leq 1$, conjectured to be dual to the 't Hooft limit of the unitary $\mathcal{W}_N$ minimal models.

- First, note that we can simply compute the scalar profile also for general mass $m^2 = \lambda^2 - 1 \rightarrow \Delta_\pm = 1 \pm \lambda$. Indeed, ϕ = trace of an SL(2,R) element!, and we can compute it via analytical continuation N $\rightarrow \lambda$ (since $\mathfrak{hs}[\lambda]$ itself can be thought of as an analytic continuation of $\mathfrak{sl}(N)$) .

$$\boxed{\phi \;=\; \mu \frac{\sinh(2\lambda \operatorname{arctanh}\sqrt{x^2})}{\lambda \sinh(2 \operatorname{arctanh}\sqrt{x^2})} \;=\; \frac{\mu \sinh \lambda\rho}{\lambda \sinh \rho}}$$

- For $\lambda > 0$ the near-boundary behaviour is $\phi \;\sim\; \beta e^{-\Delta_- \tilde{\rho}} + \alpha e^{-\Delta_+ \tilde{\rho}}\,, \qquad \beta = -\alpha = \frac{\mu}{\lambda}$

Holographic interpretation

- Imposing a boundary condition on the scalar where $\alpha = \alpha(\beta)$ corresponds in the dual theory to adding the deformation ($\beta \leftrightarrow \langle O_- \rangle$)

$$\Delta S = -N \int d^2x\, W(\mathcal{O}_-) , \qquad \text{with} \quad [\mathcal{O}_-] = \Delta_- , \qquad \frac{\partial W}{\partial \beta} = \alpha(\beta)$$

- Choosing *W* to be marginal,

$$W(\mathcal{O}_-) = \frac{f\Delta_-}{2}\mathcal{O}^{\frac{2}{\Delta_-}} , \qquad \Rightarrow \qquad f = -\left(\frac{\mu}{\lambda}\right)^{\frac{2\lambda}{\lambda-1}}$$

- As $\beta = const$, it seems the CFT description is bound to be smooth even though the solution has a big crunch! But this is a coordinate artifact. In AdS global coordinates,

$$ds^2 = -\cosh^2 r\, dt^2 + dr^2 + \sinh^2 r\, d\varphi^2$$

  the VEV is NOT smooth and constant, but rather

$$\beta = \frac{\mu}{\lambda(\cos t)^{\Delta_-}}$$

- At $t = \pi/2$ the bubble reaches the boundary and the VEV blows up, signalling that the deformation has rendered the field theory singular.

Holographic interpretation

- From the POV of the dual theory, we can understand this behaviour from the effective action for $\sigma = < O_- >$ in the deformed CFT.

$$\frac{1}{N}\Gamma[\sigma] = -\int d^2x \left(\frac{f\Delta_-}{2}\sigma^{\frac{2}{\Delta_-}} + c\frac{\partial_\mu\sigma\partial^\mu\sigma}{2\sigma^2} + \dots \right)$$

- Redefining $\sigma = \exp(c^{1/2}\psi)$ one gets the classical Liouville action,

$$\frac{1}{N}\Gamma[\sigma] = -\int d^2x \left(\partial_\mu\psi\partial^\mu\psi + \frac{f\Delta_-}{2}e^{\gamma\psi} \right) \,, \qquad \gamma = \frac{2\sqrt{c}}{\Delta_-}$$

- For $f < 0$, the effective theory is unstable and admits solutions of the form

$$\sigma = \left(\frac{(-f)\cos^2 t}{c\Delta_-} \right)^{-\frac{\Delta_-}{2}}$$

which coincides with that of the bulk profile.

- Note that in Poincaré coordinates $ds^2 = \frac{dz^2 + dy^\mu dy_\mu}{z^2}$ the solution looks like

$$\boxed{\sigma = e^{\sqrt{c}\psi} = \left(\frac{(-f)(1+y^\mu y_\mu)^2}{c\Delta_-} \right)^{-\frac{\Delta_-}{2}}}$$

a $d \rightarrow 2$ scaling limit of a Fubini instanton .

Holographic interpretation

- Interesting boundary interpretation for $\lambda = 0$, where the conjectured undeformed dual CFT is (the singlet sector of) a theory with N free fermions Ψ^a, a = 1, . . . , N.

- For $\lambda = 0$, $m^2 = -1$, and the scaling dimensions become equal, $\Delta+ = \Delta- = 1$. For this value of the mass, the profile of a free scalar has a logarithmic term in the near-boundary expansion,

$$\phi = \alpha \ln(mz)z + \beta z$$

  where z = exp(−r) and we have introduced a scale *m* to define the logarithm. On the dual side, this corresponds to the fact that the double-trace deformation with the operator dual to ϕ is only classically marginal, while quantum mechanically it has a running coupling. This limit captures effects of a running coupling in a large-N interacting fermion model.

- The operator dual to ϕ is the single-trace $\mathcal{O} = \frac{\sqrt{\pi}\bar{\Psi}^a\Psi_a}{N}.$

  and the deformed theory is a Gross-Neveu model with wrong-sign potential, free in the infrared,

$$S = \int d^2x \left[\bar{\Psi}^a\gamma^\mu\partial_m\Psi_a - \frac{\pi f}{2N}(\bar{\Psi}^a\Psi_a)^2\right]$$

Z-space monodromies and S-moduli

- The structure of the vacuum $\Phi = 0$ may be richer than it seems at first sight. Indeed, the deformed oscillator equation

$$[\widehat{S}^{(0)}_\alpha, \widehat{S}^{(0)\alpha}]_\star \ = \ -4i$$

has the structure of $X^2 = 1$. If X is valued in a purely commutative algebra → $X = \pm 1$. However, if it is vaued in a non-commutative algebra, there are more interesting possibilities:

$$X^2 = 1 \ \rightarrow \ X = 1 - 2P \ , \quad P^2 = P$$

- *Projectors in Z-space connection* → flat but non-trivial! More complicated vacua?

$$\widehat{\Phi}' \ = \ 0 \, , \quad \widehat{S}'_\alpha \ = \ z_\alpha \left(1 - 2\sum_{n=0}^{\infty} \theta_n P_n(Y,Z)\right) \ , \qquad P_n \star P_m \ = \ \delta_{nm} P_n \ , \qquad \theta_n \ = \ \{0,1\}$$

(C.I.-Sezgin-Sundell '07)

- ➢ Can also dress up non-vacuum solutions (like windings in String Theory…). Both bhs and the scalar-instanton can be decorated with these discrete S-moduli.

- ➢ Different choices for the Fock-space where the projectors act lead to different global symmetries.

Conclusions & Outlook

- Constructed a Lorentz-invariant exact solution to the 3D HS gravity coupled to matter.
  In a specific gauge, it only activates a scalar profile over a rescaled AdS metric.
  It admits a "dressing" in terms of Lorentz-invariant projectors, reminiscent of monodromies of the z-space connection. They furnish new vacua, possibly inequivalent to AdS.
  Most properties are identical to the 4D analogue found by Sezgin and Sundell.

- Attempted a geometrical description of the solution in terms of spin-2 geometry.
  A proper analysis should be performed in terms of HS observables.

- In terms of the spin-2 geometry, the solutions resemble a CDL instanton with a big crunch singularity. At the same time, there are notable differences from similar solutions of standard gravity + scalar theories.

- Attempted a holographic interpretation for generic mass of the scalar: the corresponding Dual configuration in the CFT is a D → 2 scaling limit of a Fubini instanton.