Mixed symmetry multiplets
&
higher-spin curvatures

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Scuola Normale Superiore & INFN

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SCUOLA NORMALE SUPERIORE

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➡ *free local Lagrangians, however, are usually required to be generated by 2nd order kinetic tensors*

➡ *still, free equations naturally appear in higher-derivative form, once they are formulated à la Bargmann-Wigner*

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extending it to the case of *multi-particle representations*

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extending it to the case of *multi-particle representations*

→ *alternative to more conventional single-particle equations*

→ *akin to massless hsp as emerging from tensionless strings*

Back to basics:

wave equations for particles with zero mass wave equations for particles with zero mass

two options:

gauge dependent

gauge independent

Wave equations for m=0, s=2

$\sim$

gauge dependent

Wave equations for m=0, s=2

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$$h_{\mu\nu} \sim \boxed{\mu \,|\, \nu}_{GL(D)}$$

s.t.

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$iso(D-2)$ non compact

finite spin

same tensor as
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$\boxed{\text{Fierz 1939}}$

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Connecting the two descriptions:

$$\partial_{[\lambda}\,\mathcal{R}_{\,\mu\nu],\,\rho\sigma} = 0 \quad \xrightarrow[\textit{Poincaré Lemma}]{} \quad \mathcal{R}_{\mu\nu,\,\rho\sigma}\,(h) = \partial_{\mu}\,\partial_{\rho}\,h_{\,\nu\sigma} + \ldots$$

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$$\Longrightarrow$$

- $\partial_{[\lambda}\,\mathcal{R}_{\mu\nu],\,\rho\sigma}\,(h) \equiv 0$

- $\eta^{\mu\rho}\,\mathcal{R}_{\mu\nu,\rho\sigma}\,(h) = 0$ corresponds to the vanishing of the linearised Ricci tensor, that can be written

$$\Box h_{\mu\nu} = \partial_{(\mu}\,\Lambda_{\nu)}(h)$$

so as to stress that it reduces to $P^2 = 0$ upon partial gauge fixing

Wave equations for $m = 0$, spin s

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$$\varphi \equiv \varphi_{\mu_1 \ldots \mu_s} \sim \boxed{}\boxed{}\boxed{\cdots}\boxed{}$$

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$$d \mathcal{R} = 0$$

$$\mathcal{R}' = 0$$

Wave equations for spin s

Connecting the two descriptions:

$$d\mathcal{R} = 0 \xrightarrow[\text{Generalised Poincaré Lemma}]{} \mathcal{R}_{\mu_1\nu_1,\ldots,\mu_s\nu_s} = \partial_{\mu_1}\ldots\partial_{\mu_s}\varphi_{\nu_1\ldots\nu_s} + \ldots$$

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Generalised Poincaré Lemma

$\rightarrow$

$$d\mathcal{R}(\varphi) \equiv 0$$

Wave equations for spin s

Bekaert Boulanger 2002, 2003

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$$\longrightarrow$$

- $d\mathcal{R}(\varphi) \equiv 0$

- The higher-derivative equation $\mathcal{R}' = 0$ can be proven to be equivalent to the wave equation

$$\Box\varphi = \partial\Lambda(\varphi)$$

where the r.h.s. can be gauge fixed to zero. (! *Note*: this is not the Fronsdal equation)

Goal of this talk

$\sim$

we focus on hsp curvatures:

$$\mathcal{R}_{\mu\nu,\,\rho\sigma}\,(h)$$

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standard hsp theories are ``Ricci-like"

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Spin one (and p-forms)

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Our goal:

we wish to extend the Bargmann-Wigner program

to encompass the Maxwell-like equations

$$\partial \cdot \mathcal{R}\,(\varphi) \,=\, 0$$

for all spins, in any D, i.e. including tensors with mixed symmetry

Plan

§ Maxwell-like equations à la Bargmann-Wigner

§ Curvatures & wave operators for gauge potentials

§ Reducible multiplets and tensionless strings

Based on

- *J.Phys.A: Math.Theor. 48 (2015)* *(with X. Bekaert and N. Boulanger)*

- *Class.Quant.Grav. 29 (2012)*

see also

- *Nucl.Phys. B881 (2014) 248-268* *(with S. Lyakhovic and A. Sharapov)*
- *JHEP 1303 (2013) 168* *(with A. Campoleoni)*
- *Prog.Theor.Phys.Suppl. 188 (2011)*
- *Phys.Lett. B690 (2010)*
- *J.Phys.Conf. Ser. 222 (2010)*

Maxwell-like equations à la Bargmann-Wigner

spin 2

$$h_{\mu\nu} \sim \begin{array}{|c|c|}\hline \mu & \nu \\ \hline \end{array} \longrightarrow \mathcal{R}_{\mu\nu,\rho\sigma} \sim \begin{array}{|c|c|}\hline \mu & \rho \\ \hline \nu & \sigma \\ \hline \end{array}$$

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$$P^2 = 0 \longrightarrow p_{\mu} = (p_+, 0, \ldots, 0)$$

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$$P^2 = 0 \longrightarrow p_{\mu} = (p_+, 0, \ldots, 0)$$

$$\partial^{\mu}\mathcal{R}_{\mu\nu,\rho\sigma} = 0 \longrightarrow \mathcal{R}_{-\nu,\rho\sigma} = 0$$

$$\partial_{[\lambda}\mathcal{R}_{\mu\nu],\,\rho\sigma} = 0 \longrightarrow \mathcal{R}_{ij,\,kl} = 0$$

spin 2

The only non-vanishing components of $\mathcal{R}_{\mu\nu,\,\rho\sigma}$ *are*

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i.e. they define a symmetric tensor of GL(D-2)

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➞ *In terms of particles (irreps of O(D-2)) this means*

$$\partial_{[\lambda}\mathcal{R}_{\mu\nu],\,\rho\sigma} = 0 \qquad \longrightarrow \qquad \textit{one particle with } m = 0,\ s=2$$

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Maxwell-like eqs propagate reducible multiplets

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General case: consider an arbitrary tableau in GL(D-2) and build its Bargmann-Wigner counterpart, by adding a row on its top

$$Y_{GL(D-2)} = \ydiagram{4,3,1} \qquad \longrightarrow \qquad \mathcal{R}_{GL(D)} = \ydiagram{4,4,3,1}$$

Arbitrary spin in arbitrary $\mathcal{D}$

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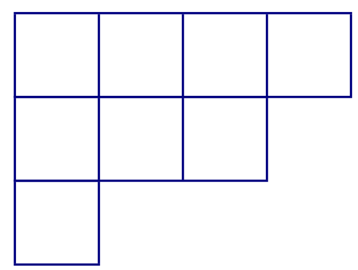
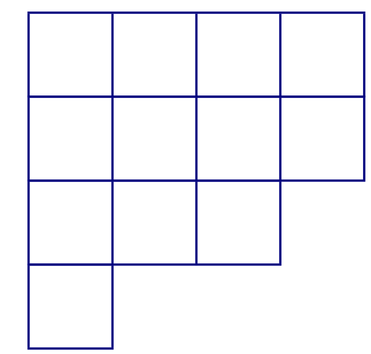
$$Y_{GL(D-2)} =$$  $$\longrightarrow \qquad \mathcal{R}_{GL(D)} =$$ 

➙ *Require* $\mathcal{R}_{GL(D)}$ *to satisfy the closure and co-closure conditions*

$$\begin{aligned} d\,\mathcal{R} &= 0 \\ d^{\dagger}\,\mathcal{R} &= 0 \end{aligned} \quad \longrightarrow \quad P^2 = 0 \quad \longrightarrow \quad p_\mu = (p_+, 0, \ldots, 0)$$

(w.r.t all rectangular blocks)

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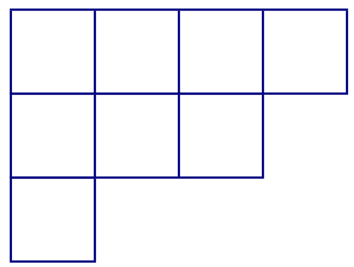
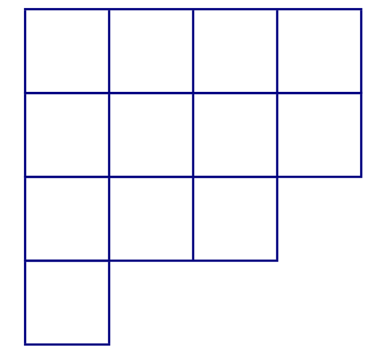
$Y_{GL(D-2)} =$  $\longrightarrow$ $\mathcal{R}_{GL(D)} =$ 

- *Require* $\mathcal{R}_{GL(D)}$ *to satisfy the closure and co-closure conditions*

$$d\,\mathcal{R} = 0 \qquad d^{\dagger}\mathcal{R} = 0 \quad \longrightarrow \quad P^2 = 0 \quad \longrightarrow \quad p_\mu = (p_+, 0, \dots, 0)$$

(w.r.t all rectangular blocks)

- *The non-vanishing components,* $\mathcal{R}_{+\,j^1_1\dots j^1_{l_1},\,\dots,\,+\,j^i_1\dots j^i_{l_i},\,\dots,\,+\,j^s_1\dots j^s_{l_s}}$ *,*

correspond to a multiplet of massless particles:

branching of the GL(D-2)-irrep in terms of its O(D-2)-components.

Curvatures & wave operators for gauge potentials

High-derivative equations from curvatures

We make contact with gauge potentials solving for the closure conditions via the Generalised Poincaré Lemma:

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(w.r.t all rectangular blocks)

where $\mathcal{R}(\varphi)$ corresponds to the irrep of *GL(D)* obtained from a given tableau *Y* by adding an extra row on top of it:

$$\varphi_Y = \ydiagram{4,3,1} \quad \longrightarrow \quad \mathcal{R}(\varphi_Y) = \begin{ytableau} \partial & \partial & \partial & \partial \\ \; & \; & \; & \; \\ \; & \; & \; \\ \; \end{ytableau}$$

High-derivative equations from curvatures

$\sim$

We go through the Bargmann-Wigner analysis again,
but now for high-derivative functions of gauge potentials

$$\mathcal{R}\,(\varphi)\;\equiv\;d^{\,1}\,d^{\,2}\,\cdots\,d^{\,s}\,\varphi$$

computing the divergence of $\mathcal{R}$

$$d_{\,1}\,\mathcal{R}\,(\varphi)\;=\;d^{\,2}\,\cdots\,d^{\,s}\,(\Box\;-\;d^{\,i}d_{\,i})\,\varphi\;\sim\;\mathcal{O}\,(d)\,M\;=\;0$$

where

$$M\;=\;(\Box\;-\;d^{\,i}d_{\,i})\,\varphi$$

is a sort of second-order
Maxwell-like wave operator

From high- to 2nd-order equations

$\sim$

Problem: determine the kernel of the operator $\mathcal{O}(d)$

two steps:

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Getting an equation for M
via the Generalised Poincaré Lemma

$$d^{\,2}\,\cdots\,d^{\,s}\,(\Box\,-\,d^{\,i}d_{\,i})\,\varphi\,=\,0$$

$$\longrightarrow$$

$$M\,=\,d^{\,i}\,d^{\,j}\,D_{\,ij}\,(\varphi)$$

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Getting an equation for M
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$$d^2 \cdots d^s \left(\Box - d^i d_i\right) \varphi = 0$$

$$\longrightarrow$$

$$M = d^i d^j D_{ij}(\varphi)$$

2

Show that the resulting equation
can be gauge fixed to $P^2 = 0$:

$$\Box \varphi = d^i \Lambda_i(\varphi)$$

$$\longrightarrow$$

$$\Box \varphi = 0 \qquad d^\dagger \varphi = 0$$

Same analysis for the ``standard'' BW trace conditions:

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$$T_{12}\,\mathcal{R}\,(\varphi) \,=\, d^{\,3}\,\cdots\,d^{\,s}\,\mathcal{F}\,\sim\,\hat{\mathcal{O}}\,(d)\,\mathcal{F}\,=\,0$$

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$$\mathcal{F}\;:=\;\Box\varphi\;-\;d^{\,i}\,d_{\,i}\,\varphi\;+\;\frac{1}{2}\,d^{\,i}\,d^{\,j}\,T_{ij}\,\varphi$$

Fronsdal-Labastida tensor

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$$M = d^i\, d^j\, D_{ij}(\varphi)$$

still higher-derivative eqs!

$$\mathcal{F} = \tfrac{1}{2}\, d^i\, d^j\, d^k\, \mathcal{H}_{ijk}(\varphi)$$

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Our analysis shows that the two ``compensator'' structures

$D_{\,ij}\,(\varphi)$ and $\mathcal{H}_{\,ijk}\,(\varphi)$

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$$M \;=\; 0 \qquad\qquad \mathcal{F} \;=\; 0$$

$$d^{\,i}\, d^{\,j} d_{\,(i}\, \Lambda_{\,j)} \;=\; 0 \qquad\qquad T_{(ij}\, \Lambda_{\,k)} \;=\; 0$$

D.F., A. Campoleoni 2013

Fronsdal-Labastida '78, '89

To summarise:

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BW trace conditions on
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_Via the Poincare' lemma
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BW trace conditions on ``curvature precursors'' describe one-particle dof

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BW transversality conditions on the same tensors describe multi-particle dof

_Via the Poincare' lemma
_upon partial gauge fixing
they reduce to

$$M := \Box\varphi - d^{\,i}\, d_{\,i}\, \varphi = 0$$

Let us compare the corresponding Lagrangian formulations

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Maxwell-like, N families:

(multi-particle spectrum)

$$\mathcal{L} \,=\, \frac{1}{2}\,\varphi\,M\,\varphi$$

$$M \,=\, (\Box \,-\, \partial^{\,i}\,\partial_{\,i})$$

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Fronsdal-Labastida, N families:

$$\mathcal{L} \;=\; \frac{1}{2}\,\varphi\,\left\{\mathcal{F} \;+\; \sum_{p\,=\,1}^{N}\,\frac{(-1)^p}{p!\,(p+1)!}\,\eta^{i_1 j_1}\;\ldots\;\eta^{i_p j_p}\,Y_{\{2^p\}}\,T_{i_1 j_1}\;\ldots\;T_{i_p j_p}\,\mathcal{F}\right\}\,,$$

$$\mathcal{F} \;=\; \left(M \;+\; \partial^{\,i}\partial^{\,j}\,T_{ij}\right)\,\varphi \qquad\qquad \begin{cases} T_{(\,ij}\,\Lambda_{\,k\,)} \;=\; 0 \\[2ex] T_{(\,ij}\,T_{kl\,)}\,\varphi \;=\; 0 \end{cases}$$

Reducible multiplets and tensionless strings

~

Massless higher spins from tensionless strings

Open bosonic string oscillators

$$[\alpha_k^\mu, \alpha_l^\nu] \;=\; k\,\delta_{k+l,0}\,\eta^{\mu\nu}$$

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Virasoro generators and their rescaling limit:

$$L_k \;=\; \frac{1}{2}\sum_{l=-\infty}^{+\infty} \alpha^\mu_{k-l}\,\alpha_{\mu l}\ , \;\to\; \begin{cases} \tilde{L}_{\,k\neq 0} \;=\; \dfrac{1}{\sqrt{\alpha'}}\,L_{\,k} \\[2ex] \tilde{L}_{\,0} \;=\; \dfrac{1}{\alpha'}\,L_{\,0} \end{cases} \xrightarrow[\alpha' \to \infty]{} \quad \begin{aligned} l_{\,k} \;&=\; p_{\,\mu}\,\alpha^{\,\mu}{}_{k} \\ l_{\,0} \;&=\; p_{\,\mu}\,p^{\,\mu} \end{aligned}$$

"tensionless" limit

Massless higher spins from tensionless strings

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Algebra with no central charge $\longrightarrow$ identically nilpotent BRST charge $\mathcal{Q}$

same charge from tensionless limit of open string BRST charge, after rescaling of ghosts

Massless higher spins from tensionless strings

$$\mathcal{L} = \frac{1}{2} \langle \psi | \, Q \, | \psi \rangle \quad \xrightarrow[\alpha' \to \infty]{\sim} \quad \text{decomposes in diagonal blocks}$$

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for ``diagonal blocks'' associated to symmetric, rank-s tensors $\varphi_{\,\mu_1\cdots\mu_s}$, (states generated by powers of $\alpha^{\,\mu}_{-1}$) the corresponding Lagrangian is

$$\mathcal{L}_{\,triplet} \,=\, \frac{1}{2}\,\varphi\,\Box\,\varphi - \frac{1}{2}\,s\,C^{\,2} - \binom{s}{2}\,D\,\Box\,D \,+\, s\,\partial\cdot\varphi\,C \,+\, 2\binom{s}{2}\,D\,\partial\cdot C$$

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equations of motion		*gauge transformations*
$\Box \, \varphi = \partial \, C$	$\varphi \to$ spin s	$\delta \, \varphi = \partial \, \Lambda$
$C = \partial \cdot \varphi - \partial \, D$	$C \to$ spin $s-1$	$\delta \, C = \Box \, \Lambda$
$\Box \, D = \partial \cdot C$	$D \to$ spin $s-2$	$\delta \, D = \partial \cdot \Lambda$

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the eom for the physical field from the tensionless string

$$M\,\varphi = 2\partial^2\,\mathcal{D}$$

are just the Maxwell-like equations with a ``compensator''

Massless higher spins from tensionless strings

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Bengtsson, Ouvry-Stern '86 Henneaux-Teitelboim '88
D.F.-Sagnotti '02, Sagnotti-Tsulaia '03
Fotopoulos-Tsulaia '08 . . .

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the eom for the physical field from the tensionless string

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[also valid for mixed-symmetry fields]

Conclusions

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$\sim$

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``Ricci = 0'' provides the backbone of gauge theories...

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Alternative option:

reducible, multi-particle theories

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when the focus is on *single-particle interactions*

Alternative option:

reducible, multi-particle theories

``Maxwell = 0'' seems to provide the proper model to this end

$$\partial^{\alpha}\mathcal{R}_{\alpha\,\mu_2\ldots\mu_s\,,\,\nu_1\ldots\nu_s} \;=\; 0$$

Why?

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$\sim$

Exploit an alternative basis of field variables

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- *for instance for the spin-2 case the self-interactions of a single field would encompass all the vertices of a scalar-tensor theory*

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what are their actual role and meaning?

in progress…

Maxwell-like geometric Lagrangians

~

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how does the Lagrangian would look in terms of the physical field only?

Maxwell-like geometric Lagrangians

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how does the Lagrangian would look in terms of the physical field only?

Integrating over the fields C and D we find

$$\mathcal{L}_{eff}(\varphi) = \frac{1}{2}\varphi(\Box - \partial\partial\cdot)\varphi + \frac{1}{2}\binom{s}{2}\partial\cdot\partial\cdot\varphi\left(\Box + \frac{1}{2}\partial\partial\cdot\right)^{-1}\partial\cdot\partial\cdot\varphi$$

Maxwell-like geometric Lagrangians

The inverse of the operator $\mathcal{O} = \Box + \frac{1}{2}\partial\,\partial\cdot$ on rank-*k* tensors is

$$\mathcal{O}^{-1}_{(k)} = \frac{1}{\Box}\{1 + \sum_{m=1}^{k}(-1)^m \frac{m!}{2^m \prod_{l=1}^{m}(1 + \frac{l}{2})}\frac{\partial^m}{\Box^m}\partial\cdot^m\}$$

and the resulting Lagrangian is

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and the resulting Lagrangian is

$$\boxed{\mathcal{L}_{eff}(\varphi) = \frac{(-1)^s}{2(s+1)}\mathcal{R}^{(s)}_{\mu_1\cdots\mu_s,\,\nu_1\cdots\nu_s}\frac{1}{\Box^{s-1}}\mathcal{R}^{(s)\,\mu_1\cdots\mu_s,\,\nu_1\cdots\nu_s}}$$

Lagrangians $\sim$ *squares of curvatures*