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T-duality of superstrings in $AdS_d \times S^d \times M^{10-2d}$

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Motivation

- AdS/CFT correspondence, which relates a string theory in an *AdS space to* a conformal field theory (CFT) on the *AdS boundary*

- Instances of the holographic dualities include *AdS5/CFT4, AdS4/CFT3, AdS3/CFT2 and AdS2/CFT1* correspondences

- The most developed and best understood is *AdS5/CFT4* holography

String on $AdS_5 \times S^5$ with $PSU(2,2|4)$ isometry v.s. $\mathcal{N}=4$, D=4 SYM with $SU(N)$

Integrability on the both sides (at $N \to \infty$) manifests itself in various features, e.g:

relation between planar scattering amplitudes and Wilson loops at strong and weak coupling in SYM theory (hidden dual $PSU(2,2|4)$ symmetry of scattering amplitudes)

On the string side dual $PSU(2,2|4)$ manifests itself in the invariance of string action under combined bosonic-fermionic T-duality (*Berkovits & Maldacena; Baisert et. al. `08*)

$$dx^m(\tau,\sigma) \to d\tilde{x}_m, \quad d\theta^\alpha(\tau,\sigma) \to d\tilde{\theta}_\alpha \quad \alpha = 1,\ldots,8$$

 (4d Minkowski boundary)

Motivation

- Manifestation and role in the AdS/CFT correspondence of the fermionic T-duality of the string sigma-models in **less susy backgrounds** is much less understood
  - $AdS_3 \times S^3 \times T^4$ preserves 16 (of 32) 10D SUSY, isometry ***G*** **=** ***PSU*****(1,1|2) ×** ***PSU*****(1,1|2)**
  - $AdS_2 \times S^2 \times T^6$ preserves 8 (of 32) 10D SUSY, isometry ***G*** **=** ***PSU*****(1,1|2)**
  - $AdS_4 \times CP^3$ preserves 24 (of 32) 10D SUSY, isometry ***G*** **=** ***OSp*****(6 |4)** - most problematic
- By now T-duality has been demostrated only for subsectors of string actions described by ***G/H*** **supercoset** sigma-models *(Adam, Dekel & Oz `09)*
  - $AdS_d \times S^d \quad (d = 2,3,5) \qquad H = SO(1, d-1) \times SO(d)$
    fermionic **kappa-symmetry** of the string actions was **always** (partially) **gauge fixed**
  - string **fluctuations along Torus directions** and **fermionic modes** associated with **broken susy have not been taken into account**

Aim of this project

- Prove invariance under combined bosonic-fermionic T-duality of the ***G/H* supercoset** sigma-models **without fixing** kappa-symmetry

- Give **evidence** for the **T-selfduality** of the **complete superstrings** in $AdS_d \times S^d \times T^{10-2d}$ by taking into account the contribution of the string fluctuations along the Tori directions and broken-susy fermionic modes

- To extend these results to string sigma-models in $AdS_d \times S^d \times S^d \times T^{10-3d}$ $(d = 2,3)$ with superisometries described by the exceptional supergroup $D(2,1;\alpha)$

Fixing kappa-symmetry issue. It can gauge away ½ of the 32 string fermionic modes

In $AdS_3 \times S^3 \times T^4$ - 16 fermions $\theta_{16} \subset G/H$ and 16 fermions υ_{16} are "non-susy"

In $AdS_2 \times S^2 \times T^6$ - 8 fermions $\theta_8 \subset G/H$ and 24 fermions υ_{24} are "non-susy"

Using k-symmetry we can only remove, **for instance**, 16 fermions υ

T-dualization of 2d sigma-models

$$S = -\tfrac{1}{2}\int d\tau\, d\sigma \left(\partial_+ x^m g_{mn}(y)\partial_- x^n + \partial_+ x^m B_{mn}(y)\partial_- x^n + \partial_+\theta^\alpha F_{\alpha\beta}(y)\partial_-\theta^\beta + L(y)\right)$$

$$\partial_\pm = \partial_\tau \pm \partial_\sigma$$

Action is invariant uder translations (commuting isometries): $x^m \to x^m + a^m, \quad \theta^\alpha \to \theta^\alpha + \varepsilon^\alpha$

- Convert the Lagrangian in the first order form:

$$L_1 = \tfrac{1}{2}\left(A_+^m g_{mn}(y)A_-^n + A_-^m B_{mn}(y)A_-^n + A_+^\alpha F_{\alpha\beta}(y)A_-^\beta + L(y)\right)$$
$$+ A_-^m \partial_+ \tilde{x}_m - A_+^m \partial_- \tilde{x}_m + A_-^\alpha \partial_+ \tilde{\theta}_\alpha - A_+^\alpha \partial_- \tilde{\theta}_\alpha$$

- Solve for (or integrate out) the auxiliary fields $A_+^m, A_-^m, A_+^\alpha, A_-^\alpha$

$$\tilde{S} = -\tfrac{1}{2}\int d\tau\, d\sigma \left(\partial_+ \tilde{x}_m \tilde{g}^{mn}(y)\partial_- \tilde{x}_n + \partial_+ \tilde{x}_m \tilde{B}^{mn}\partial_- \tilde{x}_n + \partial_+ \tilde{\theta}_\alpha \tilde{F}^{\alpha\beta}(y)\partial_- \tilde{\theta}_\beta + L(y)\right)$$

The action is "self-dual" if (upon some field redefinitions) $\tilde{g}(y) = g(y),\ \tilde{B}(y) = B(y),\ \tilde{F}(y) = F(y)$

G/H supercoset sigma-models

$$G = PSU(2,2|4),\quad PSU(1,1|2)\otimes PSU(1,1|2),\quad PSU(1,1|2)$$

$$H = SO(1,d-1)\times SO(d), \qquad d = 5,3,2$$

$$G/H \approx AdS_d(x^m, r)\times S^d(y) + 8(d-1)\ \text{fermionic directions}\ \vartheta$$

Action is constructed using worldsheet pullbacks of the Cartan forms (currents):

$$J = g^{-1}dg(x,r,y,\vartheta) = J^0 M_0 + J^2 P_2 + J^1 Q_1 + J^3 Q_3,$$

$$M_0 \subset H,\quad P_2, Q_1, Q_3 \subset G/H, \qquad 0,1,2,3\ \text{are}\ Z_4\ \text{gradings of the generators}$$

The action (*Metsaev & Tseytlin `98, Berkovits et. al. `99*)

$$S_{G/H} = -\mathrm{Str}\int d\tau d\sigma \left(J_+^2 J_-^2 + \tfrac{1}{2}(J_+^1 J_-^3 - J_-^1 J_+^3)\right)$$

T-dualization of the G/H models

- **Key steps:**
  - single out an appropriate **Abelian** subalgebra of G which is associated with translation superisometries of the Minkowski boundary of $AdS(d)$

$$ds^2_{AdS} = r^2 dx^m dx^n \eta_{mn} + \tfrac{1}{r^2} dr dr, \qquad [P_m, P_n] = 0$$

$$Q^1 = (Q + \bar{Q} - S - \bar{S}), \qquad Q^3 = i(Q - \bar{Q} + S - \bar{S}) : \qquad \{Q, Q\} = 0, \ \ [Q, P_m] = 0$$

$$\vartheta^1 = (\theta + \bar{\theta} - \xi - \bar{\xi}), \qquad \vartheta^3 = i(\theta - \bar{\theta} + \xi - \bar{\xi}) \qquad \dim Q, \theta, \ldots = 2(d-1)$$

  - select an appropriate coset element $g(x^m, r, y, \vartheta^1, \vartheta^3)$ such that $J = g^{-1} dg(x^m, r, y, \vartheta^1, \vartheta^3)$ contain x^m and θ only as dx^m, $d\theta$

$$g = e^{x^m P_m + \theta Q} e^{B(r, y, \bar{\theta}, \bar{\xi})} e^{\xi S}$$

kappa symmetry gauge: $\xi = 0 \ \ (\bar{\xi} = 0)$

T-duality of the G/H sigma-models

$$S_{G/H}=-\mathrm{Str}\int d\tau d\sigma\left(J_{+}^{2}J_{-}^{2}+\tfrac{1}{2}(J_{+}^{1}J_{-}^{3}-J_{-}^{1}J_{+}^{3})\right)$$

- Replace in the action $dx^m,\ d\theta^\alpha\to A^m, A^\alpha$
- Add the Lagrange multiplier terms $+A_{-}^{m}\partial_{+}\tilde{x}_{m}-A_{+}^{m}\partial_{-}\tilde{x}_{m}+A_{-}^{\alpha}\partial_{+}\tilde{\theta}_{\alpha}-A_{+}^{\alpha}\partial_{-}\tilde{\theta}_{\alpha}$
- Integrate out the auxiliary fields $A_{\pm}^{m}, A_{\pm}^{\alpha}$

Get the dual action (upon tedious computations and some field redefinitions):

$$\tilde{S}_{G/H}=-\mathrm{Str}\int d\tau d\sigma\left(\tilde{J}_{+}^{2}\tilde{J}_{-}^{2}+\tfrac{1}{2}(\tilde{J}_{+}^{1}\tilde{J}_{-}^{3}-\tilde{J}_{-}^{1}\tilde{J}_{+}^{3})\right)$$

$$\tilde{J}=\tilde{g}^{-1}d\tilde{g},\qquad \tilde{g}=e^{\tilde{x}^m K_m+\tilde{\theta}S}e^{B(r,y,\bar{\theta},\bar{\xi})}e^{F(\xi)}$$

$$g=e^{x^m P_m+\theta Q}e^{B(r,y,\bar{\theta},\bar{\xi})}e^{\xi S}$$

generator of conformal boosts of Minkowski boundary

$$F(\xi)=(-\xi+O(\xi^5))Q+\left(O(\xi^3)+O(\xi^7)\right)S$$

$$[K,K]=[K,S]=\{S,S\}=0$$

Green-Schwarz string actions in $AdS_d \times S^d \times T^{10-2d}$

- G/H supercoset sigma-models coupled to T^{10-2d} + $\upsilon_{8(5-d)}$ non-susy fermons in D=10 type II superspace $X^M = (x^m, r; y^{\hat p}, \varphi^{a'}), \quad \Theta = (\vartheta, \upsilon)$

$$S = -\tfrac{1}{2}\int d\tau d\sigma \left(E_+^A E_-^B \eta_{AB} + B_{+-}\right) \qquad S_{G/H} = -\mathrm{Str}\int d\tau d\sigma \left(J_+^2 J_-^2 + \tfrac{1}{2}(J_+^1 J_-^3 - J_-^1 J_+^3)\right)$$

Supervielbeins: $E_\pm^A(X,\Theta) = \partial_\pm X^M E_M^A(X,\Theta) + \partial_\pm \Theta^\mu E_\mu^A(X,\Theta), \quad A = 0,1,\ldots,9$

$B_2(X,\Theta)$ - NS-NS rank-2 tensor gauge superfield

In the action, expend the supevilebeins and B_2 in series of $\upsilon_{8(5-d)}$ (up to the 2nd order)

$$AdS_d \times S^d: \quad E^a = J^{(2)a}(x, r; y, \vartheta) - \tfrac{i}{2} D\upsilon \Gamma^a \upsilon, \qquad a = 0,1,\ldots,2d-1$$

$$T^{10-2d}: \quad E^{a'} = d\varphi^{a'} - iJ^{(1,3)}(x, r; y, \vartheta)\Gamma^{a'}\upsilon - \tfrac{i}{2} D\upsilon \Gamma^{a'}\upsilon, \quad a' = 1,\ldots,10-2d$$

$$B_2 = J^{(1)}J^{(3)} - \tfrac{i}{2} J^{(2)a} D\upsilon \Gamma_a \Gamma^{11}\upsilon - i(d\varphi^{a'} + \tfrac{1}{2} J^{(1,3)}\Gamma^{a'}\upsilon)(J^{(1,3)}\Gamma_{a'}\Gamma^{11}\upsilon + \tfrac{i}{2} D\upsilon \Gamma_{a'}\Gamma^{11}\upsilon)$$

T-dualization:

$dx^m \to d\tilde{x}_m, \qquad d\theta^\alpha \to d\tilde{\theta}_\alpha, \qquad d\varphi^i \to d\tilde{\varphi}^i$ - ½ of Torus directions

$M_{d-1} \qquad \alpha = 1,\ldots,2(d-1) \qquad i = 1,\ldots,(5-d)$

Conclusion

- Proved T-duality of string actions in $AdS_d \times S^d \times T^{10-2d}$
  - without gauge fixing kappa-symmetry
  - in the presence of "non-susy" fermionic modes υ, which require the dualization of ½ of the Torus directions (this is in accordance with results of the T-dualization of these backgrounds from the supergravity perspective (see e.g. *Colgain `12* for *d=3*)

- Proved T-duality of supercoset sigma-models in $AdS_d \times S^d \times S^d \quad (d = 2,3)$ whose superisometries are governed by the exceptional groups

  $D(2,1;\alpha)$ for d=2 and $D(2,1;\alpha) \times D(2,1;\alpha)$ for d=3

  T-dualization should involve the (complexified) coordinates of one of S^d

- Future chalanges:
  - extend results to the full superstrings in $AdS_d \times S^d \times S^d \times T^{10-3d} \quad (d = 2,3)$
  - revise the T-dualization of the type IIA superstring on $AdS_4 \times CP^3$