Towards Holographic Higher spin Interactions

Dmitry Ponomarev

Ludwig-Maximilians University

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arXiv: 1412.0016 and work in progress
[with X. Bekaert, J. Erdmenger and C. Sleight]

Motivations

- Quest for quartic interactions

[Vasiliev'90][Metsaev'91][Taronna'11][Dempster,Tsulaia'12]

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- Locality in HS theories

  Clear definition of locality/non-locality for theories with infinitely many derivatives

$$\phi^2 \frac{1}{\Box}\phi^2 \qquad \text{vs} \qquad \phi^2 \frac{1}{\Box - \Lambda}\phi^2 \qquad \text{vs} \qquad \phi^2 \exp{(\Box)}\phi^2$$

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- Non-trivial tests of the holographic higher spin conjecture

[Sezgin,Sundell'02][Klebanov,Polyakov'02]

Interactions perturbatively

- **Nonlinearity at quartic order**

$$\delta_0 S_4 + \delta_1 S_3 + \delta_2 S_2 = 0$$

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- **Proliferation of different structures**

$$\varphi^3 \quad \to \quad \sum_{n,s} a_{n,s} \varphi\, \partial_{\mu_1} \ldots \partial_{\mu_s} \varphi\, \Box^n (\varphi\, \partial^{\mu_1} \ldots \partial^{\mu_s} \varphi)$$

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- Derivatives do not commute in AdS

Higher spin holography

Bulk: **Minimal Vasiliev's theory in 4d**

$$S = \int \sqrt{g} d^4x \nabla^{\mu_1} \varphi^{\mu_2 \dots \mu_{s+1}} \nabla_{\mu_1} \varphi_{\mu_2 \dots \mu_{s+1}} + \dots$$

$$\delta \varphi_{\mu_1 \dots \mu_s} = \nabla_{\mu_1} \xi_{\mu_2 \dots \mu_s} + \dots \qquad s = 0,\ 2,\ 4,\ \dots\ \infty$$

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Boundary: Free O(N) vector model in 3d

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$$J_{\mu_1 \dots \mu_s} = \phi^a \partial_{\mu_1} \dots \partial_{\mu_s} \phi_a + \dots, \qquad \partial^{\mu_1} J_{\mu_1 \dots \mu_s} = 0$$

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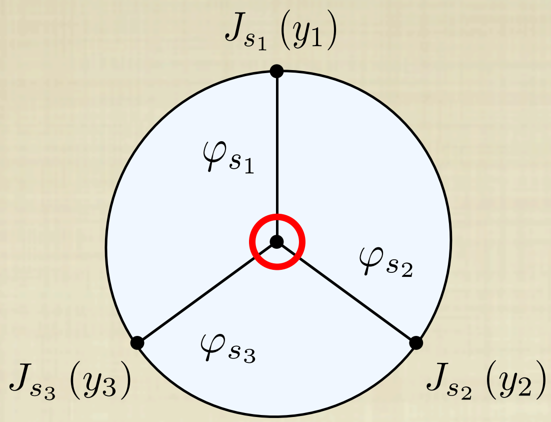
Duality: $\varphi_{\mu_1 \dots \mu_s} \quad \leftrightarrow \quad J_{\mu_1 \dots \mu_s}$

Cubic vertices



$$= \qquad \langle J_{s_1}(y_1) J_{s_2}(y_2) J_{s_3}(y_3) \rangle$$

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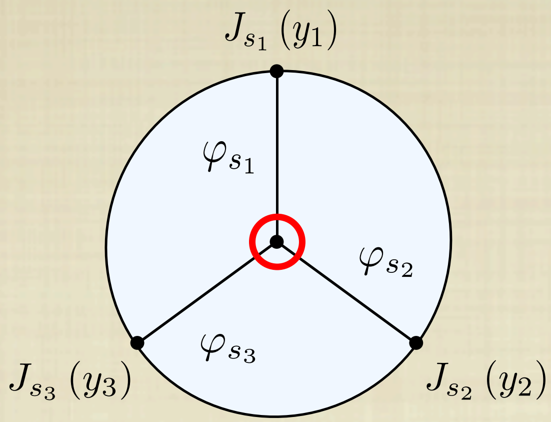

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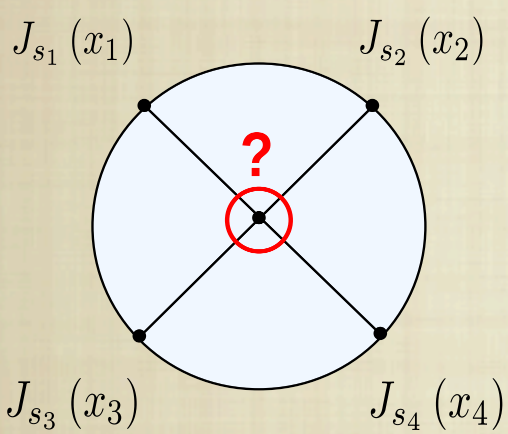
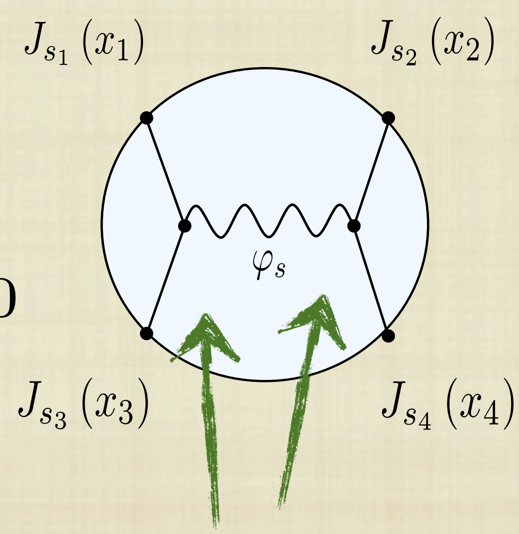


$$+ \sum_{s=0}^{\infty}$$

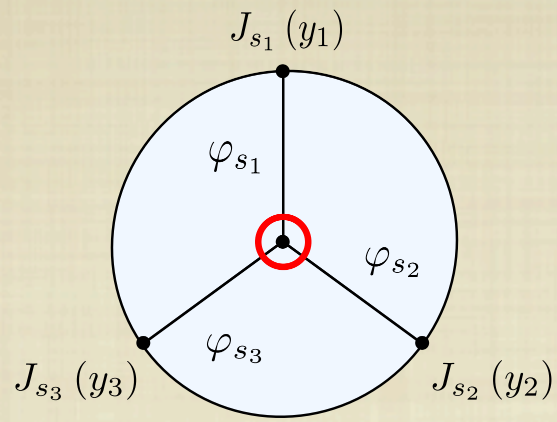


$+$ u- and t-channels

ALREADY KNOWN

$$= \quad \langle J_{s_1}(y_1) J_{s_2}(y_2) J_{s_3}(y_3) J_{s_4}(y_4) \rangle$$

Cubic vertices



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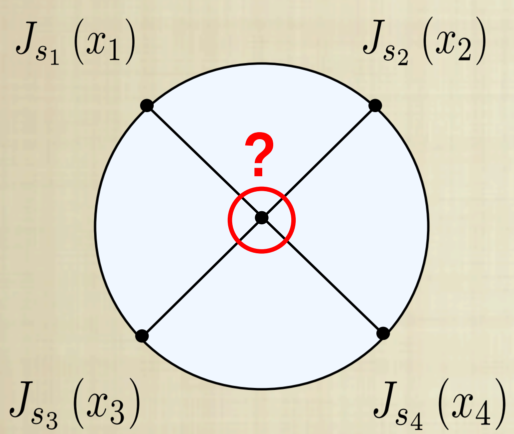

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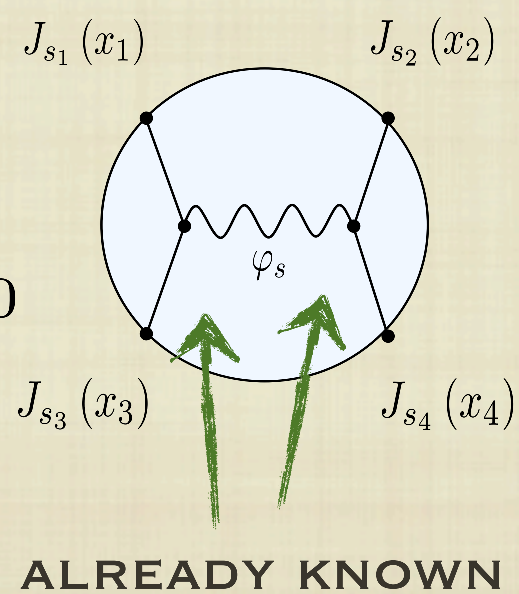

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TO START: QUARTIC VERTEX FOR SCALAR FIELDS

Ingredients

- 3-point functions and Witten diagrams
- bulk propagator
- bulk exchange
- bulk contact 4-point interaction
- 4-point CFT correlator
- putting together

3-pt functions

CFT:

$$\langle J^0(x_1) J^0(x_2) J^s_{\mu_1 \dots \mu_s}(x_3) \rangle = C_{00s} \frac{\left(\frac{x_{13}}{r_{13}} - \frac{x_{23}}{r_{23}} \right)_{\mu_1} \dots \left(\frac{x_{13}}{r_{13}} - \frac{x_{23}}{r_{23}} \right)_{\mu_s}}{r_{12}^{\frac{\Delta_1+\Delta_2-\Delta_3+s}{2}} r_{13}^{\frac{\Delta_3+\Delta_1-\Delta_2-s}{2}} r_{23}^{\frac{\Delta_3+\Delta_2-\Delta_1-s}{2}}}$$

$$J^0(x) =: \phi^a(x) \phi_a(x) : \qquad J^s_{\mu(s)}(x) =: \phi^a(x) \partial_{\mu_1} \dots \partial_{\mu_s} \phi_a(x) : + \dots$$

$$\Delta_1 = \Delta_2 = d - 2 \qquad \Delta_3 = d + s - 2$$

3-pt functions

CFT:

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$J^0(x) =: \phi^a(x)\phi_a(x) :$ $\qquad$ $J^s_{\mu(s)}(x) =: \phi^a(x)\partial_{\mu_1}\ldots\partial_{\mu_s}\phi_a(x) : + \ldots$

$\Delta_1 = \Delta_2 = d-2$ $\qquad$ $\Delta_3 = d+s-2$

Notations: $J^0 \quad \rightarrow \quad \mathcal{O}$ $\qquad$ $J^s \quad \rightarrow \quad \mathcal{J}^s$

$\Delta(\mathcal{O}) \quad \rightarrow \quad \Delta$

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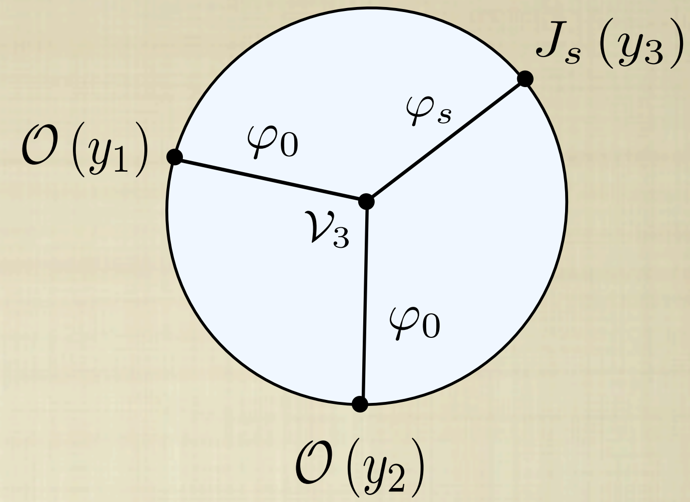
Doing Wick contractions

$$C_{00s} = \frac{2^{s/2+3/2}}{\sqrt{s!}\sqrt{N}}\frac{(d/2-1)_s}{\sqrt{(d+s-3)_s}}$$

[Diaz,Dorn'06]

3-pt Witten diagram

Bulk:



$$g_{00s}\int_{AdS} d^{d+1}x\sqrt{g}\varphi^{\mu_1\ldots\mu_s}J_{\mu_1\ldots\mu_s}$$

$$\sim 2^s g_{00s}\int_{AdS} d^{d+1}x\sqrt{g}\varphi^{\mu_1\ldots\mu_s}\varphi\nabla_{\mu_1}\ldots\nabla_{\mu_s}\varphi$$

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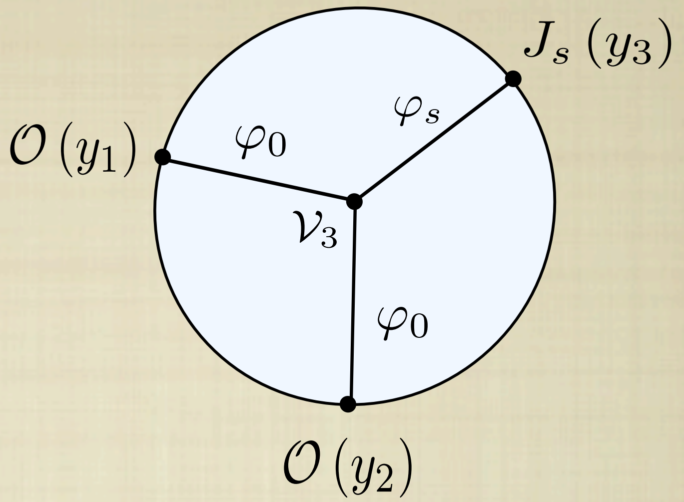

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Higher-spin bulk-to-boundary propagator

$$\Pi_s(X,W;P,Z)\sim\frac{(2(Z\cdot X)(P\cdot W)-2(W\cdot Z)(P\cdot X))^s}{(-2X\cdot P)^{\Delta_s+s}}-\text{traces}$$

[Mikhailov'02]

3-pt Witten diagram

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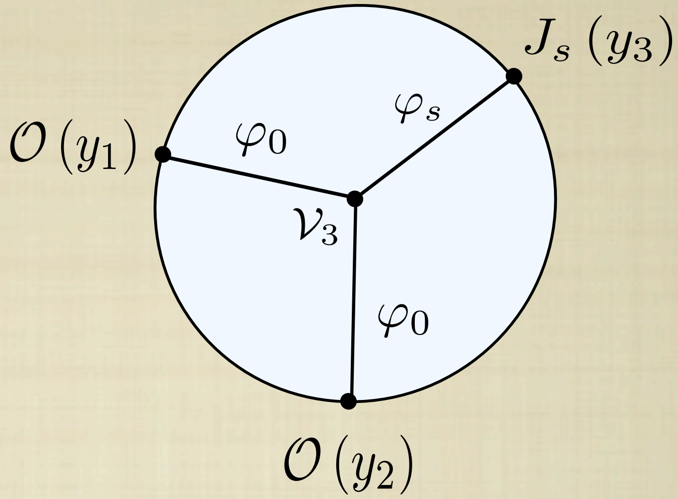

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Ambient formalism for AdS

$$\begin{aligned} AdS_{d+1} &\rightarrow \mathbb{R}^{d+2},\quad g=\text{diag}(+,+,-,\dots,-)\\ AdS_{d+1}\text{ bulk} &\rightarrow X^2=1\\ \text{boundary} &\rightarrow P^2=0,\quad P\sim\alpha P\\ W\text{ and }Z &\rightarrow \text{auxiliary vectors}\end{aligned}$$

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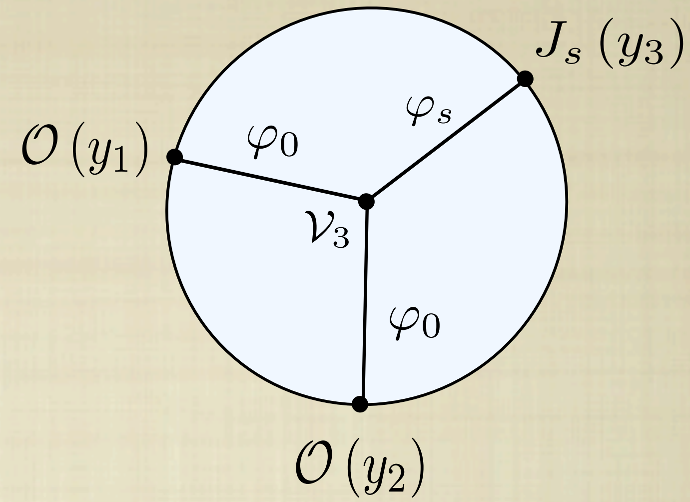

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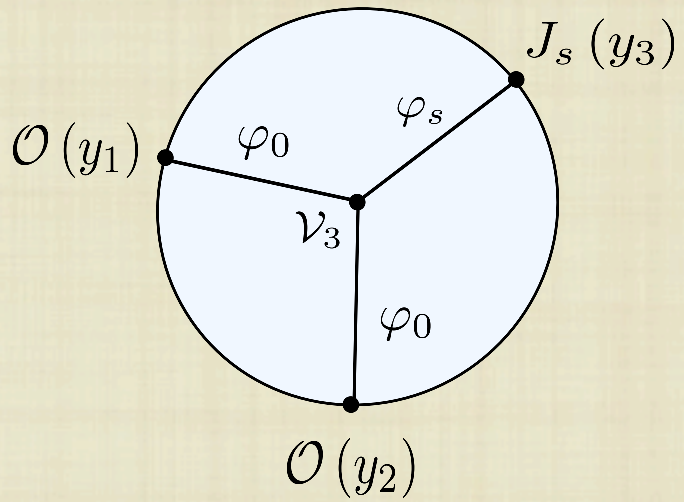
W\text{ and }Z &\to \text{auxiliary vectors}
\end{aligned}$$

Produces a well-known integral

[Paulos'11][Costa,Goncalves,Penedones'14]

Matching bulk & CFT

$$g_{00s} = \frac{2^{4-s/2}}{\sqrt{N}\Gamma(s)}$$



Bulk to bulk propagator

Warning: it is not just traceless-transverse

Scalar

$$G(v) \sim v^{-\Delta}{}_2F_1\left(\Delta, \Delta - \frac{d}{2} + \frac{1}{2}; 2\Delta - d + 1; \frac{1}{v}\right)$$

Graviton

[D'Hoker, Freedman, Mathur, Matusis, Rastelli'99]

Spin-3 & spin-4

[Leonhardt, Ruehl, Manvelyan'03]

Other results for hs

[Francia, Mourad, Sagnotti'08][Mkrtchyan'10]

Split representation

$$\Omega_\nu(X,X') \propto \int_{\partial AdS} dP \frac{1}{(-2P\cdot X)^{d/2+i\nu}} \frac{1}{(-2P\cdot X')^{d/2-i\nu}}$$

where $1/(-2P\cdot X)^{\Delta}$ is a bulk-to-boundary propagator with $m^2 = \Delta(\Delta - d)$

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$$(\Box + (d/2)^2 + \nu^2)\,\Omega_\nu = 0$$

$$\int d\nu\,\Omega_\nu(X,X') = \delta(X,X')$$

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Scalar bulk-to-bulk propagator

$$\Pi_\Delta(X,X') = \int \frac{d\nu}{\nu^2 + (\Delta - d/2)^2} \Omega_\nu(X,X')$$

Dobrev, Leonhardt, Ruehl, Manvelyan, Costa, Paulos, Penedones + other literature on Mellin amplitudes

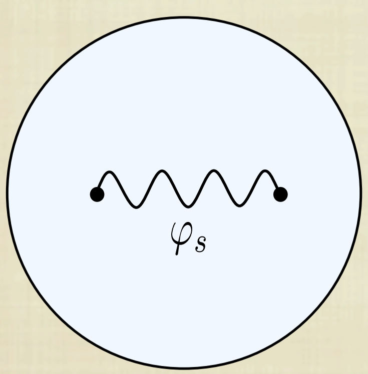
Split representation for HS

TT HS harmonic function

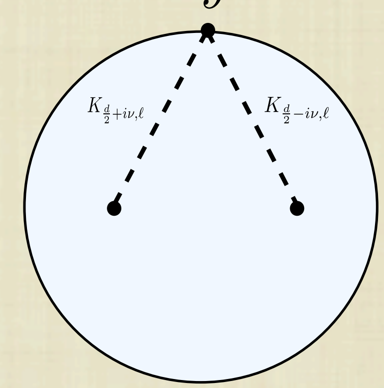
$$\Omega_{\nu,s}(X,X';W,W') \propto \int_{\partial AdS} dP\,\Pi_{d/2+i\nu}(X,P;W,\partial/\partial Z)\,\Pi_{d/2-i\nu}(X',P;W',Z)$$

Traceless basis

$$\left\{((W\cdot\nabla)(W'\cdot\nabla'))^{s-l}\Omega_{\nu,l}(X,X';W,W')\right\}_{\mathrm{Tr}\,=0}$$



$$= \sum_{\ell=0}^{\infty}\int_{-\infty}^{\infty} d\nu\, b_\ell(\nu) \int_{\partial\mathrm{AdS}} d^d y$$



[Costa,Goncalves,Penedones'14]

Complete HS massless propagator

Fronsdal equation with a source

$$\left(1 - 1/4u_1^2\partial_{u_1}\cdot\partial_{u_1}\right)\mathcal{F}_s(x_1,u_1,\nabla_1)\Pi_s(x_1,u_1,x_2,u_2) =$$
$$-\{\{(u_1\cdot u_2)^2\}\}\delta(x_1,x_2) + (u_2\cdot\nabla_2)\Lambda(x_1,u_1,x_2,u_2)$$

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$$\Pi_s = \sum_{j=0}^{[s/2]}\int d\nu g_{s,j}(\nu)(g_{AA})^j(g'_{AA})^j\Omega_{\nu,s-2j}$$

$$g_{s,0}(\nu) = \frac{1}{(h+s-2)^2+\nu^2}$$
$$g_{s,j}(\nu) = \frac{(1/2)_{j-1}}{2^{2j+3}\cdot j!}\frac{(s-2j+1)_{2j}}{(h+s-j)_j(h+s-j-3/2)_j}\times$$
$$\frac{(h/2+s/2-j+i\nu/2)_{j-1}(h/2+s/2-j-i\nu/2)_{j-1}}{(h/2+s/2-j+1/2+i\nu/2)_j(h/2+s/2-j+1/2-i\nu/2)_j}$$

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$$(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)}$$

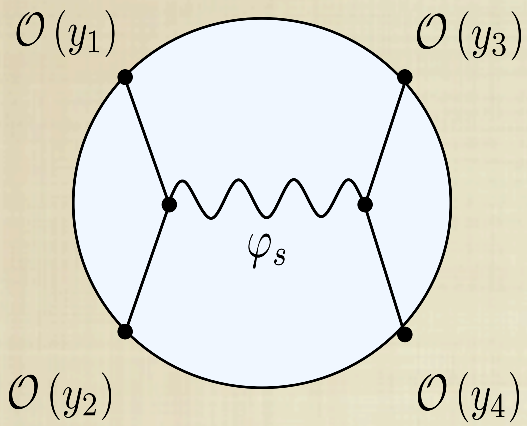
$$h = \frac{d}{2}$$

$$[(h+s-1)^2+\nu^2][(h+s-3)^2+\nu^2]\ldots$$

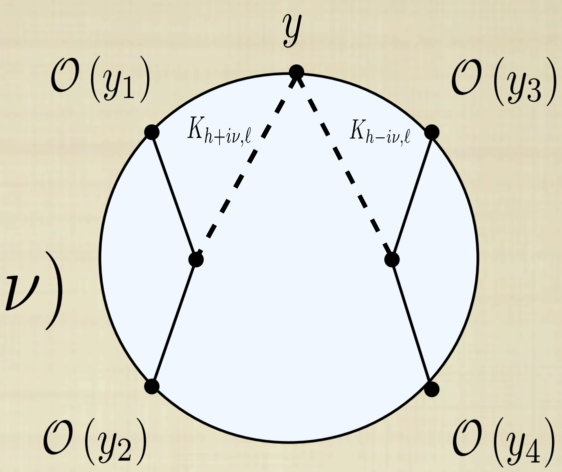
$$\Delta = d+s-1,\ d+s-3,\ \ldots$$

[Bekaert, Erdmenger, P., Sleight'14]

Exchanges. Split representation



$$= \sum_{\ell=0}^{s} \int_{\partial \mathrm{AdS}} d^d y \int_{-\infty}^{\infty} d\nu \;\; b_\ell^{s.ch}(\nu)$$



$$F_{\nu,s}(u,v) = \int_{\partial AdS} d^d y \langle \mathcal{O}_0(y_1)\mathcal{O}_0(y_2)\mathcal{O}_{h+i\nu,s}(y)\rangle\langle \mathcal{O}_{h-i\nu,s}(y)\mathcal{O}(y_3)\mathcal{O}(y_4)\rangle$$

$$F_{\nu,s}(u,v) = \kappa(\nu,s)G_{h+i\nu}(u,v) + \kappa(-\nu,s)G_{h-i\nu,s}(u,v)$$

Produces the conformal block decomposition!

[Costa,Goncalves,Penedones'14]

Refining the vertex

$$V_3 \neq \varphi^{\mu_1 \dots \mu_s} \varphi \nabla_{\mu_1} \dots \nabla_{\mu_s} \varphi$$

Consistency with gauge invariance

$$V_3 = \varphi^{\mu_1 \dots \mu_s} J_{\mu_1 \dots \mu_s}, \qquad \nabla^{\mu_1} J_{\mu_1 \dots \mu_s} = 0$$

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In flat space

$$J^{\mu_1 \dots \mu_s} = \varphi(x)(\overleftarrow{\partial}^{\mu} - \partial^{\mu})^s \varphi(x), \qquad \partial^{\mu_1} J_{\mu_1 \dots \mu_s} = 0$$

Refining the vertex

$$V_3 \neq \varphi^{\mu_1 \dots \mu_s} \varphi \nabla_{\mu_1} \dots \nabla_{\mu_s} \varphi$$

Consistency with gauge invariance

$$V_3 = \varphi^{\mu_1 \dots \mu_s} J_{\mu_1 \dots \mu_s}, \qquad \nabla^{\mu_1} J_{\mu_1 \dots \mu_s} = 0$$

In flat space

$$J^{\mu_1 \dots \mu_s} = \varphi(x)(\overleftarrow{\partial}^{\mu} - \partial^{\mu})^s \varphi(x), \qquad \partial^{\mu_1} J_{\mu_1 \dots \mu_s} = 0$$

In AdS

view J as an ambient space current

+

project to AdS hyperboloid

No compact form in intrinsic terms

[Bekaert,Meunier'10]

HS exchange in general dimensions

$$\mathcal{A}_s(y_1, y_2, y_3, y_4) = \sum_{k=0}^{[s/2]} \int d\nu \, b_{s-2k}(\nu) F_{\nu, s-2k}(u, v)$$

$$b_{s-2k}(\nu) = (g_{00s})^2 \frac{4^{s-2k} g_{s,k} \tau_{s,k}^2 \Gamma^2\left(\frac{3-2h-2(s-2k)}{2}\right)\Gamma^2(1-h-(s-2k))}{\pi^{3h+1} 2^{4h+6(s-2k)+3} \Gamma^2(2-2h-2(s-2k))\Gamma^4(\Delta+1-h)}$$

$$\tau_{s,k}(\nu) = \sum_{m=0}^{k} \frac{2^{2k} \cdot k!}{m!(k-m)!} (\Delta - h - k + m + 1/2)_{k-m} \times \left(\frac{h+s-2m+1+i\nu}{2}\right)_m \left(\frac{h+s-2m+1-i\nu}{2}\right)_m$$

[Bekaert, Erdmenger, P., Sleight'14]

HS exchange in general dimensions

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$$\times \left(\frac{h+s-2m+1+i\nu}{2}\right)_m \left(\frac{h+s-2m+1-i\nu}{2}\right)_m$$

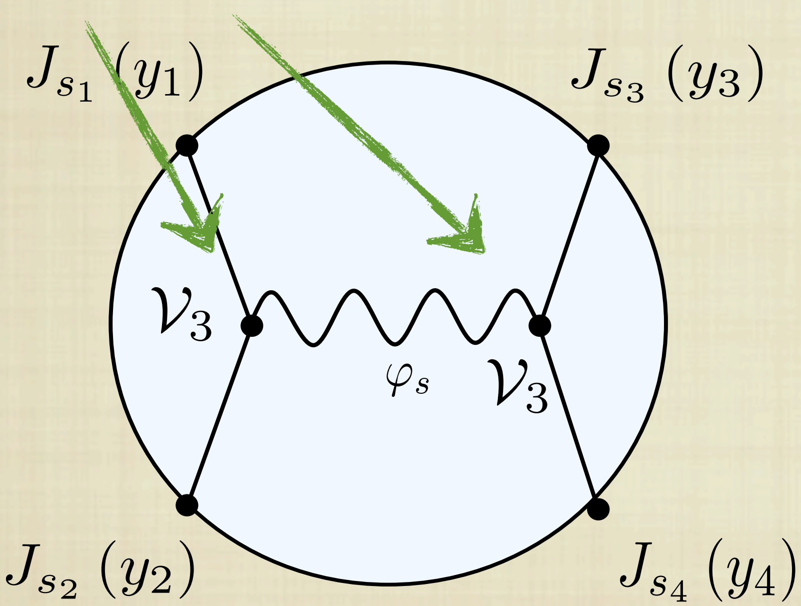
[Bekaert,Erdmenger,P.,Sleight'14]

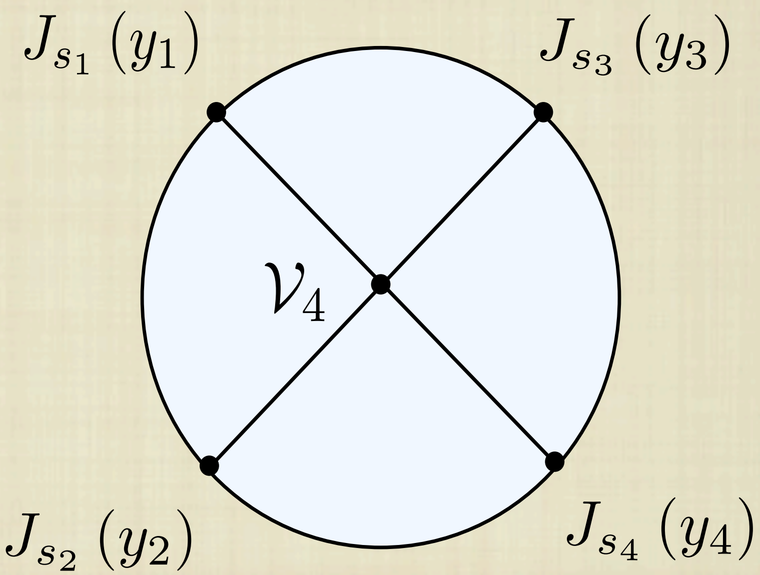
Improvements

Current improvements → **on-shell trivial vertices**

$$\mathcal{V}_3 \sim \varphi_0 \varphi_0 \Box \varphi_s$$

$$\Box \Pi_s(X, X') \sim \delta(X, X')$$





HS exchange in d=4

Simplification: currents are traceless

$$\mathcal{A}(P_1,P_2;P_3,P_4)$$

$$= \frac{g_{00s}^2}{(P_1 \cdot P_2)(P_3 \cdot P_4)} \int_{-\infty}^{+\infty} d\nu \, \frac{2^{-2i\nu+2s-3}\Gamma(i\nu+\frac{1}{2})\Gamma^2\left(\frac{1}{4}(2s+2i\nu+1)\right)}{\pi^{5/2}\Gamma(i\nu)(2i\nu+2s+1)}$$

$$\times \frac{\Gamma^2\left(\frac{1}{4}(2s-2i\nu+1)\right)}{(\nu^2+(s-\frac{1}{2})^2)} G_{h+i\nu,s}(u,v)$$

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$$\times \frac{\Gamma^2\left(\frac{1}{4}(2s-2i\nu+1)\right)}{(\nu^2+(s-\frac{1}{2})^2)} G_{h+i\nu,s}(u,v)$$

Closing the contour in the lower half plane we pick the poles at:

$$\nu = -i(2n+s+1/2) \quad \Rightarrow \quad \Delta_s = h + i\nu = 2\Delta + n + s = 2d + 2n + s - 4$$

Double-trace operators $\quad \mathcal{O}_{n,s} =: \Box^n(\phi^a\phi_a)\partial_{\mu_1}\dots\partial_{\mu_s}(\phi^b\phi_b) : + \dots$

$$\Delta = \Delta(\varphi_0) = \Delta(\phi^a\phi_a)$$

HS exchange in d=4

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$$\nu = -i(s-1/2) \quad \Rightarrow \quad \Delta_s = h+i\nu = \Delta+s = d+s-2$$

Single-trace operators (conserved currents) $\quad \mathcal{J}_s =: \phi^a\partial_{\mu_1}\dots\partial_{\mu_s}\phi_a : + \dots$

Quartic vertices

The basis: $\mathcal{V}_{n,s} = J_{\mu_1 \ldots \mu_s} \Box^n (J^{\mu_1 \ldots \mu_s}), \quad s = 2k, \quad k \geq n \geq 0$

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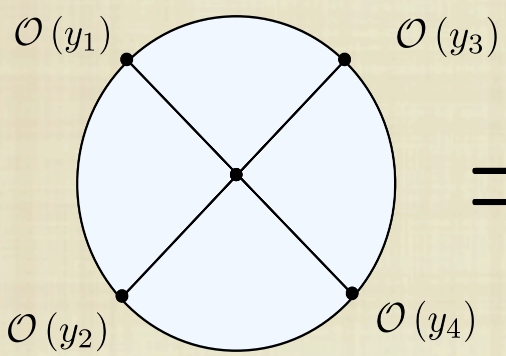
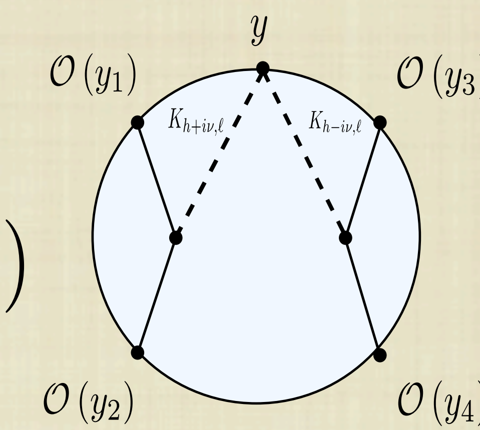
$$\mathcal{V} = J(x)\Box J(x) \;\longrightarrow\; J(x)\delta(x,x')\Box J(x') \;\longrightarrow\; J(x)\sum_s \int d\nu\,\Omega_{\nu,s}(x,x')\Box J(x')$$



$$= \sum_{\ell=0}^{s} \int_{\partial \mathrm{AdS}} d^d y \int_{-\infty}^{\infty} d\nu \; b_\ell^c(\nu)$$



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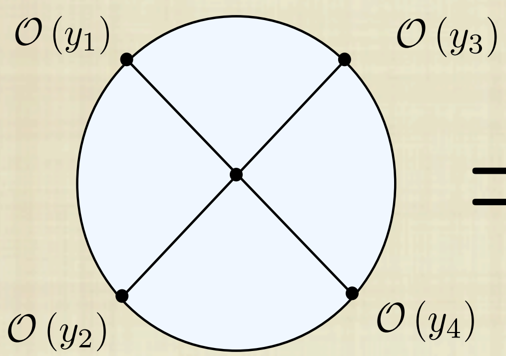

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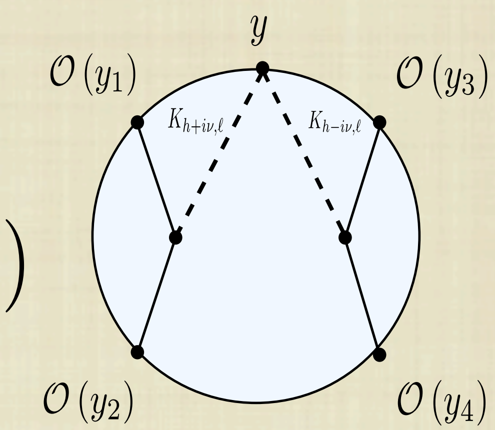

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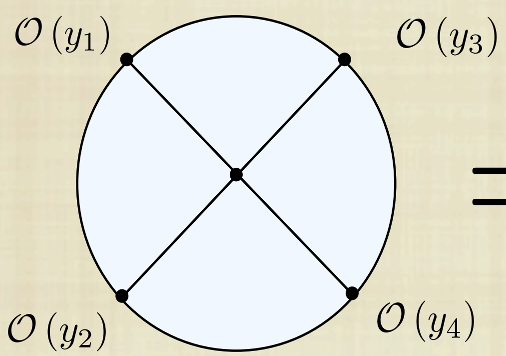

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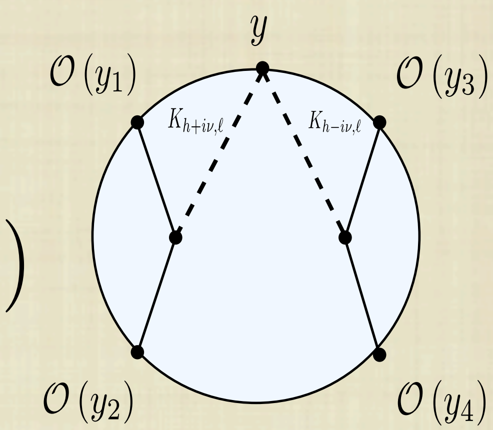

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double trace poles

Quartic vertices

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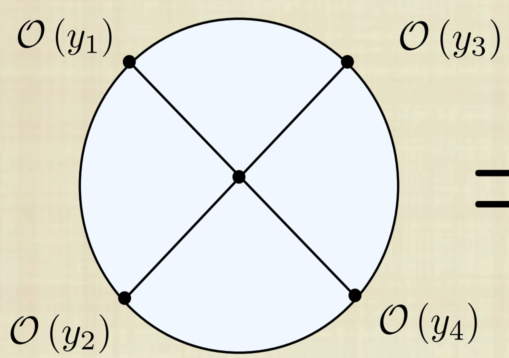

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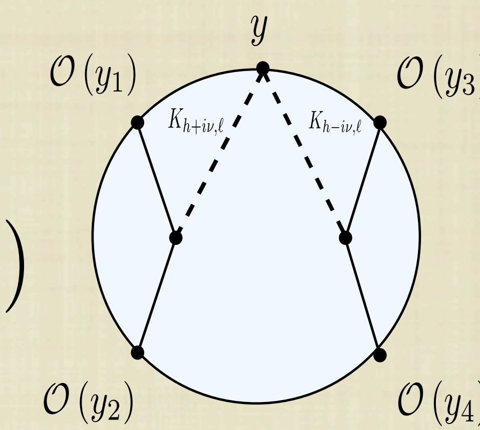

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The only difference with exchanges

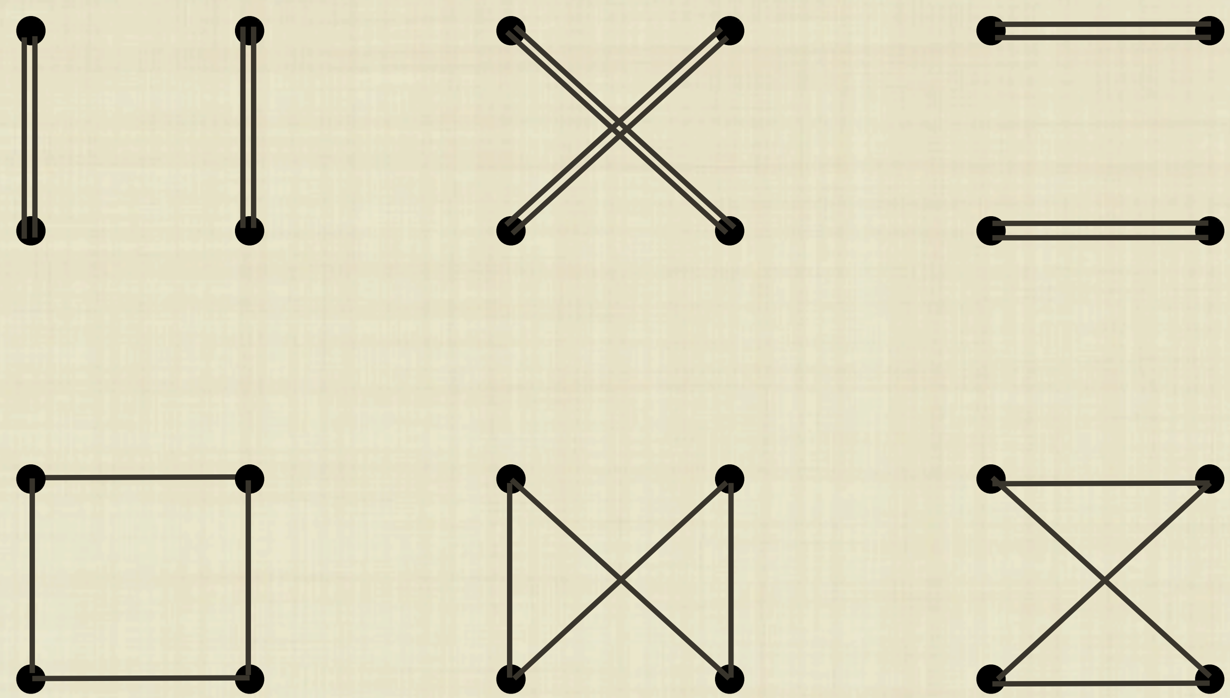
double trace poles

CFT

4-pt function via Wick contractions

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle \quad =$$

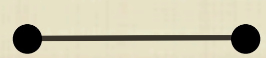
=



$\sim O(N^0)$

$\sim O(1/N)$

$\mathcal{O} = \phi^a \phi_a$



$$= \langle \phi(x)\phi(y)\rangle = \frac{1}{|x-y|^{2\delta}}$$

OPE:

$$\mathcal{O}\mathcal{O} \sim 1 + \sum_s \mathcal{J}_s + \sum_{n,s} \mathcal{O}_{n,s}$$

CFT

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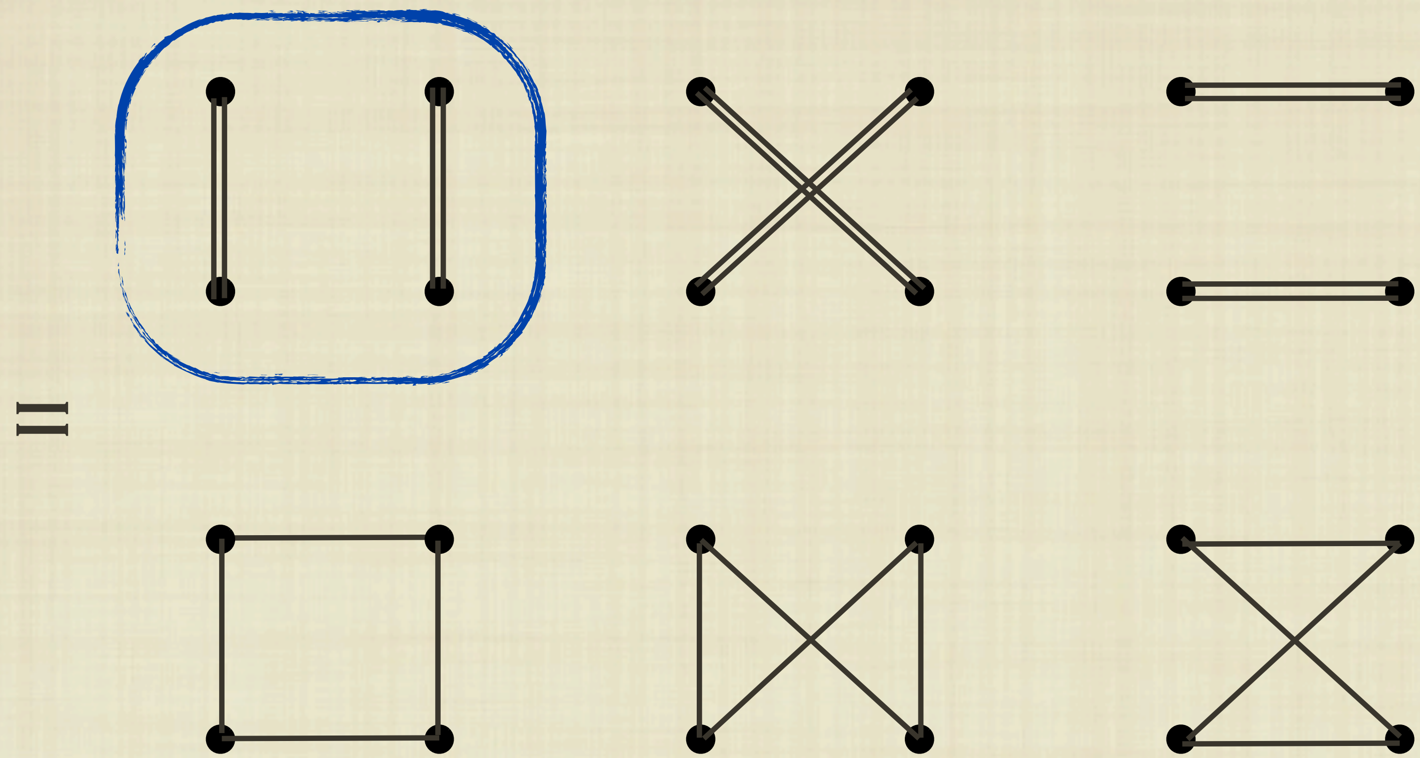

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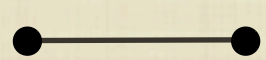

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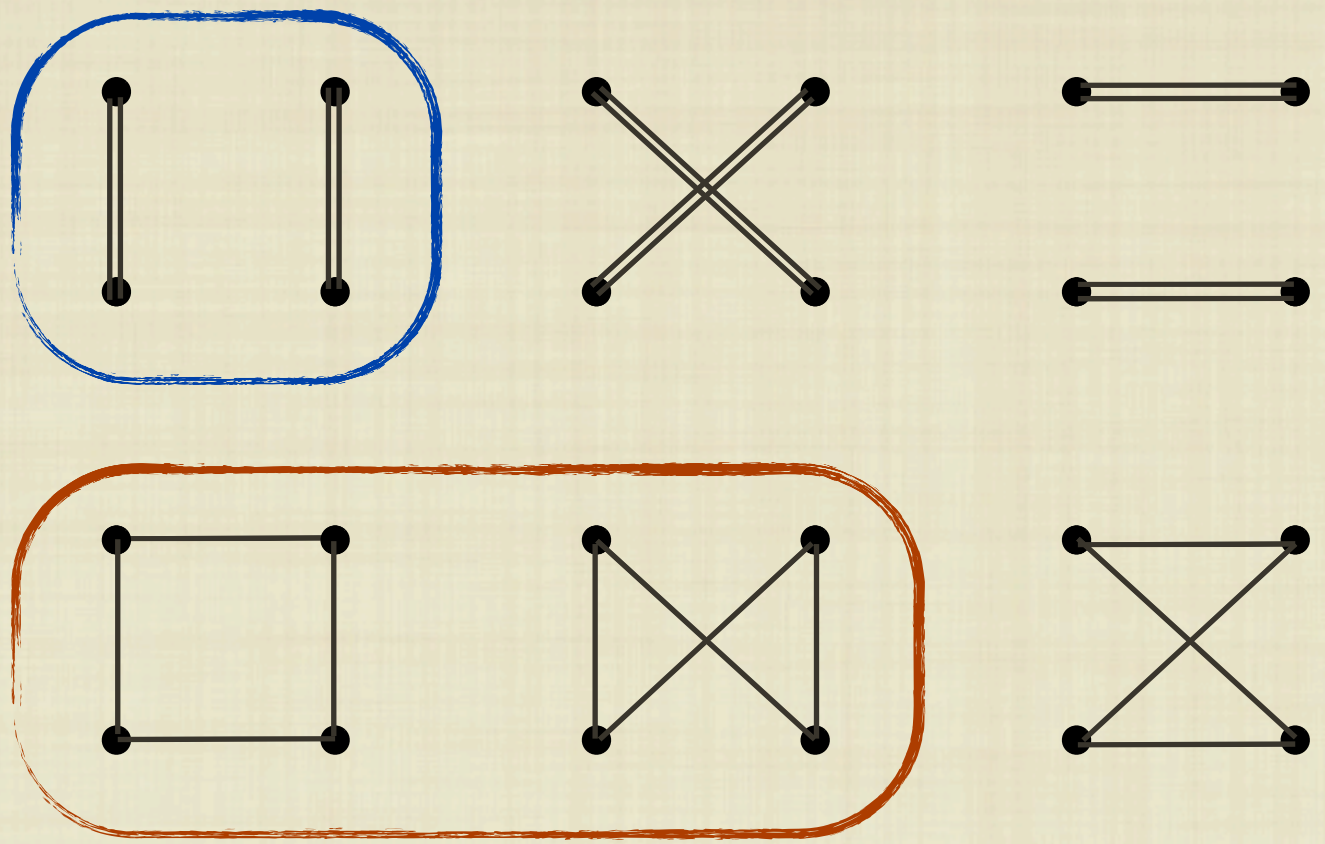
$=$



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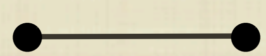

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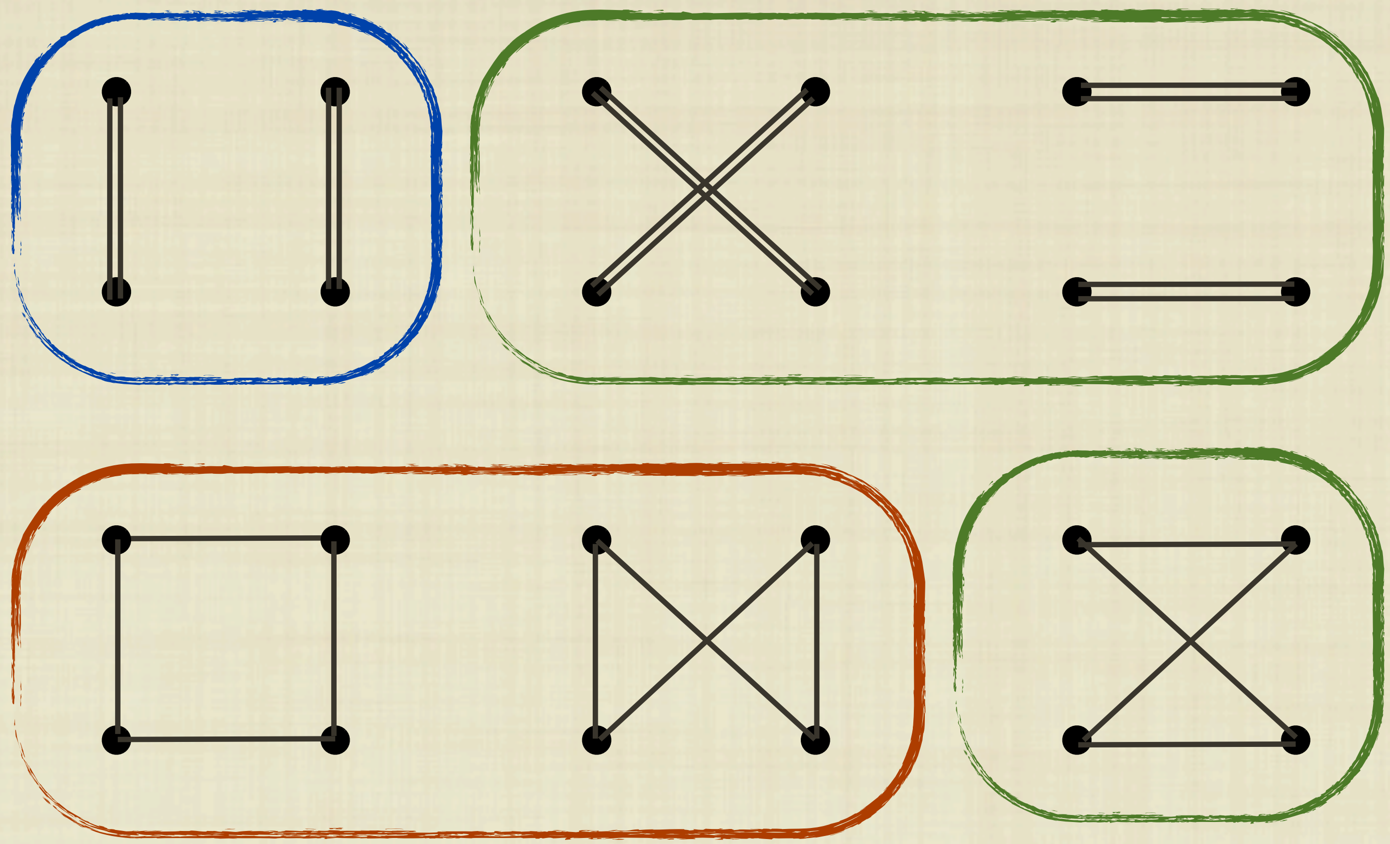

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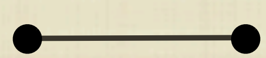 $= \langle \phi(x)\phi(y)\rangle = \dfrac{1}{|x-y|^{2\delta}}$

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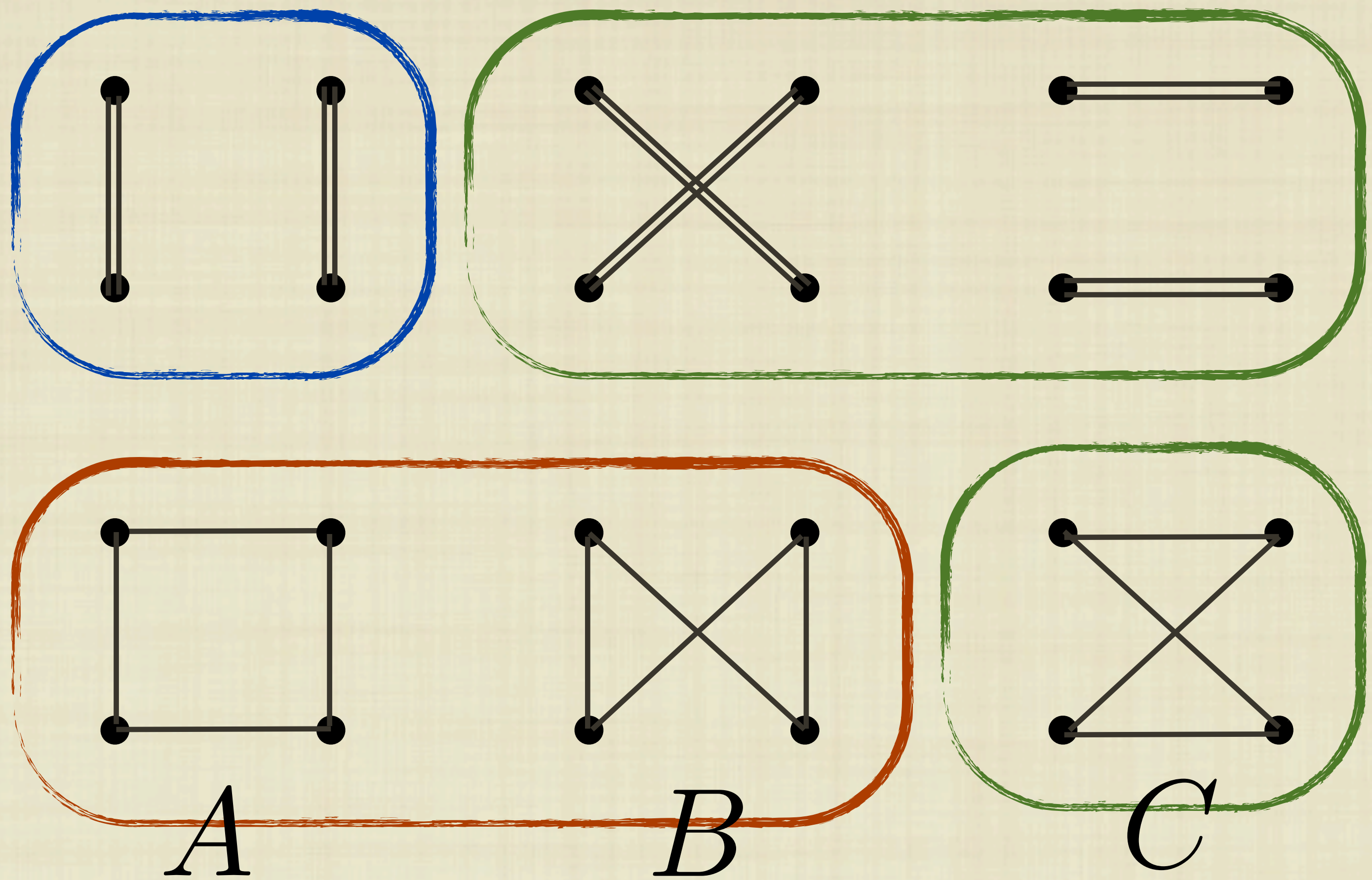

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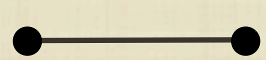

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Conformal block decomposition

Single trace part is known

[Diaz,Dorn'06]

Double trace part?

Is known in d=4 using the explicit formula for conformal blocks

[Dolan,Osborn'00]

We need d=3!

Methods in general d

- Directly through 3-pt & 2-pt functions
- Employing explicit expressions for conformal blocks
- Conglomeration

[Fitzpatrick,Kaplan'11]

Direct method

$$\mathcal{O}_{n,s} \to \mathcal{O}\partial_{\mu_1}\dots\partial_{\mu_s}\Box^n\mathcal{O} + \dots$$

?

Direct method

$$\mathcal{O}_{n,s} \to \mathcal{O}\partial_{\mu_1} \ldots \partial_{\mu_s} \Box^n \mathcal{O} + \ldots$$

?

Problem:

$$K_\mu \mathcal{O}_1 = 0, \quad K_\mu \mathcal{O}_2 = 0, \quad \Delta(\mathcal{O}_1) = \Delta_1, \quad \Delta(\mathcal{O}_2) = \Delta_2$$

Find

$$\mathcal{O}_{n,s} \to \sum_{s_1, b_1, b_2} a_{s,n}(s_1, s_2; b_1, b_2, b_{12}) \partial_{a(s_1)} \Box^{b_1} \partial^{m(b_{12})} \mathcal{O}_1 \partial_{a(s_2)} \Box^{b_2} \partial_{m(b_{12})} \mathcal{O}_2$$

such that $\quad K_\mu \mathcal{O}_{n,s} = 0$

Direct method

$$\mathcal{O}_{n,s} \to \mathcal{O}\partial_{\mu_1}\dots\partial_{\mu_s}\Box^n\mathcal{O} + \dots$$?

Problem:

$$K_\mu\mathcal{O}_1 = 0, \quad K_\mu\mathcal{O}_2 = 0, \quad \Delta(\mathcal{O}_1) = \Delta_1, \quad \Delta(\mathcal{O}_2) = \Delta_2$$

Find

$$\mathcal{O}_{n,s} \to \sum_{s_1,b_1,b_2} a_{s,n}(s_1,s_2;b_1,b_2,b_{12})\partial_{a(s_1)}\Box^{b_1}\partial^{m(b_{12})}\mathcal{O}_1\partial_{a(s_2)}\Box^{b_2}\partial_{m(b_{12})}\mathcal{O}_2$$

such that $K_\mu\mathcal{O}_{n,s} = 0$

$$a_{s,n}(s_1,s_2;b_1,b_2,b_{12}) = \frac{s!}{s_1!s_2!}\frac{(\Delta_1+s_1+2b_1+b_{12})_{s_2}}{(\Delta_2+2b_2+b_{12})_{s_2}(\Delta_1+s_1+n)_{b_1-b_2}(\Delta_2+s_2+n)_{b_2-b_1}}$$
$$\times\frac{(-1)^{s_2+b_1+b_2}n!}{2^{b_1+b_2}b_1!b_2!(n-b_1-b_2)!}\frac{(\Delta_1+s+n)_{b_1}(\Delta_2+s+n-b_1)_{b_2}}{(\Delta_1+1-h)_{b_1}(\Delta_2+1-h)_{b_2}}$$
$$\times\sum_{k=0}^{b_2}\frac{b_2!}{k!(b_2-k)!}\frac{(b_1-k+1)_k(\Delta_1+b_1-h-k+1)_k}{(\Delta_2+s+n-b_1)_k(\Delta_1+s+n+b_1-k)_k}.$$

Agrees with partial results in the literature

[Mikhailov'02][Penedones'10][Fitzpatrick,Kaplan'11]

OPE coefficients

From $\langle \mathcal{O}\mathcal{O}\mathcal{O}_{n,s}\rangle$ and $\langle \mathcal{O}_{n,s}\mathcal{O}_{n,s}\rangle$

$$C^2_{\mathcal{O}\mathcal{O}\mathcal{O}_{n,s}} = \left(1 + \frac{4}{N}(-1)^n \frac{\Gamma(s)}{\Gamma(\frac{s}{2})} \frac{\left(\frac{\Delta}{2}\right)_{n+\frac{s}{2}}}{\left(\frac{\Delta+1}{2}\right)_{\frac{s}{2}} (\Delta)_{n+\frac{s}{2}}}\right)$$
$$\times \frac{[(-1)^s + 1]\, 2^s \left(\frac{\Delta}{2}\right)_n^2 (\Delta)^2_{s+n}}{s!n! \left(s + \frac{d}{2}\right)_n (2\Delta + n - d + 1)_n (2\Delta + 2n + s - 1)_s (2\Delta + n + s - \frac{d}{2})_n}$$

Agrees with

$d = 4$ result

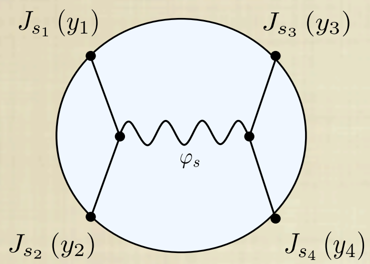
[Dolan,Osborn'00]

$O(N^0)$ part

[Fitzpatrick,Kaplan'11]

Combining the results

$$\sum_{s=0}^{\infty}$$



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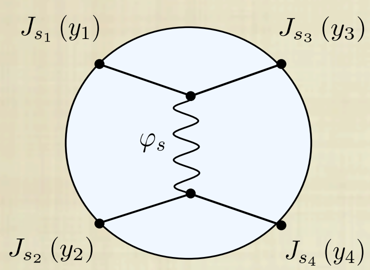

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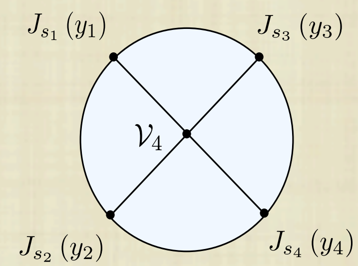
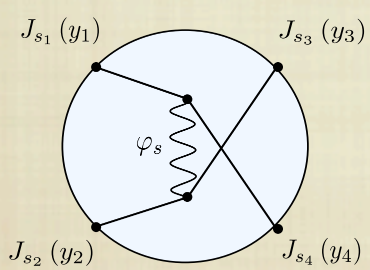


$$= \langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O} \rangle_{conn}$$

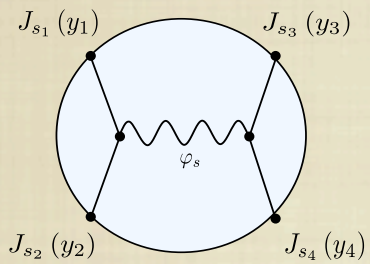
$$\sum_{s=0}^{\infty}$$



Combining the results

$G^{\mathcal{O}_{n,s}}_{12|34}$

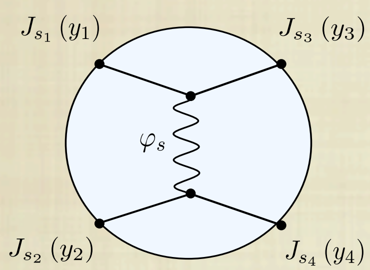
$\sum_{s=0}^{\infty}$



$G^{\mathcal{J}_s}_{12|34}$

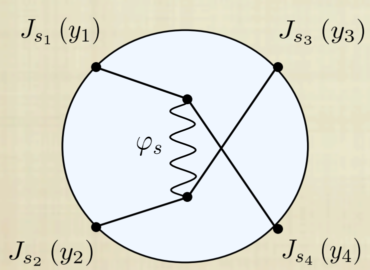
$G^{\mathcal{O}_{n,s}}_{13|24}$

$\sum_{s=0}^{\infty}$



$G^{\mathcal{J}_s}_{13|24}$

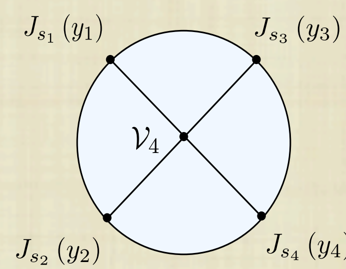
$\sum_{s=0}^{\infty}$



$G^{\mathcal{O}_{n,s}}_{14|23}$

$G^{\mathcal{J}_s}_{14|23}$

+



$G^{\mathcal{O}_{n,s}}_{any}$

=

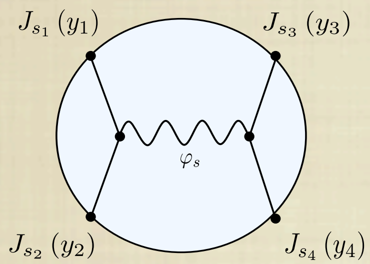
$G^{\mathcal{J}_s}_{any}$

$\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O} \rangle_{conn}$

$G^{\mathcal{O}_{n,s}}_{any}$

Simplified problem

$$\sum_{s=0}^{\infty}$$



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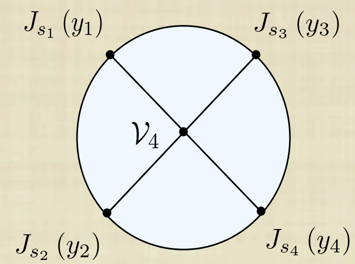
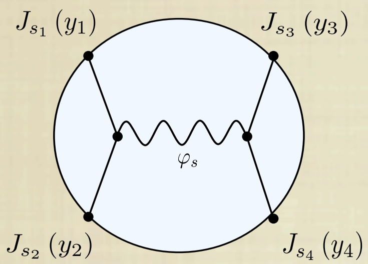


$$= \text{“} \, \frac{1}{3}\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O}\rangle \, \text{”}$$

$$\text{“} \, \frac{1}{3}\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O}\rangle \, \text{”} + \text{cross terms} = \langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O}\rangle$$

Simplified problem

$$\sum_{s=0}^{\infty}$$



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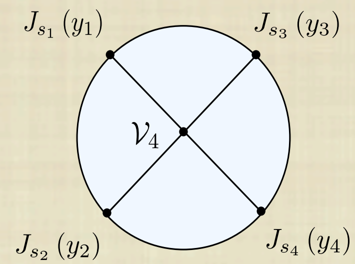

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Examples:

$$\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O}\rangle = A + B + C$$

$$\text{“}\ \frac{1}{3}\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O}\rangle\ \text{”} = A$$

$$\text{“}\ \frac{1}{3}\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O}\rangle\ \text{”} = \frac{1}{2}(A + B)$$

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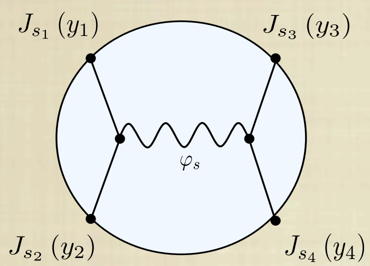
Simplified problem

$G^{\mathcal{J}_s}$ $\quad G^{\mathcal{O}_{n,s}}$ $\quad G^{\mathcal{O}_{n,s}}$ $\quad G^{\mathcal{J}_s}$

$$\sum_{s=0}^{\infty}$$



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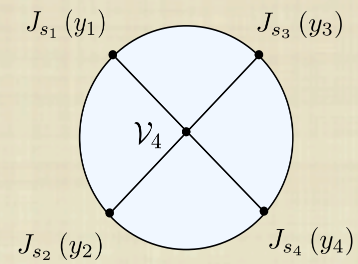

$$= \text{“}\,\frac{1}{3}\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O}\rangle\,\text{”}$$

$G^{\mathcal{O}_{n,s}}$

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Examples:

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$$\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O}\rangle = A + B + C$$

$G^{\mathcal{J}_s}$

$$\text{“}\,\frac{1}{3}\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O}\rangle\,\text{”} = A$$

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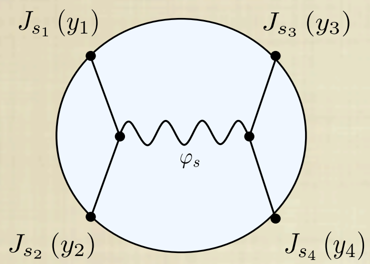

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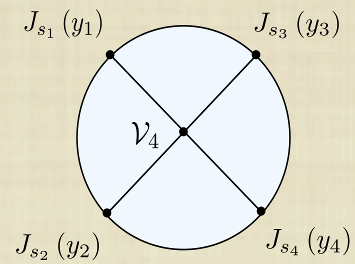

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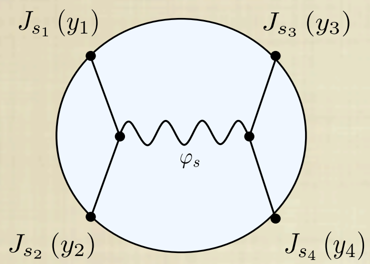
To balance single trace contributions

Simplified problem

$$\mathcal{V}_4 = \sum_{n,s} a_{n,s} J_{\mu_1 \ldots \mu_s} \Box^n J^{\mu_1 \ldots \mu_s}$$

$$\sum_{s=0}^{\infty}$$



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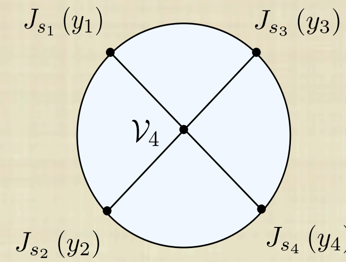

$$= \text{“} \frac{1}{3} \langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O} \rangle \text{”}$$

Use representation $\displaystyle \int d\nu \, \alpha(\nu) \, G_{h+i\nu,s}(u,v)$

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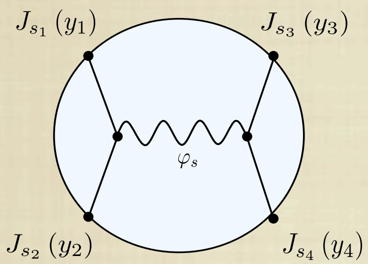

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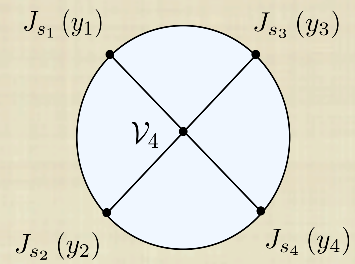

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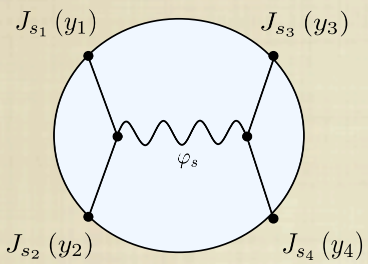

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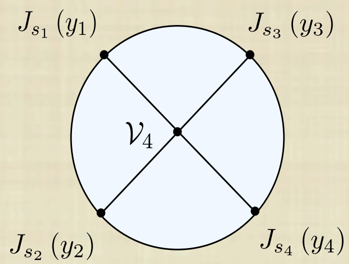

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$$f_s(\nu)$$

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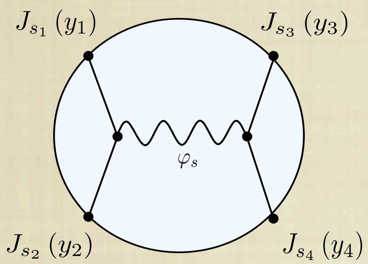
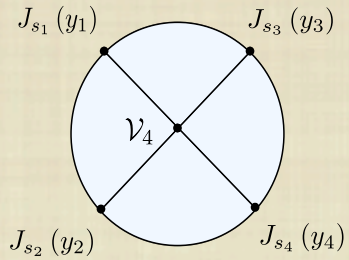
$\sum_{s=0}^{\infty}$ 

$+$ 

$=$ " $\frac{1}{3}\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O}\rangle$ "

Use representation $\int d\nu\, \alpha(\nu)\, G_{h+i\nu,s}(u,v)$?

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$$\sum_{n=0}^{\infty} a_{n,s}(\nu^2+s+\frac{9}{4})^n \Gamma^2\left(\frac{1}{4}(2s-2i\nu+1)\right)(\dots)$$

$f_s(\nu)$

Solve for $f_s(\nu)$. Expand in Taylor series at $\nu^2 = -s - 9/4$ to get $a_{n,s}$

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$$\text{“}\frac{1}{3}\langle\mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O}\rangle\text{”} = A + B - C \quad \Rightarrow \quad \text{no poles for } f_s(\nu)$$

Locality vs ∂^∞

In flat space:

Mandelstam variables

$$\mathcal{V}_4 \quad \to \quad \mathcal{A}(s,t,u)$$

Poles in $\mathcal{A}$ $\leftrightarrow$ exchanges

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Refined version: the amplitude is an entire function

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Poles in $\mathcal{A}$ $\leftrightarrow$ exchanges

Locality = Absence of poles in the corresponding amplitude

Refined version: the amplitude is an entire function

$$\phi^2 \frac{1}{\Box} \phi^2 \qquad \phi^2 \frac{1}{\Box - \Lambda} \phi^2 \qquad \phi^2 \exp\left(\Box\right) \phi^2$$

non-local — non-local — local

Leaves room for local infinite derivative interactions

Conclusion

- Preliminary results for holographic quartic interactions
  - HS propagators
  - Currents in AdS, improvements
  - Bulk amplitudes for 4pt exchanges and contact diagrams
  - OPE on the CFT side

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Outlook

- Exchanges in a crossed channel, crossing symmetry
- Vertices for fields with spin, higher vertices
- Other dualities. Bulk dual of QCD?
- Other techniques. Mellin amplitudes?