Black Hole Charge from the Unfolded Dynamics Approach

(work to appear, with V.E. Didenko and M.A. Vasiliev)

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03.06.2015

Outline

Cartan formulation of AdS_4

- AdS_4 vacuum: $\begin{cases} d\Omega^{ab} + \Omega^{ac} \wedge \Omega_c{}^b & = \lambda^2 h^a \wedge h^b; \\ dh^a + \Omega^{ac} \wedge h_c & = 0. \end{cases}$ $g^{AdS}_{\underline{mn}} = h^a_{\underline{m}} h^b_{\underline{n}} \eta_{ab}.$
- Vector-spinor dictionary:

$$V_a = \frac{1}{2} \left(\bar{\sigma}_a\right)^{\dot{\alpha}\alpha} V_{\alpha\dot{\alpha}}$$
$$A_{[ab]} \sim A_{\alpha\alpha} \oplus \bar{A}_{\dot{\alpha}\dot{\alpha}}$$
$$C_{[ab],[cd]} \sim C_{\alpha(4)} \oplus \bar{C}_{\dot{\alpha}(4)}.$$

- $o(3,2) \approx sp(4)$, $\Omega^{AB} := \begin{pmatrix} \Omega^{\alpha\beta} & -\lambda h^{\alpha\dot{\beta}} \\ -\lambda h^{\beta\dot{\alpha}} & \bar{\Omega}^{\dot{\alpha}\dot{\beta}} \end{pmatrix} \Longrightarrow$

$$d\Omega^{AB} + \frac{1}{2}\Omega^A{}_C \Omega^{CB} = 0.$$

Killing vectors

- For Killing vector V_m, Killing equation

$$D_{0m}V_n + D_{0n}V_m = 0$$

(D_0 - AdS_4-derivative) means

$$D_{0m}V_n = \varkappa_{mn}$$

where $\varkappa_{mn} = -\varkappa_{nm}$ - Papapetrou field.

- Introduce $sp(4)$-tensor $K_{AB} := \begin{pmatrix} \lambda^{-1}\varkappa_{\alpha\beta} & V_{\alpha\dot\beta} \\ V_{\beta\dot\alpha} & \lambda^{-1}\bar\varkappa_{\dot\alpha\dot\beta} \end{pmatrix}$

$$D_0 K_{AB} = 0.$$

- Any solution generates global symmetry of AdS_4 for unfolded system $D_0\Omega = 0$, $df + G = 0$:

$$\delta\Omega^{AB} = D_0 K_{AB}, \quad \delta f \sim K^{AB}\frac{\partial G}{\partial \Omega^{AB}}.$$

AdS Black Hole

- BH metric in Kerr-Schild form [Carter, 1968]:

$$g_{mn} = g^{AdS}_{mn} + \frac{2M}{r} k_m k_n .$$

- Kerr-Schild vectors k_m: $k_m k^m = 0$, $k_n D_0^n k_m = k_n \mathcal{D}^n k_m = 0$, $\frac{1}{r} = -\frac{1}{2}\mathcal{D}^m k_m = -\frac{1}{2} D_0^m k_m$.
- HS generalization [Didenko, Matveev, Vasiliev, 2008]: $\phi_{m_1 \dots m_s} = \frac{2M}{r} k_{m_1} \dots k_{m_s}$ obeys free spin-s equation in AdS background.
- $k_m = k_m(\varkappa_{ab})$ - generic AdS_4 BH is completely determined by global symmetry K_{AB} of empty AdS_4 [Didenko, Matveev, Vasiliev, 2008, 2009]
- BH Weyl tensor $C_{\alpha(4)} \sim M \varkappa_{\alpha\alpha} \varkappa_{\alpha\alpha}$.

Lagrangian extension

- HS fields: 0-form $B(Z,Y,\mathcal{K}|x)$,
  1-form $\mathcal{W} = d_x + dx^n W_n(Z,Y,\mathcal{K}|x) + dZ^A S_A(Z,Y,\mathcal{K}|x)$.
- Star product:

$$f * g = \frac{1}{(2\pi)^4}\int d^4U d^4V e^{iU_A V^A} f(Z+U;Y+U)\, g(Z-V;Y+V)$$

- Nonlinear HS equations [Vasiliev, 1990]:

$$\mathcal{W} * \mathcal{W} = -i\left(dZ_A dZ^A + \frac{\eta}{2} dz_\alpha dz^\alpha B * k * \upsilon + \frac{\bar\eta}{2} d\bar z_{\dot\alpha} d\bar z^{\dot\alpha} B * \bar k * \bar\upsilon\right)$$

$$[\mathcal{W}, B]_* = 0$$

- Lagrangian extension [Vasiliev, 2015]:

$$\mathcal{W} * \mathcal{W} = L - i\left(dZ_A dZ^A + \frac{\eta}{2} dz_\alpha dz^\alpha B * k * \upsilon + \frac{\bar\eta}{2} d\bar z_{\dot\alpha} d\bar z^{\dot\alpha} B * \bar k * \bar\upsilon\right)$$

- In our case: $L(x, dx)$ is a space-time 2-form, closed by virtue of these equations; $\delta L = d\chi$; L is assumed to support BH charge via integration over space infinity.

Linear Higher Spins

- AdS_4 vacuum solution: $W_0 = \Omega_{AB} Y^A Y^B$, $B_0 = 0$, $S_0 = Z_A dZ^A$
- Linear order: $B_1 (Z, Y, \mathcal{K}|x) = C(Y, \mathcal{K}|x)$,

$$\begin{cases} D_0\omega(Y|x) = \eta \bar{H}^{\dot\alpha\dot\alpha} \frac{\partial^2}{\partial \bar{y}^{\dot\alpha} \partial \bar{y}^{\dot\alpha}} C(0, \bar{y}|x) + \bar{\eta} H^{\alpha\alpha} \frac{\partial^2}{\partial y^{\alpha} \partial y^{\alpha}} C(y, 0, |x) + L \\ \widetilde{D}_0 C \equiv dC + W_0 * C - C * \widetilde{W}_0 = 0 \end{cases}$$

where $H^{\alpha\alpha} = h^{\alpha}{}_{\dot\alpha} h^{\alpha\dot\alpha}$ and $\widetilde{f}(y, \bar{y}) = f(-y, \bar{y})$.

HS BH in linear analysis

- If $D_0\epsilon(Y|x)=0$, then $\widetilde{D}_0\left(\epsilon*\delta^2(y)\right)=0$.
- HS BH Weyl tensors should be constructed from Papapetrou field of empty space (generalizing $s=2$ case), representing linear corrections in M: $D_0\left(K_{AB}Y^AY^B\right)=0\Longrightarrow$

$$
\begin{aligned}
C(Y|x) &= Mexp\left\{\frac{1}{2}K_{AB}Y^AY^B\right\}*\delta(y)= \\
&= \frac{M}{r}exp\left\{\frac{1}{2r^2}\varkappa_{\alpha\beta}y^\alpha y^\beta+\frac{1}{2r^2}\bar{\varkappa}_{\dot\alpha\dot\beta}\bar{y}^{\dot\alpha}\bar{y}^{\dot\beta}-\frac{1}{r^2}\varkappa_{\alpha\gamma}v^{\gamma}{}_{\dot\alpha}y^\alpha\bar{y}^{\dot\alpha}\right\}
\end{aligned}
$$

where $r^2=-\frac{1}{2}\varkappa_{\alpha\beta}\varkappa^{\alpha\beta}$.

- HS BH Weyl tensors [Didenko, Vasiliev, 2009]:
$C_{\alpha(2s)}=\frac{M}{s!2^sr}\left(\frac{\varkappa_{\alpha\alpha}}{r^2}\right)^s$, $\bar{C}_{\dot\alpha(2s)}=\frac{M}{s!2^sr}\left(\frac{\bar\varkappa_{\dot\alpha\dot\alpha}}{r^2}\right)^s$

Lagrangian 2-form

- L is completely determined by spin-1 sector, with C bilinear on Y and Y-independent ω:

$$d\omega(0|x) = 2\eta \bar{H}^{\dot\alpha\dot\alpha} \bar{C}_{\dot\alpha\dot\alpha}(x) + 2\bar\eta H^{\alpha\alpha} C_{\alpha\alpha}(x) + L.$$

- $\omega(0|x)$ can be shifted to zero by gauge transformation of L, then

$$L = -2\left(\eta \bar{H}^{\dot\alpha\dot\alpha} \bar{C}_{\dot\alpha\dot\alpha}(x) + \bar\eta H^{\alpha\alpha} C_{\alpha\alpha}(x)\right) = -M\left(cos\varphi\mathcal{F} + sin\varphi\check{\mathcal{F}}\right)$$

  where $\eta = e^{i\varphi}$.

- BH charge represents integral of L over space infinity. Conjecture - only $sin\varphi$-term contributes, because $\mathcal{F}$ is Coulomb field while $\check{\mathcal{F}}$ is magnetic monopole with ω singular at infinity [Vasiliev, 2015].

Black Hole charge

- Boyer-Lindquist coordinates for empty AdS_4 [Carter, 1973]:

$$
\begin{aligned}
ds^2 &= -\frac{\Delta_r}{\rho^2}\left[dt - \frac{a}{\Sigma} sin^2\theta d\phi\right]^2 + \frac{\rho^2}{\Delta_r} dr^2 + \frac{\rho^2}{\Delta_\theta} d\theta^2 + \\
&\quad + \frac{sin^2\theta \Delta_\theta}{\rho^2}\left[adt - \frac{(r^2+a^2)}{\Sigma} d\phi\right]^2,
\end{aligned}
$$

where

$$
\begin{aligned}
\rho^2 &= r^2 + a^2 cos^2\theta, \\
\Delta_r &= (r^2 + a^2)\left(1 + \frac{1}{3}\lambda^2 r^2\right), \\
\Delta_\theta &= 1 - \frac{1}{3}\lambda^2 a^2 cos^2\theta, \\
\Sigma &= 1 - \frac{1}{3}\lambda^2 a^2.
\end{aligned}
$$

Black Hole charge

- We start with Killing vector $V = \frac{\partial}{\partial t}$, $V^m = (1, 0, 0, 0)$.
- Corresponding Papapetrou field:
  $\varkappa_{\alpha\beta} = \frac{2}{3}\lambda^2 (r - iacos\theta) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and

$$
\begin{aligned}
L \quad = \quad & -\frac{9M}{\lambda^4 \rho^4} \left\{ \sqrt{\frac{\Delta_\theta}{\Delta_r}} sin\theta \left(cos\varphi \left(r^2 - a^2 cos^2\theta \right) - 2isin\varphi racos\theta \right) \cdot \right. \\
& \cdot \left(adr \wedge dt + \frac{r^2 + a^2}{\Sigma} d\phi \wedge dr \right) + \\
& + isin\theta \left(2icos\varphi racos\theta - sin\varphi \left(r^2 - a^2 cos^2\theta \right) \right) \cdot \\
& \left. \cdot \left(adt \wedge d\theta + \frac{r^2 + a^2}{\Sigma} d\theta \wedge d\phi \right) \right\}
\end{aligned}
$$

- Integral over $d\theta \wedge d\phi$ with $r \to \infty$ (spatial infinity) gives BH charge:

$$
Q = \frac{27\pi}{2\lambda^4 \left(3 - \lambda^2 a^2\right)} Msin\varphi.
$$

- So indeed only $\check{\mathcal{F}}$ contributes.

Conclusion

Conserved charge of HS Kerr Black Hole in AdS_4 is evaluated in the linear order. This confirmed that charge is generated by the $sin\varphi$-term of 2-form L. It would be interesting to calculate BH charge in higher orders to clarify its connection with the problem of BH entropy.