AdS/CFT Correspondence

and conformal fields

R.R. Metsaev

Lebedev Institute

HSTH-2, June, 2015

MOTIVATION

Maldacena's conjecture suggests interrelation between

IIB superstrings in AdS(5) x S(5) background

and

N=4, D=4, SYM

start with superstring field action in AdS(5) x S(5)

$$S_{AdS}(\Phi)$$

solve Dirichlet problem

$$\frac{\delta S_{AdS}(\bar{\Phi})}{\delta \bar{\Phi}} = 0$$

$$\bar{\Phi}|_{boundary} = \phi$$

$$S_{AdS}(\bar{\Phi}) = S_{CFT}(\phi)$$

$$S_{CFT}(\phi)$$

generating function of correlators of SYM

Light-cone gauge is a nice approach for study of IIB superstring field theory in flat space

Green, Schwarz

Light cone approach to fields in AdS might be very useful for studying AdS/CFT correspondence

AdS/CFT correspondence provides relations between

fields in AdS

and

boundary currents and shadow fields

conformal fields also arise in some interesting way

normal solution
massless AdS fields $\Longleftrightarrow$ boundary **conserved** currents

$$\phi_{AdS}(x,z) \sim z^{\Delta}\phi_{\mathrm{cur}}(x)$$

non-normal solution
massless AdS fields $\Longleftrightarrow$ boundary **canonical** shadows

$$\phi_{AdS}(x,z) \sim z^{d-\Delta}\phi_{\mathrm{sh}}(x)$$

normal solution
massive AdS fields $\Longleftrightarrow$ boundary **anomalous** currents

non-normal solution
massive AdS fields $\Longleftrightarrow$ boundary **anomalous** shadows

canonical shadows leads to **short** conformal fields

anomalous shadows leads to **long** conformal fields

In covariant approach

AdS field number of tensorial D.o.F and boundary fields is different

In light-cone approach

AdS field tensorial number of D.o.F and boundary fields **is the same**

Plan

1. Light-cone gauge dynamics of fields in AdS

2. Light-cone gauge description of currents and shadows

3. Light-cone gauge description conformal fields

our purpose

we demonstrate that that **light-cone gauge dynamics of fields in AdS**

leads automatically

to light-cone gauge formulation of currents, shadows, and conformal fields

Light cone gauge dynamics of fields in AdS(d+1)

Poincare coordinates

$$ds^2 = \frac{1}{z^2}(-dx^0dx^0 + dx^idx^i + dx^{d-1}dx^{d-1} + dzdz)$$

Light-cone coordinates

$$x^\pm, \qquad x^i \qquad i = 1, \ldots d-2$$

$$x^\pm \equiv \frac{1}{\sqrt{2}}(x^{d-1} \pm x^0)$$

$$\mathbf{ds^2 = \frac{1}{z^2}(2dx^+dx^- + dx^idx^i + dzdz)}$$

spin-1 in AdS(d+1). massless

Maxwell equation in AdS backround

$$D_\mu F^{\mu\nu} = 0\,, \qquad F_{\mu\nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu$$

gauge symmetry

$$\delta\phi_\mu = \partial_\mu \xi$$

$$\phi^\mu = \phi^+, \quad \phi^-, \quad \phi^i, \quad \phi^z$$

light-cone gauge

$$\phi^+ = 0$$

Use EOM

$$D_\mu F^{\mu +} = 0$$

0-

spin-1. massless

$$\phi^- = -\frac{\partial^i}{\partial +}\phi^i - \frac{1}{\partial +}(\partial_z + \frac{d-2}{z})\phi^z$$

Remaining fields

$\phi^{\mathbf{i}}$ $\phi^{\mathbf{z}}$ **dynamical**

$$D_\mu F^{\mu i} = 0 \qquad D_\mu F^{\mu z} = 0$$

spin-1. massless

Decoupled equations

$$(\Box + \partial_z^2 - \frac{1}{z^2}({\nu_{\mathbf{1}}}^2 - \frac{1}{4}))\phi^{\mathbf{i}} = 0$$

$$(\Box + \partial_z^2 - \frac{1}{z^2}({\nu_{\mathbf{0}}}^2 - \frac{1}{4})\,)\phi^{\mathbf{z}} = 0$$

$$\nu_{\mathbf{1}} = \frac{d-2}{2}\,, \qquad \nu_{\mathbf{0}} = \frac{d-4}{2}$$

spin-1. massless

$$|\phi\rangle = (\alpha^i\phi^i + \alpha^z\phi^z)|0\rangle$$

$$\alpha^I = \alpha^i, \quad \alpha^z$$

$$\phi^I = \phi^i, \quad \phi^z$$

$$|\phi\rangle = \alpha^I\phi^I|0\rangle$$

spin-1. massless

$$(\Box + \partial_z^2 - \frac{1}{z^2}A)|\phi\rangle = 0$$

$$A = \nu^2 - \frac{1}{4}$$

$$\nu = \kappa - N_z$$

$$\kappa \equiv \frac{d-2}{2}$$

$$N_z \equiv \alpha^z \bar{\alpha}^z$$

Light cone gauge Lagrangian

$$\mathcal{L} = \langle\phi|(\Box + \partial_z^2 - \frac{1}{z^2}A)|\phi\rangle$$

turns out to be valid for everything in AdS

spin-s. massless

$$|\phi\rangle \equiv \phi^{I_1\ldots I_s}\alpha^{I_1}\ldots\alpha^{I_s}|0\rangle$$

spin-s. massless

$$\mathcal{L} = \langle\phi|(\Box + \partial_z^2 - \frac{1}{z^2}A)|\phi\rangle$$

$$A = \nu^2 - \frac{1}{4}$$

$$\nu = \kappa - N_z$$

$$\kappa = \mathrm{s} + \frac{d-4}{2}$$

$$N_z = \alpha^z\bar{\alpha}^z$$

spin-s. massive

$$
\begin{aligned}
|\phi\rangle \quad &\equiv \quad \phi^{\mathbf{I_1\ldots I_s}}\alpha^{I_1}\ldots\alpha^{I_s}|0\rangle \\
&+ \quad \phi^{\mathbf{I_1\ldots I_{s-1}}}\zeta\alpha^{I_1}\ldots\alpha^{I_{s-1}}|0\rangle \\
&+ \quad \phi^{\mathbf{I_1\ldots I_{s-2}}}\zeta^{\mathbf{2}}\alpha^{I_1}\ldots\alpha^{I_{s-2}}|0\rangle \\
&+ \quad \ldots\ldots\ldots\ldots\ldots\ldots \\
&+ \quad \ldots\ldots\ldots\ldots\ldots\ldots \\
&+ \quad \phi^{\mathbf{I}}\zeta^{\mathbf{s-1}}\alpha^{I}|0\rangle \\
&+ \quad \phi\zeta^{\mathbf{s}}|0\rangle
\end{aligned}
$$

spin-s. massive

$$\mathcal{L} = \langle\phi|(\Box + \partial_z^2 - \frac{1}{z^2}A)|\phi\rangle$$

$$A = \nu^2 - \frac{1}{4}$$

$$\nu = \kappa + N_\zeta - N_z$$

$$\kappa = \mathbf{E_0} - \frac{d}{2}$$

$$N_\zeta = \zeta\bar{\zeta}\,, \qquad N_z = \alpha^z\bar{\alpha}^z$$

Space-time symmetries in AdS(d+1)

$$P^i = \partial^i\,, \qquad\qquad P^+ = \partial^+\,,$$

$$D = x^+ P^- + x^- \partial^+ + x^i \partial^i + z\partial_z + \frac{d-1}{2}\,,$$

$$J^{+-} = x^+ P^- - x^- \partial^+\,,$$

$$J^{+i} = x^+ \partial^i - x^i \partial^+\,,$$

$$J^{ij} = x^i \partial^j - x^j \partial^i + \mathbf{M^{ij}}\,,$$

$$K^i = -\frac{1}{2}(2x^+ x^- + x^i x^j + z^2)\partial^i + x^i D + \mathbf{M^{ij}} x^j - \mathbf{M^{zi}} z\,,$$

$$P^- = -\frac{\partial^i \partial^i + \partial_z^2}{2\partial^+} + \frac{1}{2z^2 \partial^+}\mathbf{A}$$

$$J^{-i} = \ldots$$

$$K^- = \ldots$$

Basic equations for operators A and M^{ij}, M^{zi}

$$2\{M^{zi}, A\} - [[M^{zi}, A], A] = 0$$

$$[M^{zi}, [M^{zj}, A]] + \{M^{il}, M^{lj}\} - \{M^{zi}, M^{zj}\}$$
$$= \delta^{ij}(-A + \frac{1}{2}M^{ij}M^{ij} + \langle \mathcal{C}_{so(d,2)} \rangle + \frac{d^2-1}{4})$$

M^{ij} generators of $so(d-2)$ algebra

M^{ij}, M^{zi} generators of $so(d-1)$ algebra

spin-1 current in $\mathbf{R^{d-1,1}}$: covariant approach

$\mathbf{J^a}$ conserved current

$$\partial^a J^a = 0$$

$$\Delta = d - 1$$

spin-1 current: light-cone gauge approach

$$J^a = J^+, \quad J^-, \quad J^i$$

$\mathbf{J^+}, \quad \mathbf{J^i}$ dynamical field

$\mathbf{J^-}$ auxiliary field

$$\partial^+ J^- + \partial^- J^+ + \partial^i J^i = 0$$

$$J^- = -\frac{\partial^i}{\partial^+} J^i - \frac{\partial^-}{\partial^+} J^+$$

∂^+ invertible operator in light-cone gauge

spin-1 current: LC approach

$$\phi^{\mathbf{i}}_{\mathsf{cur}} \equiv J^i$$

$$\phi_{\mathsf{cur}} \equiv \frac{1}{\partial^+} J^+$$

$$\Delta(\phi^i_{\mathsf{cur}}) = d - 1$$

$$\Delta(\phi_{\mathsf{cur}}) = d - 2$$

spin-1 current: LC approach

$$|\phi_{\rm cur}\rangle = (\alpha^i \phi^i_{\rm cur} + \alpha^z \phi_{\rm cur})|0\rangle$$

$$\Delta = \frac{d}{2} + \nu$$

$$\nu = \kappa - N_z$$

$$\kappa \equiv \frac{d-2}{2}$$

$$N_z \equiv \alpha^z \bar{\alpha}^z$$

spin-s current

$$|\phi_{\rm cur}\rangle \equiv \phi_{\rm cur}^{I_1\ldots I_s}\alpha^{I_1}\ldots\alpha^{I_s}|0\rangle$$

$$\Delta = \frac{d}{2} + \nu$$

$$\nu = \kappa - N_z$$

$$\kappa \equiv s + \frac{d-4}{2}$$

$$N_z \equiv \alpha^z\bar{\alpha}^z$$

spin-1 shadow : covariant approach

$\mathbf{\Phi^a}$ vector field

$$\delta\Phi^a = \partial^a\xi$$

$$\Delta = 1$$

Definition

shadow field = Φ^a/(gauge group)

spin-1 shadow : covariant approach

Two-point function

$$\Gamma = \int d^d x_1 d^d x_2 \mathcal{L}_{12}\,,$$

$$\mathcal{L}_{12} \equiv \Phi^a(x_1)\frac{\mathcal{O}^{ab}}{|x_{12}|^{2d-2}}\Phi^b(x_2)\,,$$

$$\mathcal{O} \equiv \eta^{ab} - 2\frac{x_{12}^a x_{12}^b}{|x_{12}|^2}$$

$$|x_{12}|^2 \equiv x_{12}^a x_{12}^a\,, \qquad x_{12}^a = x_1^a - x_2^a\,.$$

spin-1 shadow: LC approach

$$\Phi^a = \Phi^+, \quad \Phi^-, \quad \Phi^i$$

$\mathbf{\Phi^-}, \quad \mathbf{\Phi^i}$ dynamical field

$\mathbf{\Phi^+}$ auxiliary field

$$\delta\Phi^+ = \partial^+\xi$$

Φ^+ gauged away

∂^+ invertible operator in light-cone gauge

spin-1 shadow: LC approach

$$\phi_{\mathsf{sh}}^{\mathbf{i}} \equiv \Phi^i$$

$$\phi_{\mathsf{sh}} \equiv \partial^+ \Phi^-$$

$$\Delta(\phi_{\mathsf{sh}}^i) = 1$$

$$\Delta(\phi_{\mathsf{sh}}) = 2$$

spin-1 shadow: LC approach

Two-point function

$$\Gamma = \int d^d x_1 d^d x_2 \, \mathcal{L}_{12} \,,$$

$$\mathcal{L}_{12} = \frac{\phi_{\rm sh}^{\bf i}({\bf x_1})\phi_{\rm sh}^{\bf i}({\bf x_2})}{2|x_{12}|^{2(d-1)}} + \frac{1}{4(d-2)^2}\frac{\phi_{\rm sh}({\bf x_1})\phi_{\rm sh}({\bf x_2})}{|x_{12}|^{2(d-2)}},$$

spin-1 shadow: LC approach

$$|\phi_{\mathrm{sh}}\rangle = (\alpha^i \phi^i_{\mathrm{sh}} + \alpha^z \phi_{\mathrm{sh}})|0\rangle$$

$$\Delta = \frac{d}{2} - \nu$$

$$\nu = \kappa - N_z$$

$$\kappa \equiv \frac{d-2}{2}$$

$$N_z \equiv \alpha^z \bar{\alpha}^z$$

spin-1 shadow: LC approach

Two-point function

$$\Gamma = \int d^d x_1 d^d x_2 \mathcal{L}_{12}\,,$$

$$\mathcal{L}_{12} \equiv \langle \phi_{\rm sh}(x_1)| \frac{f_\nu}{|x_{12}|^{2\nu+d}} |\phi_{\rm sh}(x_2)\rangle\,,$$

$$f_\nu \equiv \frac{4^\nu \Gamma(\nu+\frac{d}{2})\Gamma(\nu+1)}{4^\kappa \Gamma(\kappa+\frac{d}{2})\Gamma(\kappa+1)}\,,$$

spin-s shadow

$$|\phi_{\rm sh}\rangle \equiv \phi_{\rm sh}^{I_1\ldots I_s}\alpha^{I_1}\ldots\alpha^{I_s}|0\rangle$$

$$\Delta = \frac{d}{2} - \nu$$

$$\nu = \kappa - N_z$$

$$\kappa \equiv s + \frac{d-4}{2}$$

$$N_z = \alpha^z\bar{\alpha}^z$$

spin-s shadow

Two-point function

$$\Gamma = \int d^d x_1 d^d x_2 \ \mathcal{L}_{12}\,,$$

$$\mathcal{L}_{12} \equiv \langle \phi_{\rm sh}(x_1)| \frac{f_\nu}{|x_{12}|^{2\nu+d}} |\phi_{\rm sh}(x_2)\rangle\,,$$

$$f_\nu \equiv \frac{4^\nu \Gamma(\nu + \frac{d}{2})\Gamma(\nu+1)}{4^\kappa \Gamma(\kappa + \frac{d}{2})\Gamma(\kappa+1)}\,,$$

$$\nu = \kappa + \widehat{N}$$

$$\kappa = E_0 - \frac{d}{2}$$

regularization

$$\kappa - \kappa_{\text{int}} = -2\varepsilon\,, \qquad \kappa_{\text{int}} - \text{ integer}$$

$$\nu_{\text{int}} \equiv \kappa_{\text{int}} + \widehat{N}$$

$$\frac{1}{|x|^{2\nu+d}} \quad \overset{\varepsilon}{\approx}{}^{0} \quad \frac{1}{\varepsilon}\Box^{\nu_{\text{int}}}\delta^{(d)}(x)$$

$$\Gamma \overset{\varepsilon\sim 0}{\sim} \frac{1}{\varepsilon}\int d^dx\ \mathcal{L}\,,$$

$$\mathcal{L} = \langle\phi|\Box^{\nu_{\mathsf{int}}}|\phi\rangle$$

Example. spin-2 short conformal field

$$|\phi\rangle = (\phi^{ij}\alpha^i\alpha^j + \phi^i\alpha^i\alpha^z + \phi\alpha^z\alpha^z)|0\rangle$$

$$\mathcal{L} = \phi^{ij}\Box^{d/2}\phi^{ij} + \phi^i\Box^{(d-2)/2}\phi^i + \phi\Box^{(d-4)/2}\phi$$

ϕ^{ij}, ϕ^i, ϕ fields of $so(d-2)$

Example. spin-2 short conformal field

d=4

$$\mathcal{L} = \phi^{\mathbf{ij}}\Box^{\mathbf{2}}\phi^{\mathbf{ij}} + \phi^{\mathbf{i}}\Box\phi^{\mathbf{i}} + \phi\phi$$

$$\mathbf{4} \quad + \quad \mathbf{2} \quad + \quad 0 = 6$$

light-cone gauge agrees with Fradkin and Tseytlin

Example. spin-2 long conformal field

Field content

$$\phi^{ij}$$

$$\phi^i_{-1} \qquad \phi^i_1$$

$$\phi_{-2} \qquad \phi_0 \qquad \phi_2$$

Example. spin-2 long conformal field

$$
\begin{aligned}
\mathcal{L} &= \phi^{ij}\Box^{\kappa}\phi^{ij} \\
&+ \phi^{i}_{-1}\Box^{\kappa-1}\phi^{i}_{-1} + \phi_1\Box^{\kappa+1}\phi^{i}_{1} \\
&+ \phi_{-2}\Box^{\kappa-2}\phi_{-2} + \phi_0\Box^{\kappa}\phi_0 + \phi_2\Box^{\kappa+2}\phi_2
\end{aligned}
$$

$$
\kappa = E_0 - \frac{d}{2} \qquad \textit{integer}
$$

Operator ν is known for

1. Arbitrary spin **totally symmetric massless and massive** fields in **AdS(d+1)**

2. Arbitrary spin **mixed-symmetry massless and massive** fields **in AdS(5)**

3. Type **IIB supergravity in AdS(5) x S(5)**