SCALE INVARIANT BREAKING OF CONFORMAL SYMMETRY

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Emergence of conformal symmetry

- Global rescaling is always enhanced to a local symmetry. Why?
- Big theoretical question: landscape of QFTs
    - Theory of phase transitions
    - Fixed points of the RG flow

Can we take emergence of conformal symmetry for granted?

What is conformal symmetry?

and what is scale invariance?

- Conformal field theory (CFT)

$$T_{\mu\nu} = \frac{1}{\sqrt{g}}\frac{\delta W}{\delta g^{\mu\nu}}$$ is traceless $T_{\mu\mu} = 0$ (or can be improved)

- Scale-invariant field theory (SFT)

$$T_{\mu\mu} = \partial_\mu V_\mu$$

virial current V_μ generates transformations as scale changes

What does it mean: scale $\Rightarrow$ conformal symmetry?

$$T_{\mu\mu} = \partial_\mu V_\mu$$

$$V_\mu = \partial_\mu L \quad \Rightarrow \quad T_{\mu\mu} = \partial^2 L \quad \Rightarrow$$

$$\tilde{T}_{\mu\nu} = T_{\mu\nu} + \frac{1}{d-1}(\partial_\mu \partial_\nu - g_{\mu\nu}\partial^2)L$$

- Iff virial current is a derivative, SFT is a CFT in disguise

Do Poincare invariance and unitarity imply V_μ is <u>always</u> a full derivative?

Example of a SFT

unitary, and scale- but not conformal-invariant theory

- Free massless scalar with a shift symmetry $\phi \equiv \phi + \text{const}$

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}(\partial\phi)^2 - \xi(\partial_\mu\partial_\nu - g_{\mu\nu}\partial^2)\phi^2$$

  - there is $L = \phi^2$ such that $T = \Box L$ but this operator is not present in the original theory

- Free gauge 2-form in 4d (dual to free scalar)

- Free gauge fields in d$\neq 4$

A brief historic overview

- Progress in understanding theories in 2d
  - Scale = conformal symmetry (Polchinski, 1987)
- Progress in understanding theories in 4d
  - Perturbative theories: scale=conformal (Fortin-Grinstein-Stergiou, Luty-Polchinski-Rattazzi, 2012)
  - Non-perturbative proof: scale $\Rightarrow$ conformal (Dymarsky-Komargodski-Schwimmer-Theisen 2013; Yonekura 2014; Dymarsky-Zhiboedov 2015)
- Understanding 6d, 3d...

Scale vs. conformal in 2d

- Zamolodchikov's **c**-theorem (1986)

$$c = z^4\langle T_{zz}(z)T_{zz}(0)\rangle + 2z^2|z^2|\langle T_{zz}(z)T(0)\rangle - 3|z^4|\langle T(z)T(0)\rangle$$

- Polchinski, scale vs. conformal theorem (1987)

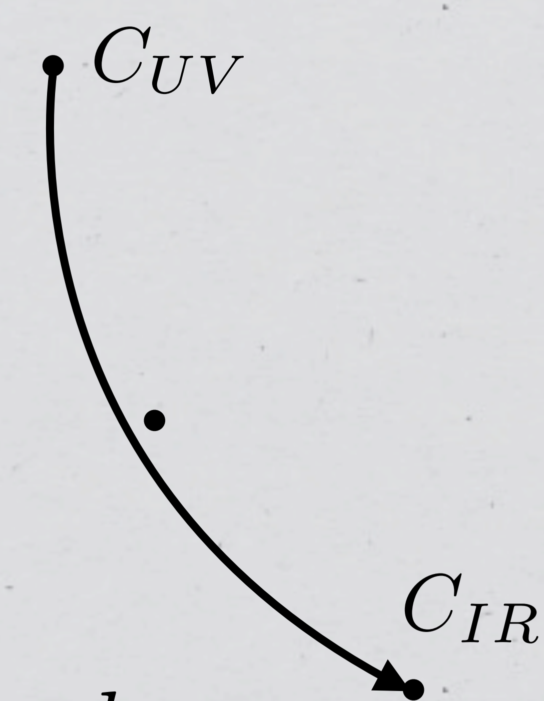
$$\frac{dc}{d\log\mu} = -x^4\langle T(x)T(0)\rangle = \alpha$$

- **c** must be μ-independent

$$\langle T(x)T(0)\rangle = 0 \Rightarrow T = 0$$

- integral form of **c**-theorem

$$c_{UV} - c_{IR} = \int \frac{d\mu}{\mu}\frac{dc}{d\log\mu}$$



a-theorem in 4d

Highlights of the Komargodski-Schwimmer (2011) approach

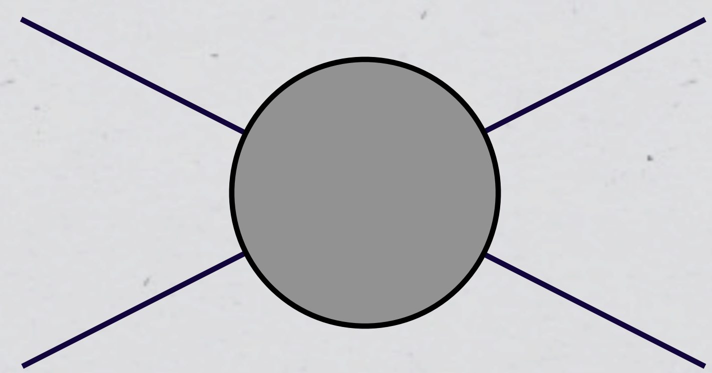
- Effective action for the dilaton

  $W[\tau] = W[g_{\mu\nu} = \eta_{\mu\nu} e^{-2\tau}]$

  $\mathcal{L} \to \mathcal{L} + \tau\, T$

  - covariance of $W[\tau]$

- Promoting dilaton to a dynamical particle and consider $2 \to 2$ scattering



$p_1 = -p_3 \quad p_2 = -p_4$

  - forward scattering depends only on $\Delta a = a_{UV} - a_{IR}$

$$W[\tau] = \int \Lambda^2 (\partial\tau)^2 e^{-2\tau} + 2\Delta a (\partial\tau)^4 + \mathcal{O}(\partial^6)$$

Sum rules for Δa

- No subtraction dispersion relation implies **a**-theorem

$$\Delta a = \int_0^\infty \frac{ds}{s^3}\, \Im \mathcal{A}_{2\to 2}(s) \geq 0 \quad \Rightarrow \quad \Delta a \geq 0$$

Komargodski-Schwimmer 2011

- On-shell dilaton effective action is exact in τ up to 4 derivatives

$$W[\tau] = \int \Lambda^2 (\partial\tau)^2 e^{-2\tau} + 2\Delta a (\partial\tau)^4 + \mathcal{O}(\partial^6)$$

- Infinite number of sum rules for $n \to n$ scattering

$$\Delta a = \int_0^\infty \frac{ds}{s^3}\, \Im \mathcal{A}_{n\to n}(s)$$

Dymarsky-Komargodski-Schwimmer-Theisen 2013

Nonperturbative result in 4d

AD-Komargodski-Schwimmer-Theisen, 2013

- sum rule for $\mathcal{A}_{n\to n}$
- positivity of $\Im\,\mathcal{A}_{n\to n}(s)$
- scale invariance $\mathcal{A}_{n\to n}(s) \sim s^2$

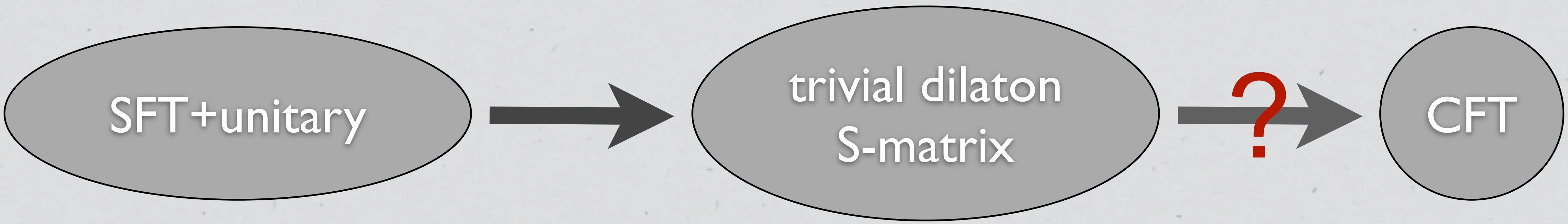
$$\Delta a = \int_0^\infty \frac{ds}{s^3}\,\Im\,\mathcal{A}_{n\to n}(s)$$

- ◉ all $n \to n$ dilaton scattering amplitudes are trivial!
- ◉ all cuts are trivial $\langle \text{anything}|\varphi \ldots \varphi\rangle = 0$
- ◉ all scattering of $\varphi-$particles is trivial

S-matrix of dilaton is trivial!

Nonperturbative result in 4d

AD-Komargodski-Schwimmer-Theisen, 2013



- all amplitudes of φ vanish on-shell $\Rightarrow$ coupling $\int \varphi T$ vanishes

on-shell: $\int \varphi \Box L$

$$T(p) = p^2 L = p^2 T'(0) + p^4 \ldots$$

- triviality of S-matrix $\Rightarrow$ after a change of variables $\varphi \to \tilde{\varphi}$ is a

trivial field

$$\int \frac{1}{2}(\partial\varphi)^2 + \varphi T + \cdots = \int \frac{1}{2}(\partial\underbrace{(\varphi + L)}_{\tilde{\varphi}})^2 - L\Box L + \ldots$$

scale-invariance in 4d implies $T = \Box L$

Our understanding so far

- In 2D, Scale$=$Conformal
- In 4D, Scale$\subset$Conformal
- In 3D, conjecture: stationarity of $F = rS' - S$ implies conformality

$$\frac{dF}{d\log r} = 0 \Rightarrow T_{\mu\mu} = \partial^2 L \ ?$$

Scale invariant breaking of conformal symmetry

- Describe scale invariant theory $\subset$ CFT
  - Traceless stress-energy tensor $T_{\mu\nu}^{\mathrm{CFT}} = 0$

$$T_{\mu\nu}^{\mathrm{CFT}} = T_{\mu\nu}^{\mathrm{SFT}} + (\partial_\mu \partial_\nu - g_{\mu\nu}\partial^2)L$$

  - Scale-invariant sector is closed under OPE

$$[S, \mathcal{O}] = 0$$

  - Conformal symmetry and symmetry defining SFT do not commute

$$[S, T_{\mu\nu}^{\mathrm{CFT}}] \neq 0$$

Dymarsky-Zhiboedov 2015

Scale invariant breaking of conformal symmetry

- Let's consider $[S, L] = X \neq 0$
- X is a free scalar, $[S, T^{\rm CFT}_{\mu\mu}] = \partial^2 X = 0$
- Consider a 4pt function

$$\langle XXXX \rangle = \frac{f(u,v)}{(x_{12}x_{34})^{d-2}}$$

EOM for X implies OPE expansion of XX contains higher spin conserved currents (higher spin symmetry)

SFT $\subset$ CFT is a theory of free fields

Dymarsky-Zhiboedov 2015

Loophole

- Other ways to define sectors closed under OPE?
  - Free scalar with shift symmetry is defines through

$$[S, \phi] = 1, \qquad [S, \mathcal{O}] = 0$$

  - Consider N free scalars

$$[S_i, \phi_j] = \delta_{ij}, \qquad [S_i, \mathcal{O}] = 0$$

  - Symmetry S_i is NOT defined within the O(N) theory

$$T_{\mu\nu}^{\mathrm{SFT}} = \partial_\mu \phi^i \partial_\nu \phi^i - \frac{g_{\mu\nu}}{2} (\partial \phi)^2$$

$$T_{\mu\nu}^{\mathrm{CFT}} = T_{\mu\nu}^{\mathrm{SFT}} - \xi (\partial_\mu \partial_\nu - g_{\mu\nu} \partial^2)(\phi^i \phi^i)$$

Conclusions

- "Scale = Conformal" holds in 4d (and 2d); an open problem in 3d and 6d
- New intuition: emergent conformal symmetry is due to irreversibility of RG flow

  `conjecture:` $dF/d\log\mu = 0 \Rightarrow$ `conformality in 3d`
- **No nontrivial SFTs** embedded within CFTs
- Scale-invariant sector of Vasiliev theory?