Holography beyond conformal invariance and AdS isometry?

A.O.Barvinsky

Theory Department, Lebedev Physics Institute, Moscow

Higher Spin Theory and Holography-2
June 2-4, 2015

Introduction

Vectorial models of *O(N)/U(N)* CFT_d dual to gauge theory on AdS_{d+1} – Vasiliev theory of higher spin gauge fields

Klebanov, Polyakov 2002
Vasiliev 1990, 1992, 2003
Bekaert, Cnockaert, Iazeolla, Vasiliev, hep-th/0503128

Tree level $O(N^1)$ and "one-loop" $O(N^0)$ checks of agreement between CFT partition functions and AdS calculations in d=3, in higher dimensions, thermal and Casimir energy parts in CFT_d and AdS_{d+1},

Giombi, Klebanov 2013
Giombi, Klebanov, Safdi 2014
Giombi, Klebanov,Tseytlin 2014

Nontrivial check of AdS_5/CFT_4: vanishing Casimir energy in odd d+1 → $\sum_s a_s = 0$ on S^4

Giombi, Klebanov, Pufu, Safdi, Tarnopolsky 2013
Tseytlin 2013

Double-trace deformations of CFT → RG flow from IR fixed point (free CFT) to UV fixed point -- holographic dual is the transition between different boundary conditions on dual massless gauge fields in AdS

Witten, hep-th/0112258
Gubser, Klebanov 2003

Something deep behind these miraculous coincidences extending beyond AdS and CFT?

A.B., Nesterov 2006
PRD 73 066012 (2006)
A.B. PRD74 084033 (2006)
A.B.JETP 147(2015)1

Plan

Introduction

Double trace deformation of CFT and the AdS/CFT correspondence

Tree level: holographic duality and the induced boundary theory

Functional determinants relations

Example of non-AdS/CFT duality – CFT driven cosmology

Conclusions

Double trace deformation of CFT and the AdS/CFT correspondence

CFT operator

$$J(x) = \Phi^i(x)\Phi^i(x)$$

$$S_{CFT}(\Phi) \to S_{CFT}(\Phi) - \frac{1}{2f}\int dx\, J^2(x)$$

Condensed DeWitt notations:

$$\hat{J}\boldsymbol{f}^{-1}\hat{J} = \int dx\, dy\, \hat{J}(x)\boldsymbol{f}^{-1}(x,y)\hat{J}(y)$$

$$\varphi\hat{J} = \int dx\, \varphi(x)\,\hat{J}(x)$$

Generating functional

source

$$Z_{CFT}(\varphi) = \int d\Phi\, \exp\left(-S_{CFT}(\Phi) + \frac{1}{2}J(\Phi)\boldsymbol{f}^{-1}J(\Phi) + \varphi J(\Phi)\right)$$

$$\frac{Z_{CFT}(\varphi)}{Z_{CFT}(0)} = \left\langle \exp\left(\frac{1}{2}\hat{J}\boldsymbol{f}^{-1}\hat{J} + \varphi\hat{J}\right)\right\rangle_{CFT} \equiv \left\langle e^{\varphi\hat{J}}\right\rangle^{f}_{CFT}$$

average in deformed CFT

“Habbard-Stratonovich” transform:

$$\left\langle e^{\varphi \hat{J}} \right\rangle^{f}_{CFT} = (\det \boldsymbol{f})^{1/2} \int d\phi \left\langle \exp\left(-\frac{1}{2}\phi\, \boldsymbol{f}\, \phi + (\phi + \varphi)\, \hat{J}\right)\right\rangle_{CFT}$$

functional
determinant in CFT

1/N-expansion:

$$\left\langle e^{\varphi \hat{J}} \right\rangle_{CFT} = 1 + \frac{1}{2}\varphi \left\langle \hat{J}\, \hat{J} \right\rangle_{CFT} \varphi + \ldots$$

$$\simeq \exp\left(\frac{1}{2}\varphi \left\langle \hat{J}\, \hat{J} \right\rangle \varphi\right) \equiv \exp\left(-\frac{1}{2}\varphi\, \boldsymbol{F} \varphi\right),$$

$\langle \hat{J} \rangle = 0$

$\langle \hat{J}\hat{J} \ldots \hat{J} \rangle \ll 1, N \to \infty$

$$\left\langle \hat{J}(x)\, \hat{J}(y) \right\rangle = -\boldsymbol{F}(x,y) \sim \frac{1}{|x-y|^{2\Delta}} \sim \frac{1}{k^{d-2\Delta}}$$

Δ – dimension of *J*

$$\left\langle e^{\varphi \hat{J}} \right\rangle^{f}_{CFT} = (\det \boldsymbol{f})^{1/2} \left(\det \boldsymbol{F_f}\right)^{-1/2} \exp\left(-\frac{1}{2}\varphi \frac{1}{\boldsymbol{F}^{-1} + \boldsymbol{f}^{-1}}\varphi\right)$$

$$\boldsymbol{F_f} \equiv \boldsymbol{F} + \boldsymbol{f}$$

Gubser, Klebanov 2003

RG flow: UV → IR

$$\langle \hat{J}(x)\hat{J}(y)\rangle^{f}_{CFT} = -\frac{1}{\boldsymbol{F}^{-1}+\boldsymbol{f}^{-1}} \rightarrow \begin{cases} -\boldsymbol{F}+\ldots, & \boldsymbol{f}^{-1}\boldsymbol{F} \ll 1 \\ -\boldsymbol{f}+\boldsymbol{f}(\boldsymbol{f}^{-1}\boldsymbol{F})^{-1}+\ldots, & \boldsymbol{f}^{-1}\boldsymbol{F} \gg 1 \end{cases}$$

$$\sim \begin{cases} \frac{1}{|x-y|^{2\Delta}} \sim \frac{1}{k^{d-2\Delta}}, & |x-y| \rightarrow 0 \qquad \text{UV, } \Delta \\ \frac{1}{|x-y|^{2(d-\Delta)}} \sim \frac{1}{k^{2\Delta-d}}, & |x-y| \rightarrow \infty \qquad \text{IR, } d-\Delta \end{cases}$$

Dual description of higher spin fields: from conserved currents to dual gauge fields

$$J = J_{\mu_1 \dots \mu_s}(x) \sim \Phi^i(x)\, \partial_{\mu_1} \dots \partial_{\mu_s} \Phi^i(x) \to \varphi = \varphi^{\mu_1 \cdots \mu_s}(x)$$

Klebanov, Polyakov 2002
Sezgin, Sundell 2005
Giombi, Yin 2010
Maldacena, Zhiboedov 2013
Didenko, Skvortsov 2013
Gelfond, Vasiliev 2013

$$AdS_{d+1} : X^A = X^1, \dots X^{d+1}, \qquad \Phi = \Phi^{A_1 \dots A_s}(X)$$

bilinear in r©

$$S_{d+1}[\Phi] = \frac{1}{2} \int_{AdS} d^{d+1} X\, \Phi \overleftrightarrow{F} \Phi$$

$$\partial(AdS_{d+1}) = R^d, S^d, S^1 \times S^{d-1}, \qquad X = e(x), \qquad x \equiv x^\mu = x^1, \dots x^d$$

$$\Phi\, | \equiv \Phi^{\mu_1 \cdots \mu_s}(e(x)) = \varphi^{\mu_1 \cdots \mu_s}(x)$$

AdS/CFT correspondence (undeformed CFT):

$$\left\langle e^{\varphi \hat{J}} \right\rangle_{CFT} = \frac{\displaystyle\int_{\Phi| = \varphi} D\Phi\, e^{-S_{d+1}[\Phi]}}{\displaystyle\int_{\Phi| = 0} D\Phi\, e^{-S_{d+1}[\Phi]}}$$

⇓

$$\left\langle e^{\varphi \hat{J}} \right\rangle^{f}_{CFT} = (\det \boldsymbol{f})^{1/2} \int d\phi\, e^{-\frac{1}{2}\phi \boldsymbol{f} \phi} \left\langle e^{(\phi+\varphi)\hat{J}} \right\rangle_{CFT}$$

$$= (\det \boldsymbol{f})^{1/2} \frac{\displaystyle\int_{\text{all } \Phi} D\Phi\, e^{-S[\Phi]}}{\displaystyle\int_{\Phi| = 0} D\Phi\, e^{-S_{d+1}[\Phi]}}$$

shift of integration variable and integral over all Φ

Total action $S[\Phi] \equiv S_{d+1}[\Phi] + \frac{1}{2}\left(\Phi| - \varphi\right)\boldsymbol{f}\left(\Phi| - \varphi\right)$

$$= \frac{1}{2}\int_{AdS} d^{d+1}X\, \Phi(X)\, \overleftrightarrow{F}(\nabla)\, \Phi(X)$$ **AdS bulk part**

$$+ \frac{1}{2}\int_{\partial(AdS)} d^{d}x\, \left(\Phi|(x) - \varphi(x)\right)\boldsymbol{f}\left(\Phi|(x) - \varphi(x)\right)$$ **boundary part**

Wronskian relation and Wronskian operator *W(r)*:

$$\int_M d^{d+1}X \left(\Phi_1 \overrightarrow{F}(\nabla)\Phi_2 - \Phi_1 \overleftarrow{F}(\nabla)\,\Phi_2 \right) = - \int_{\partial M} d^d x \left(\Phi_1\, \overrightarrow{W}\,\Phi_2 - \Phi_1\, \overleftarrow{W}\,\Phi_2 \right),$$

$$\int_M d^{d+1}X\, \Phi_1\, \overleftrightarrow{F}\,\Phi_2 = \int_M d^{d+1}X\, \Phi_1(\overrightarrow{F}\,\Phi_2) + \int_{\partial M} d^d x\, \Phi_1\, \overrightarrow{W}\,\Phi_2 \Big|$$

bilinear in r©

Saddle point of *S[Φ]*:

$$\delta S[\,\Phi\,] = \int_{AdS_{d+1}} d^{d+1}X\, \delta\Phi\, (\overrightarrow{F}\,\Phi) + \int_{M_d} d^d x\, \delta\Phi \left(\left(\overrightarrow{W} + \boldsymbol{f} \right) \Phi\,\big| - \boldsymbol{f}\varphi \right) = 0$$

$$F(\nabla)\,\Phi_f(X) = 0,$$
$$\left(\overrightarrow{W}\,(\nabla) + \boldsymbol{f} \right) \Phi_f \Big| = \boldsymbol{f}\varphi$$

depend on *f*

generalized (inhomogeneous) Neumann boundary conditions

Neumann Green's function:

$$F(\nabla)\, G_{N_f}(X,Y) = \delta(X,Y),$$

$$(\vec{W} + \boldsymbol{f}\,)\, G_{N_f}(X,Y)\Big|_{X\in\partial M_{d+1}} = 0$$

Solution:

$$\Phi_f(X) = \int_b dy\, G_{N_f}(X,y)\, \boldsymbol{f}\varphi(y) \equiv G_{N_f}|\, \boldsymbol{f}\varphi$$

On-shell action:

$$S[\,\Phi_f\,] = \frac{1}{2}\,\varphi\Big[\boldsymbol{f} - \boldsymbol{f}\, G_{N_f}\big|\big|\, \boldsymbol{f}\,\Big]\,\varphi$$

Boundary-to-boundary propagator:

$$G_{N_f}(x,y) \equiv G_{N_f}(X,Y)\,|_{\,X=e(x),\,Y=e(y)} \equiv G_{N_f}||$$

Condensed notations

Preexponential factors – functional determinants in the bulk subject to Neumann and Dirichlet b.c.:

$$\int_{\mathrm{all}\,\Phi} D\Phi\, \exp\Big(-S[\,\Phi\,] - \frac{1}{2}\big(\Phi| - \varphi\big)\boldsymbol{f}\big(\Phi| - \varphi\big)\Big)$$

$$= \big(\mathrm{Det}_{N_f} F\big)^{-1/2} \exp\Big(-\frac{1}{2}\varphi\Big[\boldsymbol{f} - \boldsymbol{f}\, G_{N_f}\big|\big|\,\boldsymbol{f}\,\Big]\varphi\Big)$$

$$\int_{\Phi|=0} D\Phi\, e^{-S_{d+1}[\,\Phi\,]} = \big(\mathrm{Det}_D F\big)^{-1/2}$$

Comparison of AdS/CFT result with double-trace deformation:

$$\Big\langle e^{\varphi\hat{J}}\Big\rangle^{f}_{CFT} = (\mathrm{det}\,\boldsymbol{f})^{1/2}\left(\frac{\mathrm{Det}_{N_f}F}{\mathrm{Det}_D F}\right)^{-1/2}\exp\Big(-\frac{1}{2}\varphi\big[\,\boldsymbol{f}-\boldsymbol{f}\,G_{N_f}\big|\big|\,\boldsymbol{f}\,\big]\varphi\Big)$$

⇕ ⇕

$$\Big\langle e^{\varphi\hat{J}}\Big\rangle^{f}_{CFT} = (\mathrm{det}\,\boldsymbol{f})^{1/2}\big(\mathrm{det}\,\boldsymbol{F_f}\big)^{-1/2}\exp\Big(-\frac{1}{2}\varphi\frac{1}{\boldsymbol{F}^{-1}+\boldsymbol{f}^{-1}}\varphi\Big)$$

$$\boldsymbol{F_f} \equiv \boldsymbol{F}+\boldsymbol{f}$$

AdS/CFT duality ⟺

$$G_{N_f}\big|\big| = \boldsymbol{F}_f^{-1},$$

$$\mathrm{Det}_{N_f}F = \mathrm{det}\,\boldsymbol{F}_f\ \mathrm{Det}_D F$$

Tree level: holographic duality and the induced boundary theory

tree level:

$$\left\langle e^{\varphi \hat{J}} \right\rangle_{CFT} = \frac{e^{-S_{d+1}[\Phi_D(\varphi)]}}{e^{-S_{d+1}[\Phi_D(0)]}}$$

$$F(\nabla)\,\phi_D(X) = 0, \qquad \phi_D\,| = \varphi(x)$$

$$\Phi_D(X) = -G_D \overleftarrow{W}\,|\,\varphi$$

Dirichlet Green's function

"Witten's calculation":

$$S_{d+1}[\Phi_D] = \frac{1}{2}\int_{AdS} dX\,\Phi_D \overleftrightarrow{F}(\nabla)\,\Phi_D$$

$$= \int d^{d+1}X\,\Phi_D(\overrightarrow{F}\Phi_D) + \int d^dx\,\Phi_D \overrightarrow{W}\Phi_D\Big|$$

($\overrightarrow{F}\Phi_D = 0$)

$$= \frac{1}{2}\varphi\Big[-\overrightarrow{W}G_D\overleftarrow{W}\,||\Big]\varphi = \frac{1}{2}\varphi \boldsymbol{F}\varphi$$

inverse propagator of induced boundary theory

$$\boldsymbol{F} = -\,\overrightarrow{W}G_D\overleftarrow{W}\,||$$

$$\left\langle e^{\varphi \hat{J}} \right\rangle_{CFT} = \exp\left(-\frac{1}{2}\varphi \boldsymbol{F}\varphi\right)$$

Functional determinants relations

$$Z = \int\limits_{\text{all}} D\Phi \, \exp\left(-S[\Phi]\right)$$

$$Z(\varphi) = \int\limits_{\Phi| = \varphi} D\Phi \, \exp\left(-S[\Phi]\right)$$

$$Z = \int d\varphi \, Z(\varphi)$$

$$S[\Phi] = \frac{1}{2} \int\limits_{M_{d+1}} dX \, \Phi(X) \overset{\leftrightarrow}{F}(\nabla) \Phi(X) + \int\limits_{M_d} dx \left(\frac{1}{2} \varphi(x) \, \boldsymbol{f}(\partial) \, \varphi(x) + j(x) \, \varphi(x) \right),$$

$$\Phi \, \Big| \equiv \Phi(X) \Big|_{\partial M_{d+1}} = \Phi(e(x)) = \varphi(x), \quad M_d = \partial M_{d+1}$$

$$F(\nabla)\,\phi_f(X) = 0$$

$$(\,\vec{W} + \boldsymbol{f}\,)\,\phi_f \Big| + j(x) = 0$$

$$Z = \left(\mathrm{Det}_{N_f} F\right)^{-1/2} \exp(-S[\Phi_f\,])$$

$$= \left(\mathrm{Det}_{N_f}\, F\right)^{-1/2} \exp\left(\frac{1}{2}\, j\, G_{N_f}||\; j\right)$$

"Witten's calculation"
+ boundary term:

$$\boldsymbol{F_f} = -\,\vec{W}\, G_D\, \overleftarrow{W}\, || + \boldsymbol{f}$$

$$Z(\varphi) = (\,\mathrm{Det}_D F)^{-1/2} \exp(-S[\Phi_D\,])$$

$$= (\,\mathrm{Det}_D\, F)^{-1/2} \exp\left(-\frac{1}{2}\,\varphi\, \boldsymbol{F}_f\, \varphi - j\,\varphi\right)$$

$$Z = \int d\varphi\, Z(\varphi)$$

$$= \left(\mathrm{Det}_D\, F\right)^{-1/2} \left(\det \boldsymbol{F_f}\,\right)^{-1/2} \exp\left(\frac{1}{2}\, j\, \boldsymbol{F_f}^{-1} j\right)$$

again Gaussian

Generic manifold with a boundary, generic second order operator:

$$F(\nabla)\, G_{N_f}(X,Y) = \delta(X,Y),$$
$$(\vec{W} + \boldsymbol{f}\,)\, G_{N_f}(X,Y)\Big|_{X\in\partial M_{d+1}} = 0$$

$$F(\nabla)\, G_D(X,Y) = \delta(X,Y)$$
$$G_D(X,Y)\Big|_{X\in\partial M_{d+1}} = 0$$

$$G_{N_f}\,||=\boldsymbol{F}_f^{-1} \equiv \Big[-\vec{W}\,G_D\,\overleftarrow{W}\,|| + \boldsymbol{f}\,\Big]^{-1}$$

$$\mathrm{Det}_{N_f} F = \det \boldsymbol{F}_f\ \mathrm{Det}_D F.$$

A.B., Nesterov 2006
PRD 73 066012 (2006)
A.B.JETP 147(2015)1

Particular case of large *N* double trace deformed CFT on AdS spacetime – RG flow from IR (*f*=1) to UV (*f*=0) fixed points:

$$\frac{\mathrm{Det}_{N_{f_1}} F}{\mathrm{Det}_{N_{f_2}} F} = \frac{\det \boldsymbol{F}_{f_1}}{\det \boldsymbol{F}_{f_2}} \equiv \frac{\det\big(\boldsymbol{1} + f_1^{-1}\,\boldsymbol{F}\big)}{\det\big(\boldsymbol{1} + f_2^{-1}\,\boldsymbol{F}\big)}$$
$$\boldsymbol{f} = f\,\delta(x,y), \quad \det\boldsymbol{f} = 1$$

Diaz, Dorn 2007
Hartman, Rastelli 2008

Extension to gauge theories

Bulk and brane gauge invariances:

$$\Phi \to \Phi^{\Xi} = \Phi + \Delta^{\Xi}\Phi, \quad \varphi \to \varphi^{\xi} = \varphi + \Delta^{\xi}\varphi, \quad \Xi^{||}\big| = \xi,$$

$$\Delta^{\Xi}\Phi^{A_1\ldots A_s}(X) = \nabla^{(A_1}\Xi^{A_2\ldots A_s)}(X),$$

$$\Delta^{\xi}\varphi^{\mu_1\ldots\mu_s}(x) = D^{(\mu_1}\xi^{\mu_2\ldots\mu_s)}$$

Bulk and brane gauge conditions and Faddeev-Popov operators:

$$H(\Phi) = H^{A_1\ldots A_{s-1}}(X) \sim \nabla_B\Phi^{BA_1\ldots A_{s-1}}(X) = 0,$$

$$\Delta^{\Xi}H(\Phi) = Q\Xi$$

$$h(\varphi) = h^{\mu_1\ldots\mu_{s-1}}(x) \sim D_\nu\varphi^{\nu\mu_1\ldots\mu_{s-1}}(x) = 0,$$

$$\Delta^{\xi}h(\varphi) = \boldsymbol{Q}\xi$$

Analogous relation for ghost determinants:

$$\mathrm{Det}_N Q = \det \boldsymbol{Q}\ \mathrm{Det}_D Q$$

A.B. PRD74 084033 (2006)

Example of non-AdS/CFT duality – CFT driven cosmology

The analogue of the thermal AdS/CFT correspondence: duality of the 4D finite temperature boundary CFT to 5D black hole thermodynamics in AdS with a boundary

Witten (1998)

vs

4D CFT cosmology: Einstein theory sourced by quantum conformal matter at finite temperature

$\Longleftrightarrow$

Brane induced gravity in 5D Schwarzschild-deSitter bulk (5D black hole in dS_5)

$$G_4,\ \Lambda_4,\ \mathcal{C}$$

primordial 4D cosmological constant

$$G_5,\ \Lambda_5,\ R_S$$

Schwarzschild radius of bulk BH

4D radiation is imitated by the BH mass $\mathcal{C} \sim G_5 M = R_S^2$

But (!):

No SUSY

De Sitter, $\Lambda_5>0$

No AdS, no group-theoretical arguments

Duality of 4D CFT driven cosmology and 5D brane induced gravity

4D side

$$S_E[\,g_{\mu\nu},\phi\,] = -\frac{1}{16\pi G}\int d^4x\, g^{1/2}\left(R - 2\Lambda\right) + S_{CFT}[\,g_{\mu\nu},\phi\,]$$

$\Lambda = \Lambda_4$ -- 4D CC

N_sÀ 1 conformal fields of spin s=0,1,1/2

5D side

$$S[\,G_{AB}(X)\,] = -\frac{1}{16\pi G_5}\int_{\rm Bulk} d^5X\, G^{1/2}\left(R^{(5)}(\,G_{AB}\,) - 2\Lambda_5\right)$$
$$-\int_{\rm brane} d^4x\, g^{1/2}\left(\frac{1}{8\pi G_5}[\,K\,] + \frac{1}{16\pi G_4}R(\,g_{\mu\nu}\,)\right).$$

5D Schwarzschild-dS solution with a bulk black hole of mass » R_s^2/G_5

$$ds^2_{(5)} = f(R)dT^2 + \frac{dR^2}{f(R)} + R^2 d\Omega^2_{(3)}$$
$$f(R) = 1 - \frac{\Lambda_5}{6}R^2 - \frac{R_S^2}{R^2}$$

embedding

$$ds^2_{(4)} = d\tau^2 + a^2(\tau)d\Omega^2_{(3)}$$

S^1£ S^3 topology

$$R = a(\tau), \quad T = T(\tau),$$
$$T'(\tau) = \frac{\sqrt{f(a) - a'^2}}{f(a)}$$

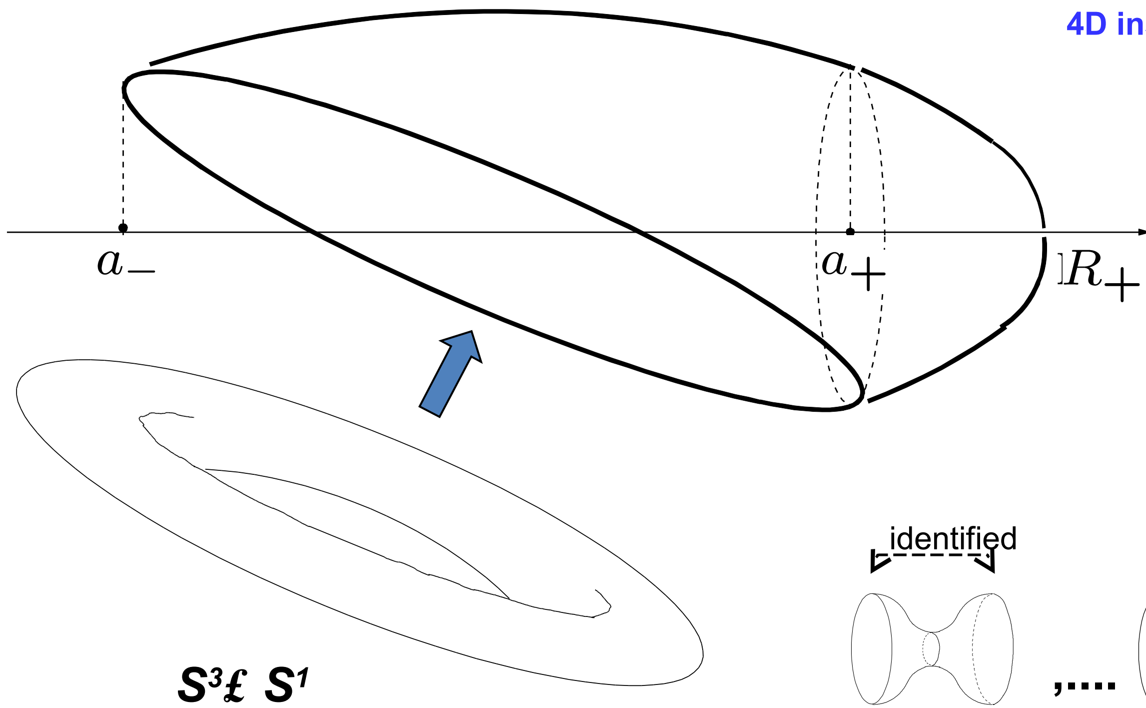
Euclidean Schwarzschild-dS “cigar” instanton:

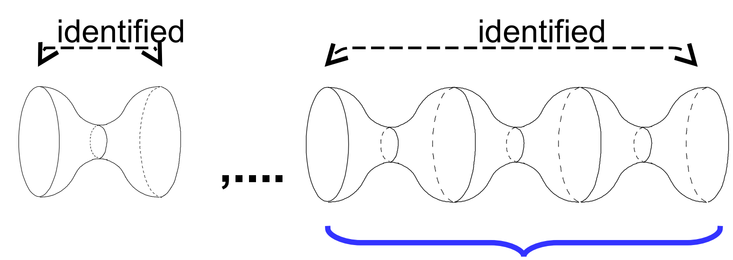
$$f(R) \geq 0, \quad R_- \leq R \leq R_+$$

$$R_\pm^2 = \frac{3}{\Lambda_5}\left(1 \pm \sqrt{1 - 2\Lambda_5 R_S^2/3}\right)$$

$$R_- < \underbrace{a_- \leq a(\tau) \leq a_+}_{} < R_+$$

4D instanton domain





1- fold, k=1

k- folded garland, k=1,2,3,…

Equation of motion of the 4D spherical shell in 5D Schwarzschild-de Sitter background $R = a(\tau)$

5D side

$$r_c^2 \left(\frac{1}{a^2} - \frac{\dot{a}^2}{a^2} \right)^2 = \frac{1}{a^2} - \frac{\dot{a}^2}{a^2} - \frac{\Lambda_5}{6} - \frac{R_S^2}{a^4}$$

$$r_c = \frac{G_5}{2\,G_4}$$

$$\left\{ \begin{array}{l} B = 2r_c^2, \quad \Lambda_4 = \dfrac{\Lambda_5}{2} \\ \mathcal{C} = R_S^2 \end{array} \right.$$

Quantum Friedmannn equation for cosmological factor

4D CFT cosmology side

$$\frac{B}{2} \left(\frac{1}{a^2} - \frac{a'^2}{a^2} \right)^2 = \frac{1}{a^2} - \frac{a'^2}{a^2} - \frac{\Lambda_4}{3} - \frac{\mathcal{C}}{a^4}$$

C » **amount of radiation** $\quad \mathcal{C} = \sum_{\omega} \frac{\omega}{e^{\omega\eta} \pm 1}$

Conclusions

Tree-level AdS/CFT » kinematical relations

One-loop AdS/CFT » functional determinants relations

Beyond one-loop » similar identities with ~» 1/*N* ?

$$\int_{\text{all}} D\Phi\, e^{-NS[\Phi]} = \int d\varphi \int_{\Phi| = \varphi} D\Phi\, e^{-NS[\Phi]}$$

$$S[\Phi] = \int_{M_{d+1}} d^{d+1}X \left(\frac{1}{2} S_{(2)}\,\Phi^2 + \frac{1}{3!} S_{(3)}\Phi^3 + \ldots\right) + \int_{M_d} d^d x \left(\frac{1}{2} \boldsymbol{f}_{(2)}\varphi^2 + \frac{1}{3!} \boldsymbol{f}_{(3)}\varphi^3 + \ldots\right)$$