On spin 5/2 in the Fradkin-Vasiliev formalism

Yu. M. Zinoviev

Institute for High Energy Physics, Protvino, Russia

10.12.2014

Outlook

1. Fradkin-Vasiliev formalism
2. Massless spin $\frac{5}{2}$
3. Partially massless spin $\frac{5}{2}$

Frame-like formalism

- Frame-like formalism: a set of one-forms Φ (physical, auxiliary and extra) fields.
- Each field has its own gauge transformation

$$\delta\Phi = D\xi + \dots$$

- For each field gauge invariant object (two-form) can be constructed

$$\mathcal{R} = D \wedge \Phi + \dots$$

- Free Lagrangian can be rewritten in explicitly gauge invariant form

$$\mathcal{L}_0 \sim \sum \mathcal{R} \wedge \mathcal{R}$$

Cubic vertices

- Three types of cubic vertices:
  - trivial: $\mathcal{L} \sim \mathcal{R} \wedge \mathcal{R} \wedge \mathcal{R}$
  - abelian: $\mathcal{L} \sim \mathcal{R} \wedge \mathcal{R} \wedge \Phi$
  - non-abelian: $\mathcal{L} \sim \mathcal{R} \wedge \Phi \wedge \Phi$
- All non-abelian vertices come from the deformation procedure:
  - quadratic deformation of curvatures: $\Delta\mathcal{R} \sim \Phi \wedge \Phi$
  - linear deformation of gauge transformations $\delta\Phi \sim \Phi\xi$
  - covariant transformations of deformed curvatures: $\delta\hat{\mathcal{R}} \sim \mathcal{R}\xi$
  - interacting Lagrangian: $\mathcal{L} \sim \sum \hat{\mathcal{R}} \wedge \hat{\mathcal{R}}$
- Vasiliev-2011: any non-trivial cubic vertex for massless completely symmetric fields with spins s_1, s_2 and s_3 having up to

$$N = s_1 + s_2 + s_3 - 2$$

derivatives can be obtained as a linear combination of abelian and non-abelian vertices.

Cubic vertices

- Three types of cubic vertices:
  - trivial: $\mathcal{L} \sim \mathcal{R} \wedge \mathcal{R} \wedge \mathcal{R}$
  - abelian: $\mathcal{L} \sim \mathcal{R} \wedge \mathcal{R} \wedge \Phi$
  - non-abelian: $\mathcal{L} \sim \mathcal{R} \wedge \Phi \wedge \Phi$
- All non-abelian vertices come from the deformation procedure:
  - quadratic deformation of curvatures: $\Delta\mathcal{R} \sim \Phi \wedge \Phi$
  - linear deformation of gauge transformations $\delta\Phi \sim \Phi\xi$
  - covariant transformations of deformed curvatures: $\delta\hat{\mathcal{R}} \sim \mathcal{R}\xi$
  - interacting Lagrangian: $\mathcal{L} \sim \sum \hat{\mathcal{R}} \wedge \hat{\mathcal{R}}$
- Vasiliev-2011: any non-trivial cubic vertex for massless completely symmetric fields with spins s_1, s_2 and s_3 having up to

$$N = s_1 + s_2 + s_3 - 2$$

derivatives can be obtained as a linear combination of abelian and non-abelian vertices.

Cubic vertices

- Three types of cubic vertices:
  - trivial: $\mathcal{L} \sim \mathcal{R} \wedge \mathcal{R} \wedge \mathcal{R}$
  - abelian: $\mathcal{L} \sim \mathcal{R} \wedge \mathcal{R} \wedge \Phi$
  - non-abelian: $\mathcal{L} \sim \mathcal{R} \wedge \Phi \wedge \Phi$
- All non-abelian vertices come from the deformation procedure:
  - quadratic deformation of curvatures: $\Delta\mathcal{R} \sim \Phi \wedge \Phi$
  - linear deformation of gauge transformations $\delta\Phi \sim \Phi\xi$
  - covariant transformations of deformed curvatures: $\delta\hat{\mathcal{R}} \sim \mathcal{R}\xi$
  - interacting Lagrangian: $\mathcal{L} \sim \sum \hat{\mathcal{R}} \wedge \hat{\mathcal{R}}$
- Vasiliev-2011: any non-trivial cubic vertex for massless completely symmetric fields with spins s_1, s_2 and s_3 having up to

$$N = s_1 + s_2 + s_3 - 2$$

derivatives can be obtained as a linear combination of abelian and non-abelian vertices.

Multispinor frame-like formalism

- Most auxiliary and extra fields are mixed symmetry (spin-)tensors (γ-traceless in fermionic cases)

$$\Phi^{a_1 \dots a_{s-1}, b_1 \dots b_k}$$

- Restriction to $d = 4 \to$ multispinor frame-like formalism: all fields are still one forms but with all local indices replaced by spinor ones $a \to (\alpha\dot{\alpha})$. Spin $\frac{5}{2}$ example:

$$\Psi^a, \quad (\gamma\Psi) = 0 \quad \Leftrightarrow \quad \Psi^{\alpha\beta\dot{\alpha}}, \Psi^{\alpha\dot{\alpha}\dot{\beta}}$$
$$\Omega^{[ab]}, \quad \gamma_a\Omega^{ab} = 0 \quad \Leftrightarrow \quad \Omega^{\alpha\beta\gamma}, \Omega^{\dot{\alpha}\dot{\beta}\dot{\gamma}}$$

- We work in $(A)dS_4$ space with background frame $e^{\alpha\dot{\alpha}}$ and covariant derivative D normalized so that ($\Lambda = -\lambda^2$)

$$D \wedge D\xi^\alpha = 2\lambda^2 E^{\alpha\beta}\xi_\beta, \qquad E^{\alpha\beta} = \frac{1}{2} e^\alpha{}_{\dot{\alpha}} \wedge e^{\beta\dot{\alpha}}$$

Kinematics

- Free Lagrangian in AdS_4;

$$\mathcal{L}_0 = \psi_{\alpha\beta\dot{\alpha}} e^{\alpha}{}_{\dot{\beta}} D \psi^{\beta\dot{\alpha}\dot{\beta}} + \frac{\lambda}{2}[3\psi_{\alpha\beta\dot{\alpha}} E^{\alpha}{}_{\gamma} \psi^{\beta\gamma\dot{\alpha}} - \psi_{\alpha\beta\dot{\alpha}} E^{\dot{\alpha}}{}_{\dot{\beta}} \psi^{\alpha\beta\dot{\beta}} + h.c.]$$

- It is invariant under the gauge transformations:

$$\delta_0 \psi^{\alpha\beta\dot{\gamma}} = D\xi^{\alpha\beta\dot{\gamma}} + e_{\gamma}{}^{\dot{\gamma}} \eta^{\alpha\beta\gamma} + \lambda e^{(\alpha}{}_{\dot{\delta}} \xi^{\beta)\dot{\gamma}\dot{\delta}}$$

- Auxiliary field $\Omega^{\alpha\beta\gamma}$:

$$\delta_0 \Omega^{\alpha\beta\gamma} = D\eta^{\alpha\beta\gamma} + \lambda^2 e^{(\alpha}{}_{\dot{\delta}} \xi^{\beta\gamma)\dot{\delta}}$$

Kinematics (cont.)

- Gauge invariant objects:

$$\mathcal{R}^{\alpha\beta\gamma} = D\Omega^{\alpha\beta\gamma} + \lambda^2 e^{(\alpha}{}_{\dot\delta}\psi^{\beta\gamma)\dot\delta}$$
$$\mathcal{T}^{\alpha\beta\dot\gamma} = D\psi^{\alpha\beta\dot\gamma} + \lambda e^{(\alpha}{}_{\dot\delta}\psi^{\beta)\dot\gamma\dot\delta} + e_\delta{}^{\dot\gamma}\Omega^{\alpha\beta\delta}$$

- Zero torsion condition

$$\mathcal{T} = 0 \quad \Rightarrow \quad \Omega = \Omega(\psi) \quad \oplus \quad \frac{\delta S}{\delta\psi} = 0$$

- Free Lagrangian can be rewritten

$$\mathcal{L}_0 = a_1 \mathcal{R}_{\alpha\beta\gamma}\mathcal{R}^{\alpha\beta\gamma} + a_2 \mathcal{T}_{\alpha\beta\dot\gamma}\mathcal{T}^{\alpha\beta\dot\gamma} + h.c.$$

Gravitational interaction

- Deformations for spin $\frac{5}{2}$ correspond to minimal substitution rules: $D \to D + \omega$, $e \to e + h$:

$$\Delta \mathcal{R}^{\alpha\beta\gamma} = c_0[\omega^{(\alpha}{}_\delta \Omega^{\beta\gamma)\delta} + \lambda^2 h^{(\alpha}{}_{\dot\alpha} \psi^{\beta\gamma)\dot\alpha}]$$
$$\Delta \mathcal{T}^{\alpha\beta\dot\alpha} = c_0[\omega^{(\alpha}{}_\gamma \psi^{\beta)\gamma\dot\alpha} + \omega^{\dot\alpha}{}_{\dot\beta} \psi^{\alpha\beta\dot\beta} + \lambda h^{(\alpha}{}_{\dot\beta} \psi^{\beta)\dot\alpha\dot\beta} + h_\gamma{}^{\dot\alpha} \Omega^{\alpha\beta\gamma}]$$

- Deformations for curvature and torsion:

$$\Delta R^{\alpha\beta} = b_0[\Omega^{(\alpha}{}_{\gamma\delta} \Omega^{\beta)\gamma\delta} + 2\lambda^2 \psi^{(\alpha}{}_{\gamma\dot\alpha} \psi^{\beta)\gamma\dot\alpha} + \lambda^2 \psi^{(\alpha}{}_{\dot\alpha\dot\beta} \psi^{\beta)\dot\alpha\dot\beta}]$$
$$\Delta T^{\alpha\dot\alpha} = 2b_0[\Omega^{\alpha}{}_{\beta\gamma} \psi^{\beta\gamma\dot\alpha} + 2\lambda \psi^{\alpha}{}_{\beta\dot\beta} \psi^{\beta\dot\alpha\dot\beta} + h.c.]$$

- Non-trivial (on-shell) part of gauge transformations:

$$\delta \hat{\mathcal{R}}^{\alpha\beta\gamma} = R^{(\alpha}{}_\delta \eta^{\beta\gamma)\delta}$$
$$\delta \hat{\mathcal{T}}^{\alpha\beta\dot\alpha} = R^{(\alpha}{}_\gamma \xi^{\beta)\gamma\dot\alpha} + R^{\dot\alpha}{}_{\dot\beta} \xi^{\alpha\beta\dot\beta}$$
$$\delta \hat{R}^{\alpha\beta} = 2b_0 \mathcal{R}^{(\alpha}{}_{\gamma\delta} \eta^{\beta)\gamma\delta}$$

Gravitational interaction

- Deformations for spin $\frac{5}{2}$ correspond to minimal substitution rules: $D \to D + \omega$, $e \to e + h$:

$$\begin{aligned} \Delta\mathcal{R}^{\alpha\beta\gamma} &= c_0[\omega^{(\alpha}{}_\delta\Omega^{\beta\gamma)\delta} + \lambda^2 h^{(\alpha}{}_{\dot\alpha}\psi^{\beta\gamma)\dot\alpha}] \\ \Delta\mathcal{T}^{\alpha\beta\dot\alpha} &= c_0[\omega^{(\alpha}{}_\gamma\psi^{\beta)\gamma\dot\alpha} + \omega^{\dot\alpha}{}_{\dot\beta}\psi^{\alpha\beta\dot\beta} + \lambda h^{(\alpha}{}_{\dot\beta}\psi^{\beta)\dot\alpha\dot\beta} + h_\gamma{}^{\dot\alpha}\Omega^{\alpha\beta\gamma}] \end{aligned}$$

- Deformations for curvature and torsion:

$$\begin{aligned} \Delta R^{\alpha\beta} &= b_0[\Omega^{(\alpha}{}_{\gamma\delta}\Omega^{\beta)\gamma\delta} + 2\lambda^2\psi^{(\alpha}{}_{\gamma\dot\alpha}\psi^{\beta)\gamma\dot\alpha} + \lambda^2\psi^{(\alpha}{}_{\dot\alpha\dot\beta}\psi^{\beta)\dot\alpha\dot\beta}] \\ \Delta T^{\alpha\dot\alpha} &= 2b_0[\Omega^\alpha{}_{\beta\gamma}\psi^{\beta\gamma\dot\alpha} + 2\lambda\psi^\alpha{}_{\beta\dot\beta}\psi^{\beta\dot\alpha\dot\beta} + h.c.] \end{aligned}$$

- Non-trivial (on-shell) part of gauge transformations:

$$\begin{aligned} \delta\hat{\mathcal{R}}^{\alpha\beta\gamma} &= R^{(\alpha}{}_\delta\eta^{\beta\gamma)\delta} \\ \delta\hat{\mathcal{T}}^{\alpha\beta\dot\alpha} &= R^{(\alpha}{}_\gamma\xi^{\beta)\gamma\dot\alpha} + R^{\dot\alpha}{}_{\dot\beta}\xi^{\alpha\beta\dot\beta} \\ \delta\hat{R}^{\alpha\beta} &= 2b_0\mathcal{R}^{(\alpha}{}_{\gamma\delta}\eta^{\beta)\gamma\delta} \end{aligned}$$

Cubic vertex

- Interacting Lagrangian

$$\mathcal{L} = a_1 \hat{\mathcal{R}}_{\alpha\beta\gamma} \hat{\mathcal{R}}^{\alpha\beta\gamma} + a_2 \hat{\mathcal{T}}_{\alpha\beta\dot{\gamma}} \hat{\mathcal{T}}^{\alpha\beta\dot{\gamma}} + a_0 \hat{R}_{\alpha\beta} \hat{R}^{\alpha\beta} + h.c.$$

- Invariance of the Lagrangian requires

$$3 a_1 c_0 = 4 a_0 b_0$$

- Cubic vertex contains terms with up to 2 derivatives

Kinematics

- Partially massless spin $\frac{5}{2}$: helicities $\pm\frac{5}{2}$, $\pm\frac{3}{2}$. Free Lagrangian:

$$\begin{aligned}\mathcal{L}_0 &= \psi_{\alpha\beta\dot\alpha} e^\alpha{}_{\dot\beta} D\psi^{\beta\dot\alpha\dot\beta} - \psi_\alpha e^\alpha{}_{\dot\alpha} D\psi^{\dot\alpha} \\ &+\frac{\alpha_1}{2}[3\psi_{\alpha\beta\dot\alpha} E^\alpha{}_\gamma \psi^{\beta\gamma\dot\alpha} - \psi_{\alpha\beta\dot\alpha} E^{\dot\alpha}{}_{\dot\beta} \psi^{\alpha\beta\dot\beta}] \\ &+3\alpha_2[\psi_{\alpha\beta\dot\alpha} E^{\alpha\beta}\psi^{\dot\alpha} - \psi_{\alpha\dot\alpha\dot\beta} E^{\dot\alpha\dot\beta}\psi^\alpha] - 3\alpha_1\psi_\alpha E^\alpha{}_\beta \psi^\beta + h.c.\end{aligned}$$

Here ${\alpha_1}^2 = \frac{\lambda^2}{4}$, ${\alpha_2}^2 = -\frac{5\lambda^2}{12}$

- Gauge transformations:

$$\begin{aligned}\delta_0\psi^{\alpha\beta\dot\alpha} &= D\xi^{\alpha\beta\dot\alpha} + \alpha_1 e^{(\alpha}{}_{\dot\beta}\xi^{\beta)\dot\alpha\dot\beta} + e_\gamma{}^{\dot\alpha}\eta^{\alpha\beta\gamma} + \alpha_2 e^{\dot\alpha(\alpha}\xi^{\beta)} \\ \delta_0\psi^\alpha &= D\xi^\alpha + 3\alpha_2 e_{\beta\dot\alpha}\xi^{\alpha\beta\dot\alpha} + 3\alpha_1 e^\alpha{}_{\dot\alpha}\xi^{\dot\alpha}\end{aligned}$$

- Auxiliary fields ($V^{\alpha\beta\gamma}$ — zero form):

$$\delta_0\Omega^{\alpha\beta\gamma} = D\eta^{\alpha\beta\gamma}, \qquad \delta V^{\alpha\beta\gamma} = 6\alpha_2\eta^{\alpha\beta\gamma}$$

Gauge invariant objects

- Each field has its own gauge invariant object:

$$
\begin{aligned}
\mathcal{R}^{\alpha\beta\gamma} &= D\Omega^{\alpha\beta\gamma} - \frac{4\alpha_2}{5} E^{(\alpha}{}_{\delta} V^{\beta\gamma)\delta} \\
\mathcal{T}^{\alpha\beta\dot{\alpha}} &= D\psi^{\alpha\beta\dot{\alpha}} + e_{\gamma}{}^{\dot{\alpha}}\Omega^{\alpha\beta\gamma} + \alpha_1 e^{(\alpha}{}_{\dot{\beta}}\psi^{\beta)\dot{\alpha}\dot{\beta}} + \alpha_2 e^{\dot{\alpha}(\alpha}\psi^{\beta)} \\
\Psi^{\alpha} &= D\psi^{\alpha} + 3\alpha_2 e_{\beta\dot{\alpha}}\psi^{\alpha\beta\dot{\alpha}} + 3\alpha_1 e^{\alpha}{}_{\dot{\alpha}}\psi^{\dot{\alpha}} - E_{\beta\gamma} V^{\alpha\beta\gamma} \\
\mathcal{V}^{\alpha\beta\gamma} &= DV^{\alpha\beta\gamma} - 6\alpha_2\Omega^{\alpha\beta\gamma}
\end{aligned}
$$

- Zero torsion conditions:

$$
\mathcal{T} = 0 \quad \Rightarrow \quad \Omega = \Omega(\psi) \quad \oplus \quad \frac{\delta S}{\delta \psi} = 0
$$

$$
\Psi = 0 \quad \Rightarrow \quad V = V(\psi) \quad \oplus \quad \frac{\delta S}{\delta \psi} = 0
$$

Lagrangian

- Lagrangian in terms of gauge invariant objects:

$$\begin{aligned}\mathcal{L}_0 &= a_1 \mathcal{R}_{\alpha\beta\gamma}\mathcal{R}^{\alpha\beta\gamma} + a_2 \mathcal{T}_{\alpha\beta\dot{\alpha}}\mathcal{T}^{\alpha\beta\dot{\alpha}} + a_3 \Psi_\alpha \Psi^\alpha \\ &+ a_4 \mathcal{V}_{\alpha\beta\gamma} E^\gamma{}_\delta \mathcal{V}^{\alpha\beta\delta} + a_5 \mathcal{T}_{\alpha\beta\dot{\alpha}} e_\gamma{}^{\dot{\alpha}} \mathcal{V}^{\alpha\beta\gamma} + h.c.\end{aligned}$$

- There is an ambiguity in the choice of coefficients due to identity:

$$\begin{aligned}0 \approx D(\mathcal{R}_{\alpha\beta\gamma}\mathcal{V}^{\alpha\beta\gamma}) &= D\mathcal{R}_{\alpha\beta\gamma}\mathcal{V}^{\alpha\beta\gamma} + \mathcal{R}_{\alpha\beta\gamma} D\mathcal{V}^{\alpha\beta\gamma} \\ &= -6\alpha_2 \mathcal{R}_{\alpha\beta\gamma}\mathcal{R}^{\alpha\beta\gamma} + \frac{12\alpha_2}{5} \mathcal{V}_{\alpha\beta\gamma} E^\gamma{}_\delta \mathcal{V}^{\alpha\beta\delta}\end{aligned}$$

Deformations

- Deformation for partially massless spin $\frac{5}{2}$ again correspond to minimal substitution rules $D \to D + \omega$, $e \to e + h$.
- Deformations for gravitational curvature and torsion are defined up to possible field redefinitions

$$h^{\alpha\dot\alpha} \Rightarrow h^{\alpha\dot\alpha} + \kappa_1 e^{\beta\dot\alpha} V^{\alpha\gamma\delta} V_{\beta\gamma\delta} + \kappa_2 e^{\alpha\dot\alpha} V^{\beta\gamma\delta} V_{\beta\gamma\delta} + \dots$$

- Non-trivial (on-shell) part of gauge transformations looks like:

$$\begin{aligned}
\delta \hat{R}^{\alpha\beta} &= 2b_1 \mathcal{R}^{(\alpha}{}_{\gamma\delta}\eta^{\beta)\gamma\delta} + b_2 \mathcal{R}^{\alpha\beta\gamma}\xi_\gamma + \dots \\
\delta \hat{\mathcal{R}}^{\alpha\beta\gamma} &= c_0 R^{(\alpha}{}_{\delta}\eta^{\beta\gamma)\delta} \\
\delta \hat{\mathcal{T}}^{\alpha\beta\dot\alpha} &= c_0 R^{(\alpha}{}_{\gamma}\xi^{\beta)\gamma\dot\alpha} + c_0 R^{\dot\alpha}{}_{\dot\beta}\xi^{\alpha\beta\dot\beta}
\end{aligned}$$

Interacting Lagrangian

- Interacting Lagrangian is just the sum of free Lagrangians with deformed curvatures plus abelian vertex:

$$\begin{aligned} \mathcal{L}_0 &= a_1 \hat{\mathcal{R}}_{\alpha\beta\gamma} \hat{\mathcal{R}}^{\alpha\beta\gamma} + a_2 \hat{\mathcal{T}}_{\alpha\beta\dot{\alpha}} \hat{\mathcal{T}}^{\alpha\beta\dot{\alpha}} + a_3 \hat{\Psi}_\alpha \hat{\Psi}^\alpha + a_4 \hat{\mathcal{V}}_{\alpha\beta\gamma} E^\gamma{}_\delta \hat{\mathcal{V}}^{\alpha\beta\delta} \\ &+ a_5 \hat{\mathcal{T}}_{\alpha\beta\dot{\alpha}} e_\gamma{}^{\dot{\alpha}} \hat{\mathcal{V}}^{\alpha\beta\gamma} + i a_0 \hat{R}_{\alpha\beta} \hat{R}^{\alpha\beta} + a_6 R_{\alpha\beta} \mathcal{V}^{\alpha\beta\gamma} \psi_\gamma + h.c. \end{aligned}$$

- Gauge invariance fixes all the coefficients in deformations as well as coefficient a_6 of abelian vertex in terms of gravitational coupling constant $c_0 \Rightarrow$ one non-trivial vertex only.
- As in the massless case cubic vertex contains terms with up to two derivatives.

Interacting Lagrangian

- Interacting Lagrangian is just the sum of free Lagrangians with deformed curvatures plus abelian vertex:

$$\begin{aligned} \mathcal{L}_0 &= a_1 \hat{\mathcal{R}}_{\alpha\beta\gamma} \hat{\mathcal{R}}^{\alpha\beta\gamma} + a_2 \hat{\mathcal{T}}_{\alpha\beta\dot{\alpha}} \hat{\mathcal{T}}^{\alpha\beta\dot{\alpha}} + a_3 \hat{\Psi}_\alpha \hat{\Psi}^\alpha + a_4 \hat{\mathcal{V}}_{\alpha\beta\gamma} E^\gamma{}_\delta \hat{\mathcal{V}}^{\alpha\beta\delta} \\ &+ a_5 \hat{\mathcal{T}}_{\alpha\beta\dot{\alpha}} e_\gamma{}^{\dot{\alpha}} \hat{\mathcal{V}}^{\alpha\beta\gamma} + i a_0 \hat{R}_{\alpha\beta} \hat{R}^{\alpha\beta} + a_6 R_{\alpha\beta} \mathcal{V}^{\alpha\beta\gamma} \psi_\gamma + h.c. \end{aligned}$$

- Gauge invariance fixes all the coefficients in deformations as well as coefficient a_6 of abelian vertex in terms of gravitational coupling constant c_0 $\Rightarrow$ one non-trivial vertex only.
- As in the massless case cubic vertex contains terms with up to two derivatives.