Off-shell Scalar Supermultiplet in the Unfolded Dynamics Approach

(based on arXiv:1301.2230, N.G. Misuna, M.A. Vasiliev)

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Outline

Wess-Zumino model

- Chiral superfield in $\mathbb{C}^{4|2}$, $\bar{D}_{\dot{\alpha}}\Phi = 0$:

$$\Phi = C\left(y\right) + \sqrt{2}\theta\chi\left(y\right) + \theta\theta F\left(y\right), \ \left\{y^m = x^m + i\theta\sigma^m\bar{\theta}; \theta^\mu\right\}$$

- General Lagrangian:

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta} K(\Phi, \bar{\Phi}, D^{(n)}\Phi, D^{(n)}\bar{\Phi}) + \left[\int d^2\theta W(\Phi) + h.c.\right]$$

- Salam-Strathdee Lagrangian:

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta}\Phi\bar{\Phi} + \left[\int d^2\theta\left(k\Phi + \frac{1}{2}m\Phi^2 + \frac{1}{3}g\Phi^3\right) + h.c.\right]$$

- Wess-Zumino Lagrangian:

$$\begin{aligned}\mathcal{L} \ &= \ i\partial_n\bar{\chi}\bar{\sigma}^n\chi + \bar{C}\Box C + \bar{F}F + \left[m\left(CF - \frac{1}{2}\chi\chi\right) + \right.\\ &\left. +g\left(CCF - \chi\chi C\right) + kF + h.c.\right]\end{aligned}$$

Unfolded equation

- Unfolded equations:

$$dW^{\Omega}(x) + G^{\Omega}(W(x)) = 0,$$
$$G^{\Omega}(W^{\Upsilon}) := \sum_{n=1}^{\infty} f^{\Omega}{}_{\Upsilon_1 \dots \Upsilon_n} W^{\Upsilon_1} \dots W^{\Upsilon_n}.$$

- Consistency condition:

$$d^2 \equiv 0 \Rightarrow Q^2 \equiv 0, \; Q = G^{\Upsilon}(W) \frac{\delta}{\delta W^{\Upsilon}} .$$

- Gauge symmetries:

$$\delta W^{\Omega} = d\varepsilon^{\Omega} - \varepsilon^{\Upsilon} \frac{\delta G^{\Omega}(W)}{\delta W^{\Upsilon}} .$$

Unfolded action

- Unfolded action of the system $dW^{\Omega} + G^{\Omega}(W) = 0$:

$$S = \int\limits_{M^d} \mathcal{L}(W) .$$

- Space of nontrivial gauge-invariant Lagrangians:

$$\mathcal{L} = H^{d}(Q) , \; Q = G^{\Upsilon}(W)\frac{\delta}{\delta W^{\Upsilon}} .$$

SUSY-vacuum

- $D = 4$ $N = 1$ SUSY 1-form connection:

$$\Omega_0 = e^a P_a + \frac{1}{2}\omega^{a,b} M_{ab} + \phi^\alpha Q_\alpha + \bar{\phi}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}}.$$

- Flat SUSY-background $d\Omega_0 + \Omega_0\Omega_0 = 0$:

$$D^L e^a + 2i\phi^\alpha \bar{\phi}^{\dot{\alpha}} (\sigma^a)_{\alpha\dot{\alpha}} = 0,$$

$$D^L \phi^\alpha = 0,\ D^L \omega^{a,b} = 0,\ D^L \bar{\phi}_{\dot{\alpha}} = 0.$$

Lorentz-covariant derivative $D^L = d + \omega$.

Flat superspace

- Transition to superspace $x^{\underline{m}} \to z^{\underline{M}} = \{x^{\underline{m}}, \theta^{\underline{\mu}}, \bar{\theta}^{\underline{\dot{\mu}}}\}$:

$$e^a_{\underline{m}}(x)dx^{\underline{m}} \to E^a_{\underline{M}}(z)dz^{\underline{M}}, \qquad \omega^{a,b}_{\underline{m}}(x)dx^{\underline{m}} \to \Omega^{a,b}_{\underline{M}}(z)dz^{\underline{M}},$$
$$\phi^{\alpha}_{\underline{m}}(x)dx^{\underline{m}} \to E^{\alpha}_{\underline{M}}(z)dz^{\underline{M}}, \qquad \bar{\phi}^{\dot{\alpha}}_{\underline{m}}(x)dx^{\underline{m}} \to \bar{E}^{\dot{\alpha}}_{\underline{M}}(z)dz^{\underline{M}}.$$

- Flat superspace:

$$DE^a + 2iE^{\alpha}\bar{E}^{\dot{\alpha}}(\sigma^a)_{\alpha\dot{\alpha}} = 0,$$

$$D\Omega^{a,b} = 0, \;\; DE^{\alpha} = 0, \;\; D\bar{E}_{\dot{\alpha}} = 0.$$

Unfolded scalar and spinor

- Massless scalar field:

$$D^L C^{a(k)} + e_b C^{a(k)b} = 0.$$

- Space of unfolded fields (fixing $e_{\underline{m}}{}^{a} = \delta_{\underline{m}}{}^{a}$; $\omega^{a,b} = 0$):

$$C^{a_1 \dots a_k} = (-1)^k \, \partial^{a_1} \dots \partial^{a_k} C.$$

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- Traslessness leads to KG equation:

$$\partial^a \partial^a C = C^{aa} \to \Box C = 0.$$

Unfolded scalar and spinor

- Massless scalar field:

$$D^L C^{a(k)} + e_b C^{a(k)b} = 0.$$

- Massless spinor field:

$$\begin{cases} D^L \chi_\alpha^{a(k)} + e_b \chi_\alpha^{a(k)b} = 0, \\ (\bar{\sigma}_b)^{\dot{\alpha}\alpha} \, \chi_\alpha^{a(k-1)b} = 0. \end{cases}$$

Scalar supermultiplet

On-shell scalar supermultiplet (Ponomarev, Vasiliev, 2012, arXiv:1012.2903):

$$\begin{cases} D^L C^{a(k)} + e_b C^{a(k)b} - \sqrt{2}\phi^{\alpha}\chi_{\alpha}^{a(k)} = 0, \\ D^L \chi_{\alpha}^{a(k)} + e_b \chi_{\alpha}^{a(k)b} - \sqrt{2}i\bar{\phi}^{\dot{\alpha}} (\sigma_b)_{\alpha\dot{\alpha}} C^{a(k)b} = 0. \end{cases}$$

Scalar supermultiplet in superspace

On-shell scalar supermultiplet (Ponomarev, Vasiliev, 2012, arXiv:1012.2903):

$$\begin{cases} DC^{a(k)}(z) + E_b C^{a(k)b}(z) - \sqrt{2} E^{\alpha} \chi_{\alpha}^{a(k)}(z) = 0, \\ \\ D\chi_{\alpha}^{a(k)}(z) + E_b \chi_{\alpha}^{a(k)b}(z) - \sqrt{2} i \bar{E}^{\dot{\alpha}} (\sigma_b)_{\alpha\dot{\alpha}} C^{a(k)b}(z) = 0 \,. \end{cases}$$

Off-shell formulation

- Off-shell scalar supermultiplet:

$$\begin{cases} DC^{a(k)} + E_b C^{a(k)b} - \sqrt{2}E^{\alpha}\chi_{\alpha}^{a(k)} = 0, \\ D\chi_{\alpha}^{a(k)} + E_b\chi_{\alpha}^{a(k)b} - \sqrt{2}i\bar{E}^{\dot{\alpha}}(\sigma_b)_{\alpha\dot{\alpha}}C^{a(k)b} - \sqrt{2}E_{\alpha}F^{a(k)} = 0, \\ DF^{a(k)} + E_b F^{a(k)b} - \sqrt{2}i\bar{E}_{\dot{\alpha}}(\bar{\sigma}_b)^{\dot{\alpha}\alpha}\chi_{\alpha}^{a(k)b} = 0. \end{cases}$$

- "Dynamical" equations:

$$\bar{D}_{\dot{\alpha}}C^{a(k)} = 0, \quad D_{\alpha}F^{a(k)} = 0\,.$$

Operator Q

- General structure

$$
\begin{aligned}
Q &= Q_\Omega + \hat{Q}, \\
Q_\Omega &= \Omega^{a,c}\Omega_c{}^{,b}\frac{\partial}{\partial\Omega^{a,b}} + \Omega^{a,b}E_b\frac{\partial}{\partial E^a} + \dots, \\
\hat{Q} &= 2iE^\alpha\left(\sigma^a\right)_{\alpha\dot{\alpha}}\bar{E}^{\dot{\alpha}}\frac{\partial}{\partial E^a} + E_a\hat{q}^a + \sqrt{2}E_\alpha\hat{q}^\alpha + \sqrt{2}\bar{E}_{\dot{\alpha}}\hat{\bar{q}}^{\dot{\alpha}},
\end{aligned}
$$

Operator Q

- General structure

$$Q = Q_\Omega + \hat{Q},$$
$$Q_\Omega = \Omega^{a,c}\Omega_c{}^{,b}\frac{\partial}{\partial\Omega^{a,b}} + \Omega^{a,b}E_b\frac{\partial}{\partial E^a} + \dots,$$
$$\hat{Q} = 2iE^\alpha\left(\sigma^a\right)_{\alpha\dot{\alpha}}\bar{E}^{\dot{\alpha}}\frac{\partial}{\partial E^a} + E_a\hat{q}^a + \sqrt{2}E_\alpha\hat{q}^\alpha + \sqrt{2}\bar{E}_{\dot{\alpha}}\hat{\bar{q}}^{\dot{\alpha}},$$

- Unfolded 'superderivatives' on the space of 0-form

$$\hat{q}^b = C^{a(k)b}\frac{\partial}{\partial C^{a(k)}} + \chi_\alpha^{a(k)b}\frac{\partial}{\partial\chi_\alpha^{a(k)}} + F^{a(k)b}\frac{\partial}{\partial F^{a(k)}} + h.c.,$$
$$\hat{q}^\alpha = (\chi^\alpha)^{a(k)}\frac{\partial}{\partial C^{a(k)}} - F^{a(k)}\frac{\partial}{\partial\chi_\alpha^{a(k)}} - \dots$$
$$\hat{\bar{q}}^{\dot{\alpha}} = -\left(\bar{\chi}^{\dot{\alpha}}\right)^{a(k)}\frac{\partial}{\partial\bar{C}^{a(k)}} + \bar{F}^{a(k)}\frac{\partial}{\partial\bar{\chi}_{\dot{\alpha}}^{a(k)}} - \dots$$

4-superform Lagrangian

- General solution:

$$\mathcal{L} = E_a E_b \left(\bar{\sigma}^{ab}\right)^{\dot{\alpha}\dot{\beta}} \bar{E}_{\dot{\alpha}} \bar{E}_{\dot{\beta}} L + \frac{\sqrt{2}}{6} \epsilon^{abcd} E_a E_b E_c \bar{E}_{\dot{\alpha}} \left(\bar{\sigma}_d\right)^{\dot{\alpha}\alpha} \hat{q}_\alpha L + \\ + \frac{i\sqrt{2}}{16} E_a E_b E_c E_d \epsilon^{abcd} \hat{q}_\alpha \hat{q}^\alpha L + h.c.$$

$$L = L\left(C^{m(k)}, \bar{F}^{m(k)}\right),\ \bar{L} = \bar{L}\left(\bar{C}^{m(k)}, F^{m(k)}\right) \text{ and } L \neq \hat{q}_a f^a.$$

- Wess-Zumino Lagrangian:

$$L = i2\sqrt{2}\left(C\bar{F} + kC + \frac{m}{2}C^2 + \frac{g}{3}C^3\right).$$

Integral form Lagrangian

- Integral form solution:

$$
\begin{aligned}
S \;=\; & \int \epsilon^{abcd} E_a E_b E_c E_d \delta^2 \left(E_\alpha\right) \delta^2 \left(\bar{E}_{\dot{\alpha}}\right) K + \\
& + \left[\int \delta^2 \left(E_\alpha\right) \epsilon^{abcd} E_a E_b E_c E_d W \left(C^{m(k)}, \bar{F}^{m(k)} \right) + h.c. \right]
\end{aligned}
$$

$K \neq \hat{q}^{\alpha} f_\alpha + h.c.,\ W \neq \hat{q}^a g_a.$

- Salam-Strathdee Lagrangian:

$$
K = C\bar{C},\ W = kC + \frac{m}{2} C^2 + \frac{g}{3} C^3 .
$$

Conclusion

1. Superspace off-shell formulation of Wess-Zumino model is built starting from KG and Weyl equations.
2. All superLagrangians in the form of superform and integral form are found.
3. Application to maximal SYM and SUGRA?