with Wenliang Li and Massimo Taronna [1406.2335]

On partially massless gravity

Euihun JOUNG

Imperial College London

Graviton: massless spin-② particle

$$S = \int d^D x - \frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h \partial^\lambda h$$

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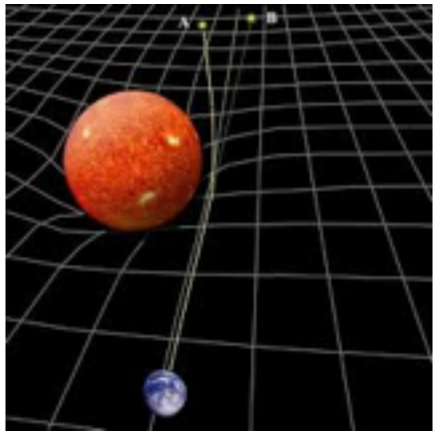

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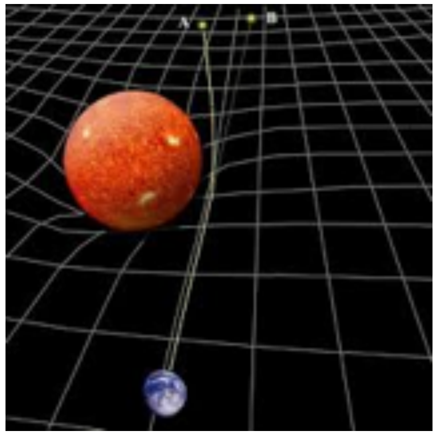

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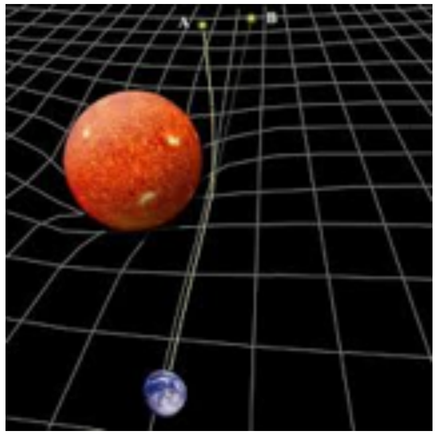

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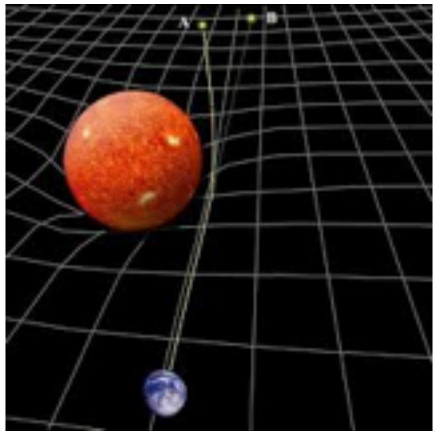
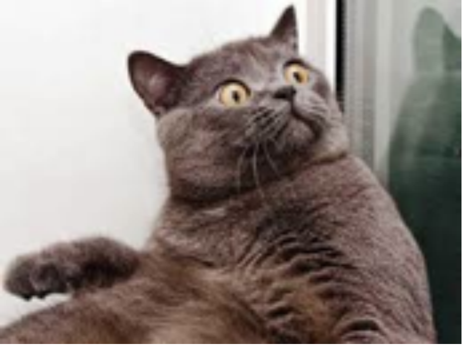
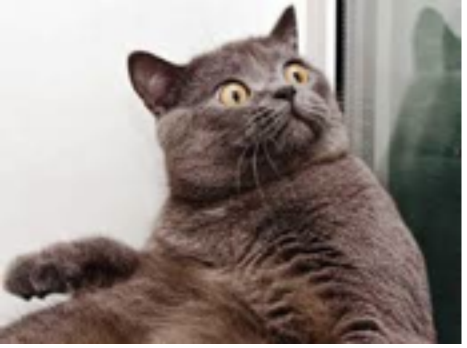




Surprises in constant curvature background



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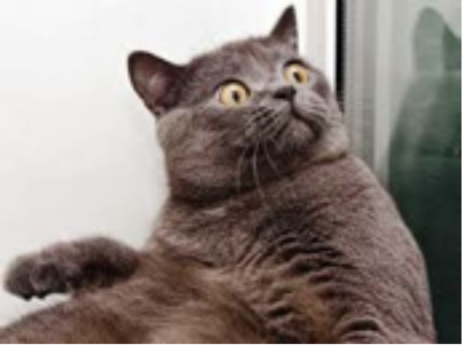

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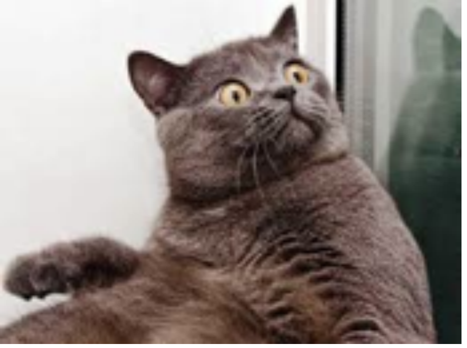

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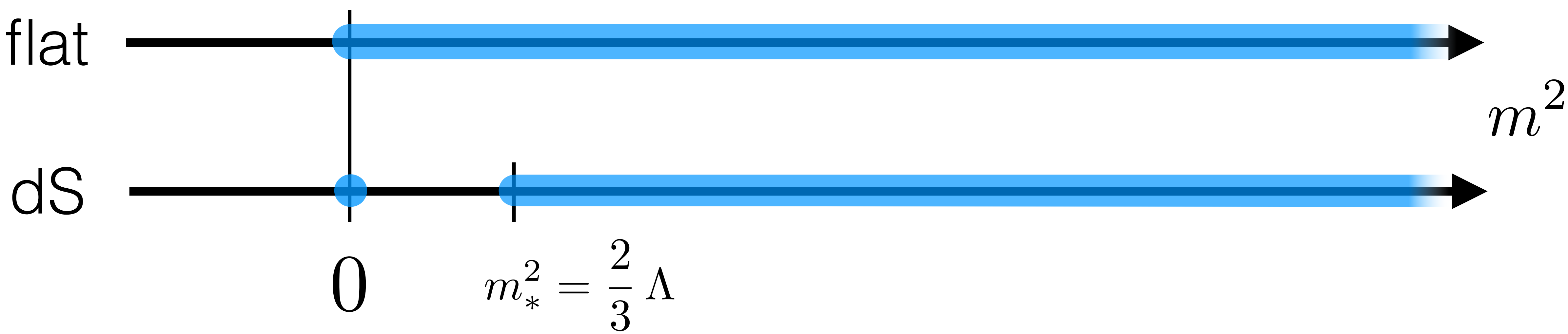

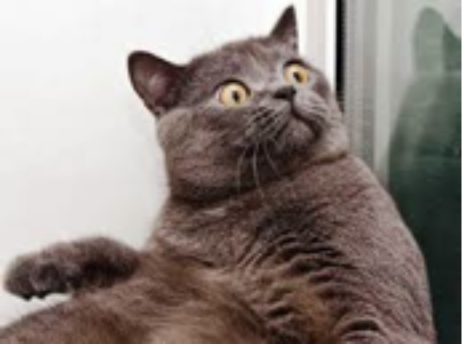

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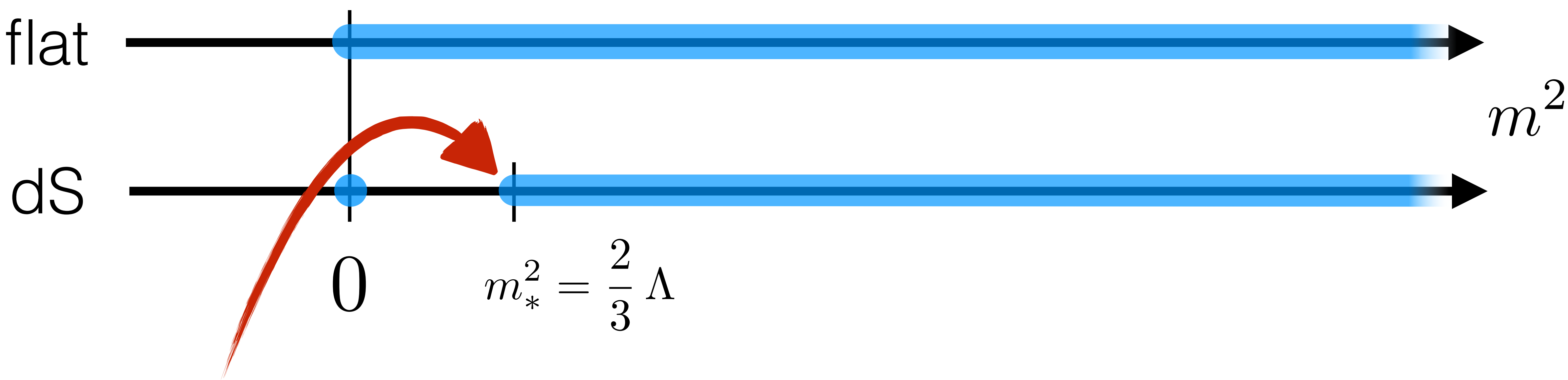

the end point is special! ❷ → ❷ + ⓪

Partially massless spin-2 field

Deser, Waldron

Partially massless spin-❷ field

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- Does this spectrum appear in some theories?

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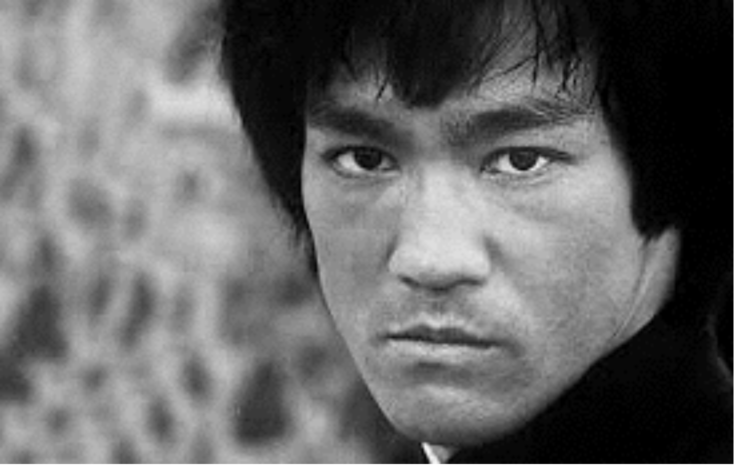
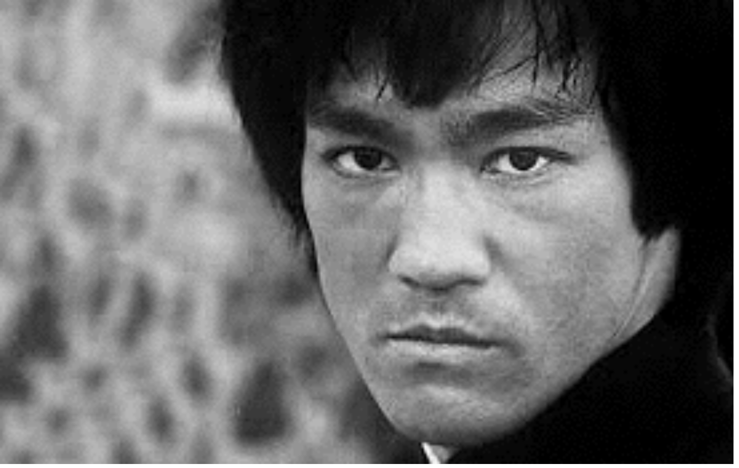
Does it decouples from Massive gravity / Bi-gravity
when $m^2 \to m_*^2$?

maybe yes *vs* maybe no



a **definite answer** to this question



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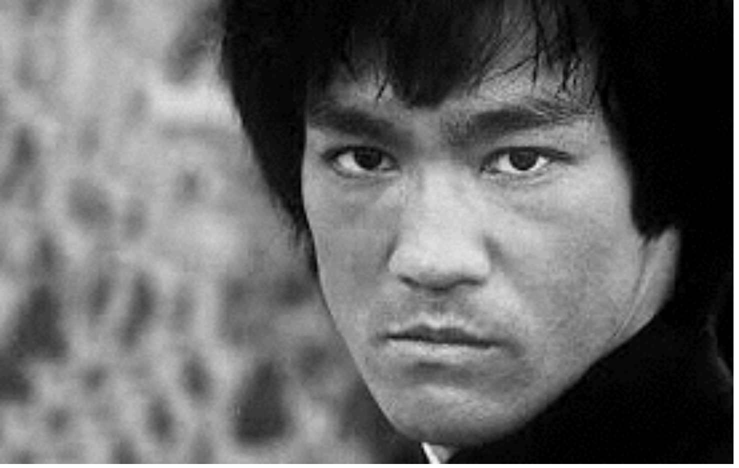

a **definite answer** to this question

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Let us reformulate the task:

Find out **all possible interacting theories**

for partially massless ❷ and massless ② !

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invariant under gauge transf. $\delta_\varepsilon \chi_i$

Berends, Burger, van Dam;
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<u>*strategy 1*</u>

expansion in χ_i :

$$S = S^{[2]} + S^{[3]} + \cdots \,, \qquad \delta_\varepsilon = \delta_\varepsilon^{[0]} + \delta_\varepsilon^{[1]} + \cdots \,,$$

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Admissibility condition Vasiliev

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Zinoviev

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- α, β, γ: coupling constants

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$\boxed{\delta^{[1]} \text{ part of gauge symmetry}}$

$$\delta^{[1]} h_{\mu\nu} = \alpha\left(2\,\bar{\nabla}_{(\mu}\bar{\xi}^{\rho}\,h_{\nu)\rho} + \bar{\xi}^{\rho}\,\bar{\nabla}_\rho h_{\mu\nu}\right) - 2\beta\,\partial^\rho\bar{\alpha}\left(2\,\bar{\nabla}_{(\mu}\varphi_{\nu)\rho} - \bar{\nabla}_\rho\varphi_{\mu\nu}\right)$$

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$$+\,2\,\sigma\gamma\,\partial^\rho\bar{\alpha}\left(\bar{\nabla}_\mu\varphi_{\nu)\rho} - \bar{\nabla}_\rho\varphi_{\mu\nu}\right)$$

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- so far, coupling constants α, β, γ are not determined

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Global symmetry

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$$\delta^{[1]}_{\bar{\varepsilon}}\,\delta^{[1]}_{\bar{\eta}} - \delta^{[1]}_{\bar{\eta}}\,\delta^{[1]}_{\bar{\varepsilon}} = \delta^{[1]}_{[\![\bar{\eta},\bar{\varepsilon}]\!]} + \delta^{[0]}_{[\bar{\eta},\bar{\varepsilon}]^{[1]}} + C^{[0]}_{ij}(\bar{\eta},\bar{\varepsilon})\,\frac{\delta S^{[2]}}{\delta\chi_i}\,\frac{\delta}{\delta\chi_j}$$

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$$\delta^{[1]} \varphi_{\mu\nu} = \sigma\beta \quad + \sigma\gamma$$

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$$\big[\,M^{AB}\,,\,K^{CD}\,\big] = 2\sigma\beta\,\eta^{C[B}\,K^{A]}$$

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❖ Global sym: $\begin{cases} SO(2,4) & \text{for} \quad \sigma\,\Lambda < 0 \\ SO(1,5) & \text{for} \quad \sigma\,\Lambda > 0 \end{cases}$

Admissibility condition

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$$\mathcal{A}_\mu = \sigma\,\alpha^2\,\tfrac{2\,\Lambda}{3}\,\bar\alpha_{[1}\,\partial_\mu\bar\alpha_{2]}$$

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- When $\sigma = +1$, no solution for real α, γ

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- When $\sigma = -1$, we recover **Conformal Gravity**