Schwarzschild:

$$ds^2 = -\left(1-\frac{2M}{r}\right)dt^2 + \frac{dr^2}{1-\frac{2M}{r}} + r^2 d\Omega^2$$

Horizons? $r = 2M$?

Minkowski: $ds^2 = -dt^2 + dr^2 + r^2 d\Omega_2^2$

$$dx^2 + dy^2 = dr^2 + r^2 d\phi^2$$

$\phi \in [0, 2\pi]$

$$\begin{cases} x = r\cos\phi \\ y = r\sin\phi \end{cases}$$

$r = 0$ — координатная особенность

$$ds'^2 = \nu\, dr^2 + r^2 d\phi^2, \quad \nu \geq 1, \quad r \geq 0$$

$$\frac{ds'^2}{\nu} = dr^2 + \frac{r^2}{\nu} d\phi^2, \quad \tilde{\phi} = \frac{\phi}{\sqrt{\nu}}$$

$$ds^2 \sim dr^2 + r^2 d\tilde{\phi}^2 \qquad \tilde{\phi} = \left[0, \frac{2\pi}{\sqrt{\nu}}\right]$$

$r = 0$ коническая сингулярность

Диаграммы Пенроуза

Пространство Минковского:

$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega_2^2$$

Theorem: $g'_{\mu\nu}(x) = g_{\mu\nu}(x)\cdot e^{2\omega(x)}$ имеют одинаковые нулевые геодезические

timelike/spacelike кривые в $g_{\mu\nu}$ $\Rightarrow$,,, в $g'_{\mu\nu}$

$g'_{\mu\nu}$ и $g_{\mu\nu}$ — конформно эквивалентны

Выберем $\omega(x) \to \infty$, $x \to \infty$: $x = \infty \to x = x_0$ в $g'_{\mu\nu}$

– конформная компактификация

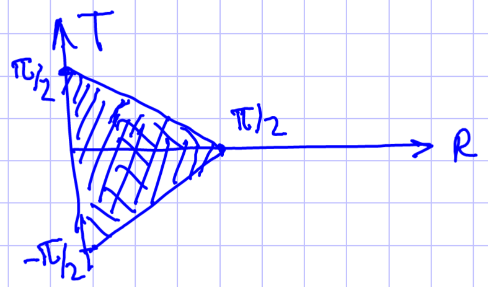
$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega_2^2, \quad d\Omega_2^2 = \sin^2\theta\, d\varphi^2 + d\theta^2$$

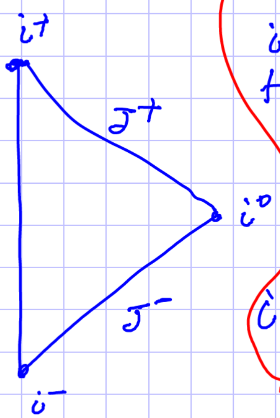
Бесконечность : $r \to \infty$, $|t| \to \infty$

$$\boxed{\begin{aligned} T+R &= \operatorname{arctg}(t+r) \\ T-R &= \operatorname{arctg}(t-r) \end{aligned}} \qquad \begin{aligned} &|T \pm R| < \frac{\pi}{2} \\ &R \geq 0 \end{aligned}$$

$$ds^2 = \frac{1}{\cos^2(T+R)\cos^2(T-R)}\left(-dT^2 + dR^2 + \left(\frac{\sin 2R}{2}\right)^2 d\Omega_2^2\right)$$

Boundary: $|T \pm R| = \pi/2$





$i^{\mp}$ – past/future timelike infinity

$J^{\mp}$ – past/future null infinity

i^0 – spatial infinity

i^- : $R=0$, $T = -\pi/2$ $\Rightarrow$ $2t = \operatorname{tg}(T+R) + \operatorname{tg}(T-R) = 2\operatorname{tg}(-\pi/2)$

$t = -\infty$, $\forall r$

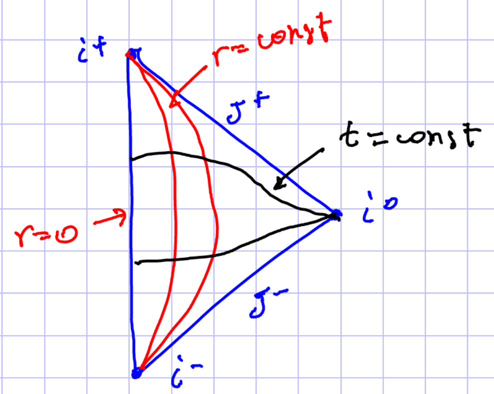
i^+ : $R=0$, $T = \pi/2$, $t = \operatorname{tg}\frac{\pi}{2} = +\infty$ $\forall r$

J^- : $T = -\pi/2 + R$ $\to$ $2t = \operatorname{tg}(2R - \pi/2) + \operatorname{tg}(-\pi/2) = -\infty$

$t = -\infty$ $\quad R > 0$

J^+ : $T = \pi/2 - R$ $\quad t = +\infty$

i^0 : $T = 0$, $R = \pi/2$ $\to$ $r = \infty$, t – любое



de Sitter:

$$ds^2 = -d\tau^2 + \cosh^2\tau \, d\Omega_3^2 \,, \quad d\Omega_3^2 = d\chi^2 + \sin^2\chi \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)$$

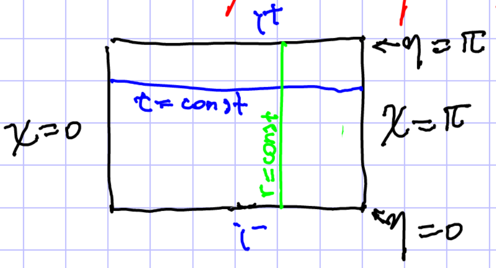
$$\tau \in (-\infty, +\infty) \,, \quad \chi \in [0, \pi] \,, \quad \theta \in [0, \pi] \,, \quad \varphi \in [0, 2\pi)$$

$$\sin\eta = \operatorname{sech}\tau \qquad \left(\tan\frac{\eta}{2} = e^{\tau}\right) \,, \quad \eta \in [0, \pi]$$

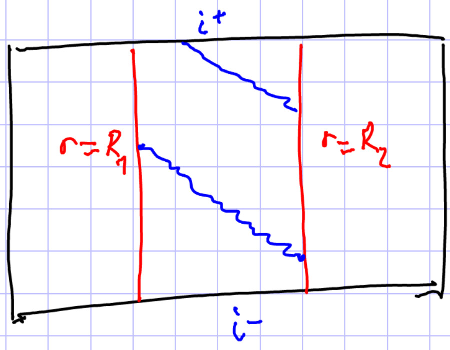
$$\Rightarrow ds^2 = \frac{1}{\sin^2\eta}\left(-d\eta^2 + d\chi^2 + \sin^2\chi \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)\right)$$

Boundary: $\eta = 0, \pi$ (i^-, i^+)

- Нет i^0, $\mathcal{J}^\pm$
- $i^\pm$ – spacelike



Horizons:



anti-de Sitter ($\Lambda < 0$):

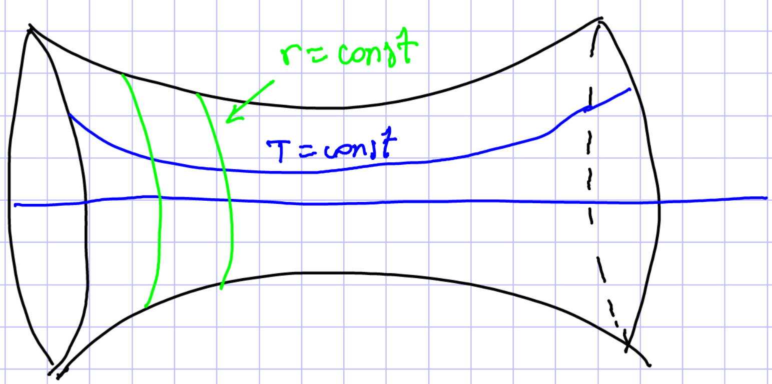
$$ds^2 = -\cosh^2 r \, dT^2 + \left(dr^2 + \sinh^2 r \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)\right)$$

$r \in [0, \infty)$

$\theta \in [0, \pi]$, $\varphi \in [0, 2\pi)$

Координ. сингулярност.:

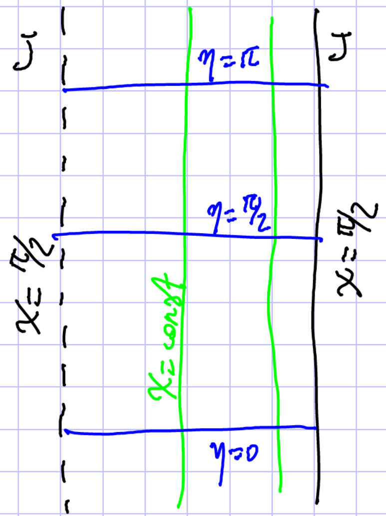
$r = 0$, $\theta = 0, \pi$



Conformal structure:

$$\operatorname{tg}\chi = \sinh r \Rightarrow ds^2 = \frac{1}{\cos^2\chi}\left(-dT^2 + d\chi^2 + \sin^2\chi \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)\right)$$

Boundary: $\chi = \pi/2$, $T = \pm\infty$ $\quad (T \in [0,\pi] \times \infty$ copies$)$

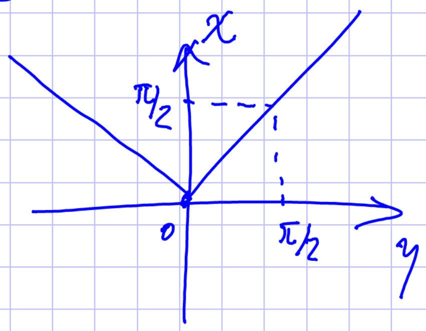


J и i^0 – timelike

$\chi \in [0, \pi/2]$

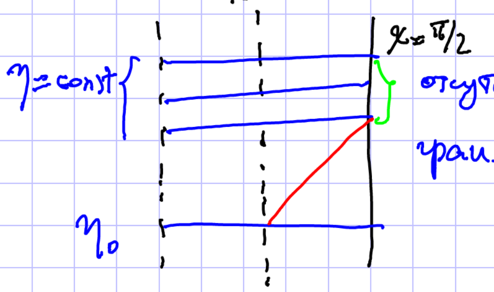
$\chi \pm \eta = c$

$\chi = c \pm \eta$



- Отсутствует глобальная гиперболичность:

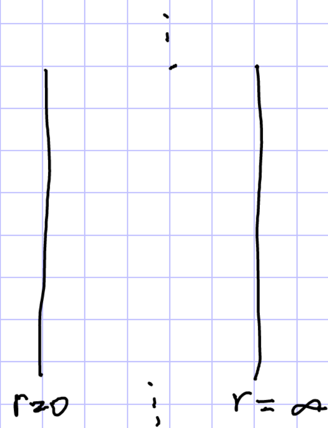
Нулевая геодезическая не пересекает $\eta > \eta_0$:



отсутствие предсказания: необходимы гран. условия на границе.

Эквивалентная форма метрики:

$$ds^2 = -(1+r^2)\,dt^2 + \frac{dr^2}{1+r^2} + r^2 d\Omega_2^2$$



Black holes (Kruskal coordinates)

$$ds^2 = -\left(1-\frac{2M}{r}\right)dt^2 + \frac{1}{1-\frac{2M}{r}}dr^2 + r^2 d\Omega_2^2 \qquad 2M = 1$$

$r_* = r + \log(r-1)$ $\quad$ (радиальные null geodesics $t = \pm r_* + C$)

$$\left[\ \frac{dr}{dt} = 1 - \frac{1}{r} \qquad \frac{dt}{dr} = \frac{1}{1-1/r} = 1 + \frac{1}{r-1}\right.$$

$$\left. t = r + \log(r-1) + C \ \right]$$

$$\begin{cases} u = -e^{\frac{r_* - t}{2}} \\ v = e^{\frac{r_* + t}{2}} \end{cases}$$

$$ds^2 = -\frac{4}{r}e^{-r}\,du\,dv + r^2 d\Omega_2^2$$

$uv = -e^{r_*} = (1-r)e^{r}$

Сингулярность: $uv = 1$

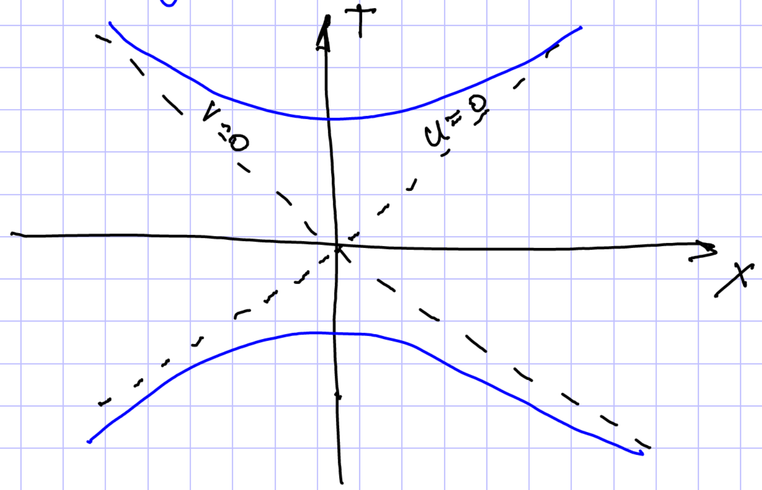
горизонт: $u=0$ или $v=0$

$$\begin{cases} u = T - X \\ v = T + X \end{cases} \Rightarrow ds^2 = \frac{4}{r} e^{-r}\left(-dT^2 + dX^2\right) + r^2 d\Omega_2^2$$

$$\boxed{X^2 - T^2 > 1}$$

$T' \pm X' = \operatorname{arctg}(T \pm X)$

$\operatorname{tg}(T'+X')\,\operatorname{tg}(X'-T') > 1$

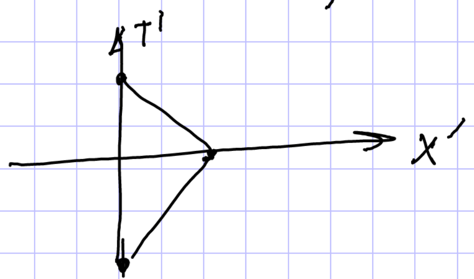


$|X' \pm T'| < \pi/2$

Напомним Минковский

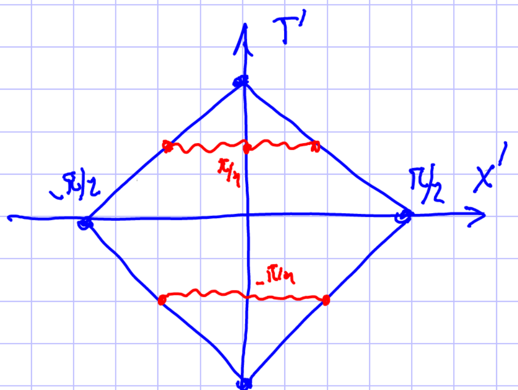
$|X' \pm T'| < \pi/2$, $X' > 0$

$\Rightarrow$

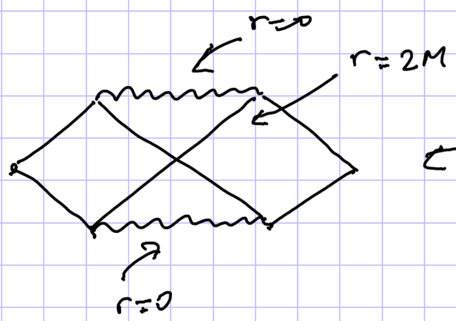


Теперь, $|X' \pm T'| < \pi/2$, нет: $X' > 0$

$X^2 - T^2 > 1 \Rightarrow |T'| < \pi/4$

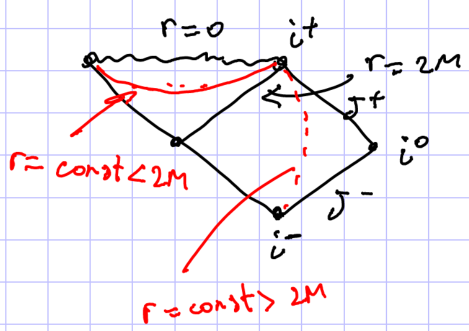


$\Rightarrow$



Аналитич. продолжение

Schwarz:



Hawking temperature

$$ds^2 = +\left(1-\frac{2M}{r}\right)dt^2 + \frac{dr^2}{1-\frac{2M}{r}} + r^2 d\Omega_2^2$$

Euclidian BH

Рассмотрим область вблизи горизонта: $r - 2M = \frac{x^2}{8M}$

$$ds_E^2 = \underbrace{(\varkappa x)^2 d\tau^2 + dx^2}_{\text{Euclidian Rindler}} + \frac{1}{4\varkappa^2} d\Omega_2^2 \ , \quad \varkappa = \frac{1}{4M}$$

Вспомним 2d Minkowski: $ds^2 = dr^2 + r^2 d\phi^2$, $\phi = \phi + 2\pi$

$\Rightarrow \quad \tau\varkappa = \tau\varkappa + 2\pi$, $\tau \sim \tau + \frac{2\pi}{\varkappa}$

Функциональный интеграл $\to$ $Z = \mathrm{tr}\, e^{-\beta H}$, $\beta = \frac{1}{T}$ – период

$\beta = \frac{2\pi}{\varkappa}$ $\Rightarrow$ $\boxed{T_H = \frac{\varkappa}{2\pi} = \frac{1}{8M\pi}}$ – температура излучения Хокинга.

NOTE ON a black brane in $d=4$.

- HS theory

— free fields $\Rightarrow$ Fronsdal action, representation theory, ...

— Cubic vertices

— Vasiliev equations

— AdS / CFT

- GR black holes

— Micro states

Examples of BHs in AdS_4:

$$ds^2 = -\left(k - \frac{M}{r} + \frac{r^2}{\ell^2}\right)dt^2 + \frac{dr^2}{k - \frac{M}{r} + \frac{r^2}{\ell^2}} + \frac{r^2}{\ell^2}\, d\Sigma_k^2$$

$\ell \rightarrow$ AdS radius

$k = 0, \pm 1$ Carter parameter (ε)

$$d\Sigma_k^2 = \begin{cases} \ell^2 d\Omega_2^2 & k = +1 \\ dx^2 + dy^2 & k = 0 \\ \ell^2 dH_2^2 & k = -1 \end{cases}$$

$M = 0,\ k = 0 \Rightarrow$ Poincare patch of AdS_4

$$ds^2 = \frac{1}{z^2}\left(-dt^2 + dx^2 + dy^2 + dz^2\right), \quad z = 1/r$$

unfolded view on BH's

(V.D, A. Matveev, M. Vasiliev)

- Unfolding in general:

$dW^A = F^A(W)$ – first order eqs, differential forms, many fields (∞)

- In case of a BH – the system is not dynamical $\Rightarrow$ finite amount of fields.

Some key observations:

- **Riemann is made out of a Killing form:**

$$C_{ab,cd} \sim (\varkappa_{ab}\varkappa_{cd})^{\boxplus} \quad \text{(Wald)} \quad (\epsilon_{abcd})$$

$$\varkappa_{ab} = -\varkappa_{ba} = D_a V_b$$

V_a – Killing vector

ANSATZ

BH unfolded system has only two 0-forms

$$W^A \to \begin{matrix} x_{ab} \\ V_a \end{matrix}$$

Example:

Let D be AdS covariant differential

$$D^2 \sim h \wedge h$$

$$\begin{cases} D x_{ab} = V_a h_b - V_b h_a \\ D V_a = x_{ab} h^b \end{cases} \quad \longleftrightarrow \quad D_0 \hat{K}_{MN} = 0$$

$$(x_{ab}, V_a) \to \hat{K}_{MN}$$

In $d=4$ $\quad o(3,2) \sim sp(4,R)$

$$\underset{o(3,2)}{\hat{K}_{[MN]}} \sim \underset{sp(4)}{K_{(AB)}} \qquad D_0 K_{AB} = 0$$

$$K_{AB} = \begin{pmatrix} x_{\alpha\beta} & V_{\alpha\dot{\beta}} \\ V_{\beta\dot{\alpha}} & \bar{x}_{\dot{\alpha}\dot{\beta}} \end{pmatrix}$$

Invariants: $\begin{cases} C_1 = \mathrm{Tr}\, K^2 \\ C_2 = \mathrm{Tr}\, K^4 \end{cases}$ ← "Casimir" operators

Scalars: $\quad G_1 = x_{ab} x^{ab} \qquad G_2 = x_{ab} x_{cd} \epsilon^{abcd}$

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DEFORMATION

$$\begin{cases} Dx_{ab} = V_a h_b - V_b h_a \\ DV_a = f_a(x, h) \end{cases}$$

Integrability $\quad D^2 = R, \quad DR = 0$

Such deformation was found to exist in $d=4$ and $d=5$ explicitly.

d=4:

Free parameters: $M, N, e, g \quad (\lambda)$

First integrals: I_1, I_2

Hidden symmetry: $Dx_{ab} \sim \left(\square \oplus \begin{smallmatrix}\square & \square \\ \square & \end{smallmatrix}\right) h$

absent

Killing-Yano tensor

(only in D-type metrics)

Scalars: $G, \bar{G}$ (Canonical Carter variables)

Integrating flow: $\left.\begin{array}{l} \vec{\mu} = (M, N, e, g) \\ \left[\frac{\partial}{\partial \vec{\mu}}, d\right] = 0 \end{array}\right\}$

$$\boxed{g_{\mu\nu} = g^0_{\mu\nu} + h_{\mu\nu}(K_{AB}, \vec{\mu})}$$

Inequivalent solutions

- as many as conjugacy classes of $sp(4)$

$$K_{AB} \sim G K G^{-1}, \quad G \in Sp(4)$$

(Recall HBTZ)

times $\vec{\mu}$

- parameter space:

$\vec{\mu}$, $\quad C_2 = \mathrm{Tr}\, K^2 = 0, \pm 1$ (Carter parameter)

$C_4 = \mathrm{Tr}\, K^4$ (rotation parameter)

$$ds^2 = \frac{\Delta_r}{r^2+y^2}\left(d\tau + y^2 d\psi\right)^2 - \frac{\Delta_y}{r^2+y^2}\left(d\tau - r^2 d\psi\right)^2 - \frac{r^2+y^2}{\Delta_r} dr^2 - \frac{r^2+y^2}{\Delta_y} dy^2$$ (Carter - Plebanski)

$$\begin{cases} \Delta_r = 2Mr - e^2 - g^2 + r^2(\lambda^2 r^2 + \epsilon) + a^2 \\ \Delta_y = 2Ny + e^2 + g^2 + y^2(\lambda^2 y^2 - \epsilon) + a^2 \end{cases}$$

K_{AB} - generic

- If K_{AB} is not Hermetian $K_{AB} \neq K^{\dagger}_{AB}$

$\Rightarrow$ Plebanski - Demianski (never published)

- Global symmetries originate from centralizer K_{AB}

$$[\xi, K] = 0$$ At least two Killings K, K^{-1}

MOST SYMMETRIC CASES

One may impose the following AdS-invariant condition

$$\boxed{K_A{}^C K_C{}^B = -k\,\delta_A{}^B}\,, \quad k = 0, \pm 1$$

if also K_{AB} is timelike then

- $k = 1$ — Schwarzschild
- $k = 0$ — planar BH
- $k = -1$ — hyperbolic BH

$$\begin{cases} D x_{\alpha\beta} = -\frac{1}{2}\left(h_\alpha{}^{\dot\gamma} v_{\beta\dot\gamma} + h_\beta{}^{\dot\gamma} v_{\alpha\dot\gamma}\right) \\ D v_{\alpha\dot\alpha} = -\frac{1}{2}\rho(r) h_\alpha{}^{\dot\gamma} \bar{x}_{\dot\alpha\dot\gamma} - \frac{1}{2}\rho(r) h^\gamma{}_{\dot\alpha} x_{\gamma\alpha} \end{cases} \qquad \underline{k = 0}$$

$$x^2 = -r^2\,, \quad \rho(r) = 1 + \frac{1}{2}\frac{M}{r^3}$$

HS extension

Weyl tensor: $C_{\alpha\beta\gamma\delta} = \frac{M}{r} \varkappa^{-1}_{(\alpha\beta} \varkappa^{-1}_{\gamma\delta)}$

linear in M

$\Rightarrow$ BH is a solution of Fronsdal eqs.

Can be seen from Kerr-Schild form:

$$g_{\mu\nu} = \overset{\circ}{g}_{\mu\nu} + \underbrace{\frac{M}{u(x)} k_\mu k_\nu}_{h_{\mu\nu}:\quad D h_{\mu\nu} + \ldots = 0}$$

Fronsdal fields:

$$\phi_{\mu_1 \ldots \mu_s} = \frac{M_s}{u(x)} k_{\mu_1} \ldots k_{\mu_s}$$

$$D \phi_{\mu_1 \ldots \mu_s} + \ldots = -2(s-1)(s+1)\lambda^2 \phi_{\mu_1 \ldots \mu_s}, \quad s \geq 0$$

free Vasiliev eqs. in $d=4$

Fields:

$$\left.\begin{array}{ll} \text{1-forms} & \omega_{\alpha_1,\dots,\alpha_n,\dot\beta_1,\dots\dot\beta_m} \\ \text{0-forms} & C_{\alpha(n),\dot\beta(m)} \end{array}\right\} \rightarrow$$

packed into generating functions:

$$W(y,\bar y|x) = \sum_{m,n}^{\infty} \frac{1}{m!\,n!}\, \omega_{\alpha(m),\dot\beta(n)} (y^\alpha)^m (\bar y^{\dot\beta})^n$$

$$B(y,\bar y|x) = \sum_{m,n}^{\infty} \frac{1}{m!\,n!}\, C_{\alpha(n),\dot\beta(m)} (y^\alpha)^n (\bar y^{\dot\beta})^m$$

Star-product: $[Y_A, Y_B]_* = 2i\,\epsilon_{AB}$

$$Y_A = (y_\alpha, \bar y_{\dot\alpha})$$

AdS_4 Vacuum: $W_0 = \omega_{\alpha\beta} y^\alpha y^\beta + \bar\omega_{\dot\alpha\dot\beta} \bar y^{\dot\alpha} \bar y^{\dot\beta} +$
$+ h_{\alpha\dot\alpha} y^\alpha \bar y^{\dot\alpha}$

$$\boxed{dW_0 + W_0 * W_0 = 0}$$

free field equations

$$\begin{cases} dB + W_0 * B - B * \pi(W_0) = 0 & , \pi(y,\bar{y}) = (-y,\bar{y}) \\ dW + [W_0, W] = h \wedge h \frac{\partial^2}{\partial y^2} B\Big|_{\bar{y}=0} + c.c.\Big|_{y=0} & \end{cases}$$

Observe: if $f(Y|x)$: $df + [W_0, f]_{*} = 0$

$$\Rightarrow \boxed{B = c_1 f(Y|x) * \delta(y) + c_2 f(Y|x) * \delta(\bar{y})}$$

is a solution.

Take $f(Y|x) = F(K_{AB} Y^A Y^B)$

Weyl tensors:

$$C_{\alpha(2s)} = c_1(s) \frac{1}{r} \varkappa^{-1}_{\alpha\alpha} \ldots \varkappa^{-1}_{\alpha\alpha} \qquad \varkappa_{\alpha\beta} = K_{\alpha\beta}$$

$$\bar{C}_{\dot\alpha(2s)} = c_2(s) \frac{1}{r} \hat{\varkappa}^{-1}_{\dot\alpha\dot\alpha} \ldots \hat{\varkappa}^{-1}_{\dot\alpha\dot\alpha} \qquad \hat{\varkappa}_{\dot\alpha\dot\beta} = (K^{-1})_{\dot\alpha\dot\beta}$$

Reality condition mixes these together

To disentangle one uses projector Π

$$B = \Pi\left(f(Y) * \delta(y)\right) + c.c.$$

$$\Rightarrow B = \sum_{s=0}^{\infty} B_s$$

$$B_s = \frac{m_s}{r} e^{i\bar{x}^{-1\dot{\alpha}\dot{\beta}} v_{\alpha\dot{\beta}} \bar{y}_{\dot{\alpha}} y^{\alpha}} \times \operatorname{Res} \frac{1}{z^{1-s}} e^{\frac{1}{2}(az - \frac{b}{z})}$$

$$a = \left(x_{\alpha\beta} + \bar{x}^{-1}{}_{\dot{\alpha}\dot{\beta}} v_{\alpha}{}^{\dot{\alpha}} v_{\beta}{}^{\dot{\beta}}\right) y^{\alpha} y^{\beta}$$

$$b = \bar{x}^{-1}{}_{\dot{\alpha}\dot{\beta}} \bar{y}^{\dot{\alpha}} \bar{y}^{\dot{\beta}}$$

No such problem when $K_{AB}^2 = -k\,\delta_{AB}$

- **$k=1$** – Schwarzschild (NUT)

admits extension to HS to all orders (V.D., M. Vasiliev)

$$P = 4 \exp \tfrac{1}{2} K_{AB} Y^A Y^B, \qquad P * P = P$$

$$B = M \cdot P * \delta(y)$$

(C. Iazeolla, P. Sundell)

$$B = \sum M_n P_n \star \delta(y), \qquad P_n = L_n(x)\, e^{x}$$

- $k = -1$ never been considered (hyperbolic BH)

- $k = 0$ HS generalization of a black brane

$$B = \sum_{s \geq 0}^{\infty} m_s B_s, \qquad \forall m_s$$

$$B_s = \frac{m_s}{r}\, e^{-i x^{-1}{}_{\alpha}{}^{\beta} v_{\beta\dot{\alpha}} y^{\alpha} \bar{y}^{\dot{\alpha}}} \times \frac{\left(x^{-1}_{\dot{\alpha}\dot{\beta}} \bar{y}^{\dot{\alpha}} \bar{y}^{\dot{\beta}}\right)^{s}}{s!}$$

NONLINEAR LEVEL

Some formal exact solution can be written down for

$$m_s = m, \quad \forall s$$

(Used 'Kerr-Schild' trick to linearize the solution)

FAIL!

- The solution does not exist in physical sector to the 2^{nd} order.

- For finite amount of HS charges

$$m_{s_1}, m_{s_2}, \ldots, m_{s_N}$$

the solution does not exist either

- Some delicate fine tuning of infinite number of charges

$$m_s \qquad s = s_1, s_2, \ldots \infty$$

may save the day from divergencies.

Conclusion

- It is shown that AdS_4 planar black hole admits the unfolded description in terms of time-like AdS global symmetry parameter

$$g_{\mu\nu} = g^0_{\mu\nu} + f_{\mu\nu}(K_{AB}), \quad K_A{}^C K_C{}^B = 0$$

- There is a natural bosonic lift to HS that generalizes black brane at least at free level.

- It is not clear whether the solution exists beyond free level, For finite amount of charges it does not.