Higher Spin Theory and Holography, December 8, 2014
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Quantum Coulomb solution & black hole microstates

1) Claim: microstates for BH entropy related to non-proper gauge dof rather than physical gravitons and should be quantized as such

2) Arguments:

   a) linearized Schwarzschild solution involves longitudinal dof

   b) no physical gravitons in 3d but BTZ black hole

   c) observable: surface charge, connected with gauge trsfs

3) BRST quantization of free Maxwell field

4) Quantum Coulomb solution

2 a & b) ADM 1962, Dynamics of GR, gr-qc/0405109

- Hamiltonian formulation of linearized GR = PF of massless spin 2 fields on Minkowski background

$$S_{PF} = \int dt \int d^{D-1}x \, \left(\pi^{mn} \dot{h}_{mn} - n^m \mathcal{H}_m - n \mathcal{H}_\perp - \mathcal{H}_{PF} \right)$$

$$H_{PF} = \int d^{D-1}x \, \Big(\pi^{mn} \pi_{mn} - \frac{1}{D-2} \pi^2 + \frac{1}{4} \partial^r h^{mn} \partial_r h_{mn}$$
$$- \frac{1}{2} \partial_m h^{mn} \partial^r h_{rn} + \frac{1}{2} \partial^m h \partial^n h_{mn} - \frac{1}{4} \partial_m h \partial^m h \Big)$$

$$\mathcal{H}_m = -2 \partial^n \pi_{mn} , \quad \mathcal{H}_\perp = \Delta h - \partial^m \partial^n h_{mn} , \quad h_{00} = -2n$$
$$h_{0i} = n_i$$

- Decomposition of symmetric rank 2 tensors:

	# of components D=4	D=3
$h_{mn} = h^{TT}_{mn} + h^{T}_{mn} + h^{L}_{mn}$		
$h^{L}_{mn} = \partial_m A_n + \partial_n A_m$	3	2
$h^{T}_{mn} = \frac{1}{2} (\delta_{mn} \Delta - \partial_m \partial_n) C$	1	1
$h^{TT}_{mn} = h_{mn} - h^{T}_{mn} - h^{L}_{mn}$	2	0

$$A_m = \Delta^{-1}\left(\partial^m h_{mn} - \frac{1}{2}\Delta^{-1}\partial_m\partial^k\partial^l h_{kl}\right)$$

$$C = \Delta^{-1}\left(h - \Delta^{-1}\partial^m\partial^n h_{mn}\right)$$

Orthogonality: $\int d^{D-1}\, \dot{h}_{mn}\, \pi^{mn} = \int d^{D-1}\left(\dot{h}^{TT}_{mn}\pi^{mn}_{TT} + \dot{h}^{L}_{mn}\pi^{mn}_{L}\right.$
$\left. + \dot{h}^{T}_{mn}\pi^{mn}_{T}\right)$

canonical pairs:

$(h^{TT}{}_{mn}, \pi^{mn}_{TT})$, $(h^{L}{}_{mn}, \pi^{kl}_{L})$, $(h^{T}{}_{mn}, \pi^{mn}_{T})$

$\mathcal{H}_m = 0 \Leftrightarrow \pi^{kl}_{L} = 0$ gauge fixation: $h^{L}{}_{mn} = 0$

$\mathcal{H}_\perp = 0 \Leftrightarrow h^{T}{}_{mn} = 0$ " $\pi^{mn}_{T} = 0$

reduced Hamiltonian: $H^R = 0$ $D = 3$

$$H^R = \int d^3x\left(\pi^{mn}_{TT}\,\pi^{TT}_{mn} + \frac{1}{4}\partial_r h^{TT}_{mn}\partial^r h^{mn}_{TT}\right)$$

canonical change of variables

$(h^{TT}{}_{mn}, \pi^{kl}_{TT})$, (C, π^C), (A_m, π^m)

Coupling to a massive particle

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Coupling to sources

$$S_T = \frac{1}{16\pi G} S^{PF} + S^J \,, \qquad S^J = \int d^D x \; h_{\mu\nu} T^{\mu\nu}$$

$$T^{\mu\nu}(x) = \delta^\mu_0 \delta^\nu_0 M \delta^3(x) \,,$$ mass at rest at origin

$\Rightarrow$ only Hamiltonian constraint is affected

$$\mathcal{H}_\perp = 16\pi G \, M \delta^3(x) \quad | \; \Delta^{-1} \qquad \Delta \frac{1}{r} = -4\pi \delta^3(x)$$

$$\mathcal{H}_\perp = \Delta^2 C \qquad \Delta C = GM \frac{4}{r}$$

all EOM solved by $\begin{cases} C = 2GMr \,, \quad n = -GM\frac{1}{r} = -\frac{1}{2} h_{00} \\ A_m = 0 = n^m = h^T_{mn} \,, \quad \pi^{mn} = 0 \end{cases}$

$$\Rightarrow h_{mn} = GM\left(\delta_{mn} + \frac{x_m x_n}{r^3}\right)$$

standard form: gauge trsf with parameter

$$\xi^m = GM\left(-\frac{1}{2}\frac{x^m}{r}\right) \Rightarrow h_{mn} = 2GM \frac{x_m x_n}{r^3}$$

spherical coordinates: $h_{rr} = \frac{2GM}{r} = h_{00}$

no π variables involved!

2c) observable. ADM mass = surface charge

$$16\pi G\, P = -\oint_{S^\infty} d\sigma_m\, \partial^m \Delta C = \oint d\sigma_m \left(\partial_n h^{mn} - \partial^m h \right)$$

$$= M$$

↑ on-shell

exactly as for Gauss law in EM

quantum level → study EM first

simpler but similar physics

3) EM first order

$$S^{EM} = \int dt\, d^3x \left[\dot{A}_\mu \pi^\mu - \mathcal{H}_0 - \lambda^1 \pi^0 + (\lambda^2 + A_0)\partial_i \pi^i \right]$$

$$(A_\mu, \pi^\mu),\ \lambda^1, \lambda^2 \qquad H_0 = \int d^3x \left(\tfrac{1}{2}\pi^i \pi_i + \tfrac{1}{4} F_{ij} F^{ij} \right), \quad E^i = -\pi^i$$

$\phi_1 = \pi^0$ first class constraints
$\phi_2 = -\partial_i \pi^i$

(A_0, π^0) among canonical variables

$$A_i = A_i^T + A_i^L \qquad A_i^L = \partial_i A, \quad A = \Delta^{-1}\partial_i A^i$$

(A_i^T, π^i_T) physical

$(A_i^L, \pi^i_L), (A_0, \pi^0)$ unphysical

- Quantization: either reduced phase space quantization
  - transverse dof in positive def. Hilbert space
- in the presence of charges, quantize transverse fluctuations around classical charged solution

what about quantum mechanics of background solution?

<u>or</u> quantize all polarizations in indef. metric Hilbert space

2) options (i) Gupta-Bleuler $[\hat{a}_\mu(\vec{k}), \hat{a}^+_\nu(\vec{k}')] = \delta^3(\vec{k}-\vec{k}')\,\eta_{\mu\nu}$

physical state condition $(\partial_\mu \hat{A}^\mu)^+(x)\,|\psi\rangle^{phys} = 0$

null states decouple

(ii) BRST quantization (follow HT, Quantization of Gauge Systems, chapter 19)

fermionic dof $(\mathcal{P}, \eta)$, $(\bar{C}, \rho)$

cancel contributions from longit. & temporal γ's

BRST charge $\Omega = \int d^3x\,(\phi_1 \rho + \phi_2 \eta)$, $\{\Omega, \Omega\} = 0$

$H_K = H_0 + \{\Omega, K\}$, $K = \int d^3x\,\left(-i\bar{C}\,\partial_k A^k - \mathcal{P} A_0 + \left(\frac{i}{2}\alpha \bar{C}\pi^0\right)\right)$

$PI = \int \mathcal{D}(A_\mu \pi^\mu\, \eta\, \mathcal{P}\, \bar{C}\, \rho)\, e^{i\int d^3x\,(\dot{A}_\mu \pi^\mu + \dot{\eta}\mathcal{P} + \dot{\bar{C}}\rho - \mathcal{H}_K)}$

eliminate momenta $\rightarrow$ standard FP PI.

canonical quantization

$$A_0(\vec{x}) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3\vec{k}}{\sqrt{2\omega(\vec{k})}} \left[a_0(\vec{k})\, e^{i\vec{k}\cdot\vec{x}} + h.c. \right]$$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ ↱ polarization tensor

$$A_i(\vec{x}) = \quad\quad a_m(\vec{k})\, e^m_i(\vec{k})$$

$$\pi^i(\vec{x}) = -i \quad\quad a_m(\vec{k})\, e^m_i(\vec{k}) + a_0(\vec{k})\, e^3_i(\vec{k}) - h.c.$$

$$\eta \quad\quad\quad\quad c + h.c.$$

$$\bar{C} \quad\quad\quad\quad \bar{c} - h.c.$$

$$\bar{\rho} \quad\quad\quad\quad \bar{c} + h.c.$$

$$\rho \quad\quad\quad\quad c - h.c.$$

$$a = a_3 + a_0 \,, \quad b = \frac{1}{2}(a_3 - a_0)$$

non-vanishing CCR:

$$[\hat{a}_a(\vec{k}), \hat{a}^+_b(\vec{k}')] = \delta_{ab}\, \delta^3(\vec{k} - \vec{k}') \quad a = 1,2 \text{ transverse}$$

$$[\hat{a}(\vec{k}), \hat{b}^+(\vec{k}')] = \delta^3(\vec{k} - \vec{k}')$$

$$[\hat{c}(\vec{k}), \hat{\bar{c}}^+(\vec{k}')] = \delta^3(\vec{k} - \vec{k}')$$

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{pmatrix} \to \left(\begin{array}{cc|cc} 0 & 1 & & 0 \\ 1 & 0 & & \\ \hline & 0 & 1 & 0 \\ & & 0 & 1 \end{array}\right)$$

$$\hat{\Omega} = \int d^3\vec{k} \, \left[\hat{c}^+(\vec{k}) \, \hat{a}(\vec{k}) + \hat{a}^+(\vec{k}) \, \hat{c}(\vec{k}) \right]$$

$$H_K^{\alpha=1} = \int d^3\vec{k} \;\; \omega(\vec{k}) \; \hat{a}^{+a}(\vec{k}) \, \hat{a}_a(\vec{k}) + [\hat{\Omega}, \hat{K}_\omega]$$

$$\hat{K}_\omega = \int d^3\vec{k} \;\; \omega(\vec{k}) \left(\hat{c}^+(\vec{k}) \, b(\vec{k}) + \hat{b}^+(\vec{k}) \, \bar{c}(\vec{k}) \right)$$

physical state condition : $\hat{\Omega} |\psi\rangle^{phys} = 0$ BRST closed

(in particular : $\hat{\Omega} |0\rangle = 0$)

BRST exact states decouple

$${}^{phys}\langle \psi, \Omega | \chi \rangle = 0 \quad \forall |\chi\rangle, \quad (\hat{\Omega} = \hat{\Omega}^+)$$

quartet mechanism $H(\hat{\Omega}, \text{Fock}) \simeq$ Fock of $\hat{a}_a^+(\vec{k})$

ghosts & unphysical bosonic dof drop out from cohomology

4) Coupling to a source

$$S^T = S^{EM} - \int d^3x \; j^\mu(x) A_\mu(x) \,, \qquad j^\mu = \delta^\mu_0 \, Q \, \delta^3(x)$$

→ only Gauss law is modified:

$$\phi_2 \to \phi_2^Q = -\partial_i \pi^i + j^0$$

$$\hat{\Omega}^Q = \int d^3\vec{k} \left[\hat{c}^+(\vec{k}) \left(\hat{a}(k) - q(\vec{k}) \right) + \left(\hat{a}^+(\vec{k}) - q(\vec{k}) \right) \hat{c}(\vec{k}) \right]$$

$$q(\vec{k}) = \frac{Q}{(2\pi)^{3/2} \sqrt{2} \; \omega(\vec{k})^{3/2}} \qquad \text{c-number}$$

$$\Omega^Q |0\rangle \neq 0$$

but new vacuum $|0\rangle^Q$: $\left(\hat{a}(\vec{k}) - q(\vec{k})\right) |0\rangle^Q = 0$

$$\hat{b}(\vec{k}),\ \hat{c}(\vec{k}),\ \hat{\bar{c}}(\vec{k}),\ \hat{a}^a(\vec{k}) \; |0\rangle^Q = 0$$

$$\Rightarrow \Omega^Q |0\rangle^Q = 0$$

new vacuum in terms of old:

$$|0\rangle^Q = \prod_{k} e^{q(\vec{k}) \hat{b}^+(\vec{k})} |0\rangle = e^{\int d^3\vec{k}\, q(\vec{k})\, \hat{b}^+(\vec{k})} |0\rangle$$

coherent state of null photons

NB: unusual properties ${}^{Q}\langle 0|0\rangle^{Q} = 1$

physical coherent states: $|\alpha_1\rangle = e^{\alpha_1 \hat{a}_1^{+}}$ $\quad \langle \alpha_1^{*}|\alpha_1\rangle = e^{\alpha_1^{*}\alpha_1}$

Ehrenfest theorem:

$${}^{Q}\langle 0| \hat{A}_\mu |0\rangle^{Q} = \delta^{0}_{\mu} \frac{Q}{4\pi r} \qquad \text{(gauge dependent)}$$

$${}^{Q}\langle 0| \pi^{i}(x) |0\rangle = \frac{Q x^{i}}{4\pi r^{3}}, \qquad {}^{Q}\langle 0| \vec{\nabla}\times\vec{A} |0\rangle^{Q} = 0$$

(gauge invariant)

NB: IR regularisation is implicit:

$$FT\left(\frac{1}{\partial^2+\mu^2}\right) = \frac{e^{-\mu r}}{4\pi r}, \quad \mu \to 0^{+}$$

Perspectives:

1) Compute at fixed potential instead of fixed (macroscopic charge Q)

Evaluate $Z(\beta, \alpha) = \mathrm{Tr}_{\text{transverse}}\, e^{-\beta H_{phys}}\, \mathrm{Tr}_{\text{quartets}}\, e^{-\alpha Q}$

$$Q = \int_S d^2\sigma_i\, \Pi^i_L$$

contribution to entropy goes like surface

2) apply to AdS_3 gravity