Effective actions, nonrelativistic diffeomorphism invariance and some applications

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based on Phys. Rev. D 89, 064012 (2014),

arXiv: 1408.7031 [hep-th]

Plan

- motivations
- nonrelativistic diffs.
- the $c \to \infty$ limit
- adding $U(1)$ gauge field
- implications for viscosity tensor
- Hořava-Lifshitz gravity with $U(1)$ gauge symmetry

a nonrelativistic theory living on a curved d-dim. manifold with the metric g_{ij}

$$e^{iS_{eff}(A,g)} = \int \mathcal{D}\psi \, e^{iS_0(\psi, A, g)}$$

background fields $\quad A_\mu = (A_0, A_i) \quad\quad x^\mu = (t, x^i)$

g_{ij}

- transformations $\quad \delta t = 0 \quad \delta x^i = \xi^i(t, x)$

then

$$\delta A_0 = -\partial_k A_0 \xi^k - A_k \dot{\xi}^k$$
$$\delta A_i = -\partial_k A_i \xi^k - A_k \partial_i \xi^k - m g_{ik} \dot{\xi}^k$$
$$\delta g_{ij} = -\partial_k g_{ij} \xi^k - g_{kj} \partial_i \xi^k - g_{ik} \partial_j \xi^k$$

Son - Wingate (2006)

- $U(1)$ gauge transformations

$$\delta A_0 = -\dot{\alpha} \quad\quad \delta A_i = -\partial_i \alpha \quad\quad \delta g_{ij} = 0$$

* a system of noninteracting spin-zero particles

$$S = \frac{1}{2}\int dt\, dx \left[i\psi^\dagger \overleftrightarrow{\partial_t} \psi + 2A_0 \psi^\dagger \psi - \frac{g^{ij}}{m} (\partial_i \psi^\dagger + i A_i \psi^\dagger)(\partial_j \psi - i A_j \psi) \right]$$

with $\psi^\dagger \overleftrightarrow{\partial_t} \psi = \psi^\dagger \dot{\psi} - \dot{\psi}^\dagger \psi$ & $\delta\psi = -i\alpha\psi - \partial_k \psi \xi^k$

Alternatively,

$$g_{\mu\nu} = \begin{pmatrix} -1 + \frac{2A_0}{mc^2} & \frac{A_i}{mc} \\ \frac{A_i}{mc} & g_{ij} \end{pmatrix}$$

$$\delta g_{\mu\nu} = -\partial_\lambda g_{\mu\nu}\xi^\lambda - g_{\lambda\nu}\partial_\mu \xi^\lambda - g_{\mu\lambda}\partial_\nu \xi^\lambda$$

with $\xi^\lambda = \left(-\frac{\alpha}{mc}, \xi^k\right)$ & $x^0 = ct$

then take $c \to \infty$

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Example : $3 \to 2+1$

$$S = mc\int d^3x \sqrt{g^{(3)}}\, R^{(3)}$$

$$\downarrow \; c \to \infty$$

$$S = \int dt\, d^2x \sqrt{g}\left[\frac{m}{8}\left(\dot{g}^{ij}\dot{g}_{ij} + (g^{ij}\dot{g}_{ij})^2\right) + \frac{1}{\sqrt{g}}\varepsilon^{\mu\nu\lambda}\omega_\mu \partial_\nu A_\lambda - \frac{1}{4m}B^2\right]$$

$B = e^{ij}\partial_i A_j$, $e^{ij} = \frac{\varepsilon^{ij}}{\sqrt{g}}$, $\omega_0 = \frac{1}{2}\varepsilon^{ab} e^j_a \dot{e}^b_j$, $\omega_i = \frac{1}{2}\varepsilon^{ab} e^j_a \nabla_i e^b_j$

e^a_i – zweibein, $g_{ij} = \delta^{ab} e^a_i e^b_j$

Another example in 2 + 1

$$S = \int dt\, d^2x \left[\varepsilon^{\mu\nu\lambda} \omega_\mu \partial_\nu \omega_\lambda - \frac{1}{2m} \sqrt{g}\, R\, B \right]$$

$\omega_\mu = (\omega_0, \omega_i)$

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if Σ is a Riemann surface of non-zero curvature R

$$S = \int dt\, d^2x \sqrt{g} \left[RB - \frac{1}{R} g^{ij} \partial_i R \left(2m(\dot{\omega}_j - \partial_j \omega_0) + \partial_j B \right) \right]$$

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Problem with the Chern-Simons action

$$S = \int dt\, d^2x\, \varepsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda \quad \underbrace{\qquad\qquad}_{\text{corrections}}$$

it's not invariant $(\delta S \neq 0)$

3d action that allows one $S^{(3d)} \xrightarrow[c \to \infty]{} S$?

introduce a $U(1)$ gauge field in 3d such that

$$\mathcal{A}_\mu = \left(\frac{\mathcal{A}_0}{c}, \mathcal{A}_i\right) \qquad \delta\mathcal{A}_\mu = -\partial_\nu \mathcal{A}_\mu \xi^\nu - \mathcal{A}_\nu \partial_\mu \xi^\nu$$

$$\delta\mathcal{A}_\mu = -\partial_\mu \Lambda$$

in the $c \to \infty$ limit

$$\delta\mathcal{A}_0 = -\partial_k \mathcal{A}_0 \xi^k - \mathcal{A}_k \dot{\xi}^k \qquad \delta\mathcal{A}_i = -\partial_k \mathcal{A}_i \xi^k - \mathcal{A}_k \partial_i \xi^k$$

$$\delta\mathcal{A}_0 = -\dot{\Lambda} \qquad \delta\mathcal{A}_i = -\partial_i \Lambda$$

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We want to transform one gauge theory $\underset{\Lambda,\,\xi^i}{(\mathcal{A}_0, \mathcal{A}_i)}$

to another $\underset{\alpha,\,\xi^i}{(A_0, A_i)}$ by the Seiberg-Witten map

$$\mathcal{A}(A) + \delta_{\xi'}\mathcal{A}(A) = \mathcal{A}(A + \delta_\varepsilon A)$$

the solution is

$$\begin{cases} \mathcal{A}_0 = A_0 - \frac{1}{2} m v_i v^i + \frac{1}{B} m^2 J^n_i v^i d v_n + \dots \\ \mathcal{A}_i = A_i + m v_i - \frac{1}{B} m^2 J^n_i d v_n + \dots \end{cases}$$

Son-Hoyos
ε-expansion

where

$$v^i = -e^{ij}\frac{E_j}{B}, \qquad J^n_m = e_{mk} g^{kn}$$

the inverse map is simply

$$\begin{cases} A_0 = \mathcal{A}_0 + \frac{1}{2} m V_i V^i \\ A_i = \mathcal{A}_i - m V_i \end{cases} \qquad \text{with } V_i = -e_{ij} \frac{\mathcal{E}^j}{B}$$

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with these solutions

$$\Lambda = \alpha$$

Finally,

$$S = \int dt\, d^2x \sqrt{g} \left[\frac{\varepsilon^{\mu\nu\lambda}}{\sqrt{g}} A_\mu \partial_\nu A_\lambda + m \left(B + \frac{1}{2} m \Omega \right) v_i v^i - m^2 e^{ij} v_i d v_j + \dots \right]$$

$\Omega = e^{ij} \partial_i v_j$ (vorticity)

Physical implications for Quantum Hall

- Continuity equation (another motivation for this symmetry)

$$S(A_0, A_i, g_{ij}) \qquad J^\mu = \frac{1}{\sqrt{g}}\frac{\delta S}{\delta A_\mu}$$

$$\partial_t J^0 + \frac{1}{2}\partial_t \ln g\, J^0 + \nabla_i J^i = 0$$

$$m\partial_t J_i + \frac{1}{2} m\,\partial_t \ln g\, J_i + \nabla_j T_i^j = -J^0 E_i + \epsilon_{ij} J^j B$$

$$T^{ij} = \frac{2}{\sqrt{g}}\frac{\delta S}{\delta g_{ij}}$$

if there is no more than one particle (quasiparticle)

$$T^{0i} = m J^i$$

- conductivity tensor $\sigma_{ij}(\omega, q)$

q^2-part at $\omega = 0$

$$\sigma_{ij}^{(2)}(q) \sim \frac{1}{B^2}\left(\eta^H - \kappa_{in} + 2i\frac{m}{B}(\omega\eta^{sh})\Big|_{\omega=0}\right) q^2 \epsilon_{ij} + \ldots$$

η^H - Hall viscosity $\qquad \eta^H = \frac{B}{4\pi}$ (up to coefficient)

Bradlyn - Goldstein - Read (2012)

$$\sigma_{\nu\beta}(q,\omega) = \frac{in\,\delta_{\nu\beta}}{m\omega^+} + \frac{1}{\omega^+}\int_0^\infty dt\, e^{i\omega^+ t}\int d^2x\, e^{-iq\cdot x}\langle [J_\nu(x,t), J_\beta(0,0)]\rangle$$

η^H is due to the Wen-Zee term

$$S = \int dt\, d^2x \; \varepsilon^{\mu\nu\lambda} \omega_\mu \partial_\nu A_\lambda$$

it's not invariant

$$S = \int dt\, d^2x \; \varepsilon^{\mu\nu\lambda} \omega_\mu \partial_\nu A_\lambda + \underbrace{\frac{1}{2} B \Omega}_{\text{the desired correction to the conductivity}} + \dots$$

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the viscosity tensor η_{ijnm}

$$T_{ij} = \eta_{ijnm} \dot{u}_{nm} + \dots$$

$$\eta_{ijnm} = \eta^S_{ijnm} + \eta^A_{ijnm}$$

$$\eta^A_{ijnm} = \frac{1}{2} \eta^H \left(\delta_{jn} \varepsilon_{im} + \delta_{im} \varepsilon_{jn} + \delta_{in} \varepsilon_{jm} + \delta_{jm} \varepsilon_{in} \right)$$

$$\eta^H \sim \frac{1}{4\pi} \Big(B + \underbrace{m\Omega - 2 \frac{m}{B} \varepsilon_{ij} \partial_i B v_j}_{\text{corrections due to non-uniform magnetic field}} \Big)$$

Hořava-Lifshitz gravity with U(1) gauge symmetry

ADM $\quad ds^2 = -N^2 c^2 dt^2 + g_{ij}(dx^i + N^i c\,dt)(dx^j + N^j c\,dt)$

here $\underbrace{\delta t = f(t), \; \delta x^i = \xi^i(t,x)}_{FPD}$

$$K_{ij} = \frac{1}{2} N^{-1}(\dot g_{ij} - \nabla_i N_j - \nabla_j N_i) \qquad K = g^{ij} N_{ij}$$

$$S = \frac{1}{\kappa^2}\int dt\, d^2x \sqrt{g}\, N\left[K_{ij}K^{ij} - \lambda K^2 + \beta(R - 2\Lambda)\right]$$

S - inv. wrt nonrel. diff but not wrt U(1)!

λ, β - couplings

In the case of interest,

$$N = \sqrt{1 - \frac{2A_0}{mc^2} + \frac{A_i A^i}{m^2c^2}} \qquad N_i = \frac{A_i}{mc}$$

$$S = \int dt\, d^2x \sqrt{g}\Big[-\frac{1}{4}\dot g^{ij}\dot g_{ij} - \frac{\lambda}{4}(g^{ij}\dot g_{ij})^2 - \frac{1}{m}(\lambda A^i \partial_t \nabla_i \ln g - \dot g^{ij}\nabla_i A_j)$$
$$+ \frac{1}{m^2}\Big((1-\lambda)(g^{ij}\nabla_i A_j)^2 + \frac{1}{4}F_{ij}F^{ij} - \frac{1}{2}A_iA^i R\Big)\Big]$$

U(1) gauge symmetry if $\lambda = 1$ & term $\frac{R}{m}\left(\frac{1}{2m}A_iA^i - A_0\right)$ is added

Hořava-Melby-Thompson (2010)